GCMP

GREATER CLEVELAND MATHEMATICS PROGRAM

Teacher's Edition
(Printed in 1962)

TEACHER'S GUIDE

2

S R A

Reorder No. 3-644

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GCMP

GREATER CLEVELAND MATHEMATICS PROGRAM

Teacher's Guide for Second Grade

Prepared by the staff of
the Educational Research Council
of Greater Cleveland,
assisted by teachers from
the participating Council Schools.

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S R A

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INTRODUCTION

The skillful teacher will always seek the best way to guide children in their understanding of the logical structure of mathematics. She will continuously search for instructional materials that not only help children to gain insight into the important principles underlying this structure, but also help them to appreciate its simplicity. She has long realized that understanding of basic principles permits greater flexibility of application and ease of retention, and provides a better opportunity for developing powers of reasoning than does memorization of a great number of unrelated facts and procedures. She has accepted as her basic role the responsibility of helping each child learn better, make maximum use of his potentialities, increase his own obligation for learning, develop his sense of inquiry and creativity, and improve his effectiveness in thinking critically.

The teacher of the child in a primary grade is especially sensitive to the fact that this youngster is very aware of the world around him and has an eagerness to know more about it. She makes every effort to create a learning environment that will keep alive this active interest. She designs experiences that encourage the child to explore for himself, for she knows that this young child is always at his best when in the process of discovery. Every effort is made to encourage him to question, to wonder. The teacher realizes, however, that this approach necessitates a great deal of well-planned guidance. Rigid adherence to narrowly-prescribed bounds and learning facts in neat categories may seem efficient, but this very efficiency may well destroy enthusiasm for learning and stifle intellectual curiosity.

As the child makes generalizations about his discoveries, the teacher will want to lead him to test and apply these generalizations in many new situations. She will encourage him to search for new patterns and new ideas and to find relationships among these ideas. The child will learn that arithmetic symbols, like language symbols, can be used as a means of communicating these ideas and relationships.

To provide a well-formulated learning program which will help the child utilize to the fullest his powers of reasoning, the Greater Cleveland Mathematics Program (GCMP) emphasizes the orderly structure of mathematics and the discovery approach. The importance of patterns and relationships is stressed as the child moves from concrete experiences to abstract ideas and as he applies these ideas to new situations. Basic mathematical concepts are developed, then tested and extended again and again. The child learns that not all problems have ready solutions, that problems may have more than one possible solution. He is taught to test alternate approaches. Continuity and creativity are an integral part of the program.

The materials of GCMP are designed to provide maximum flexibility in planning to meet the needs of each child. Challenge as well as the security of understanding are provided in each phase of the program. The student work pages develop the major mathematical concepts. In many instances, a key sentence or two placed at the bottom of selected work pages makes direct reference to basic concepts being developed. The Teacher's Guide offers a rich variety of suggestions for effective teaching of these concepts.

Organization of the Teachers's Guide

Each lesson is organized to develop specific concepts. The teaching goals are clearly stated in terms of the desired learning outcomes for the child. The teacher will find it a distinct advantage to refer to the *Objectives* as she makes her daily lesson plans. She will want to evaluate each child's achievement in terms of these objectives.

Basic mathematical ideas which serve as background information for the teacher are presented in the *Fundamentals* section. Thorough understanding of these concepts should be considered as prerequisite to teaching each lesson. At the end of many of the *Fundamentals* sections, reference is made to those chapters in KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER in which the ideas are presented in greater depth and in relation to other ideas.

It is generally recognized that the greatest understanding comes when the child is ready for a particular concept. Before beginning a new lesson, the teacher should carefully evaluate the child's readiness for the concepts to be developed. The specific knowledge and skills needed for optimal achievement are listed under *Readiness for Understanding*. The teacher must be prepared to stop periodically to review and perhaps to reteach some of the concepts previously presented. She may wish to use this section to help in planning small-group activities and individual work.

Through many carefully-structured activities and guided discussions, the teacher can help the child develop an understanding of the concepts presented in each lesson. In the *Developmental Experiences* section, the teacher will find a variety of suggestions for leading the child from known concepts to the discovery of new ideas. This material has been worked out with great care to insure systematic and correct development of number concepts. One of its major characteristics is the utilization of a "why-and-how" approach. The teacher is encouraged to ask questions that begin with "why" and "how" far more often than those that begin with "what." Needed materials and vocabulary are listed, and specific directions are given for the use of the student work pages.

Experiences designed to strengthen and extend the child's understanding of the concepts developed in the lessons are presented in *Supplemental Experiences.* No attempt has been made to label a particular activity "enrichment" or "remedial," for it has been found that what serves as review for one child may very well bring about initial understanding to another. The teacher must take the responsibility of selecting and adapting particular activities to meet the needs of her group.

The Second-Grade Program

A good elementary arithmetic program makes provision for the fact that a multitude of differences exist in pupil aptitude and readiness for learning. It must provide for these differences through carefully-designed instructional materials. At second-grade level, as at all grade levels, GCMP has been deliberately structured to meet the needs of children with different degrees of ability and backgrounds of understanding. (1) It provides for those children who have not had the program at first-grade level. (2) It provides for those who have had first-grade GCMP, but whose potential has limited their rate of progress. (3) It provides for those whose superior ability has allowed them to achieve at maximum level with GCMP materials. (4) It provides for those children who have progressed in an average way using the GCMP.

Work pages S-1 through S-30 present a complete development of addition and subtraction combinations related to sums through ten. The structural properties of commutativity and associativity are carefully developed. The inverse relation between the operations of addition and subtraction is treated extensively. The concept of different numerals for a number is thoroughly taught. Ability to determine the correct operational sign needed to make a true mathematical statement is tested.

For those children who have not had GCMP in first grade, developmental and supplemental experiences provided in relation to work with addition and subtraction combinations through ten should adequately meet their needs. For those children of average and limited abilities, who have satisfactorily achieved in relation to their potential when undertaking GCMP work in the area of the easy combinations in first grade, work pages S-1 through S-30 will offer an opportunity to move easily into the second-grade program and to build up a feeling of confidence during the first weeks of work. Children who advanced rapidly in the first-grade GCMP presentation of the easy addition and subtraction combinations may need only a brief review of the fundamental ideas and basic skills developed in the first thirty work pages. Work pages S-3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 21, 22, 25, 26, 27, 28, 29, and 30 should orient them quickly to the work at second-grade level.

Work pages S-38 through S-60 are designed to develop the addition and subtraction combinations related to sums of eleven through nineteen. Problems such as $12 + 3 = 15$ are presented, and the use of the associative property in justifying such additions is explored in detail. Continued application of the inverse concept is made. Great emphasis is given to the understanding of place value and the concept of a different numeral for a number. Expanded notation, as well as vertical form, is utilized. Numbers are compared and the relation between them analyzed. Use of parentheses to clarify ideas in equations is developed.

All children new to GCMP should thoroughly understand these concepts before any attempt is made to advance to work with the harder combinations—two single-digit addends whose sums range from eleven through eighteen. The developmental and supplemental experiences structured to insure maximum coverage of these essential ideas should be thoughtfully utilized. For average and limited achievers, these work pages, S-38 through S-60, will once more allow them to move forward with a feeling of security in their work at second-grade level. Children who have achieved at a superior level in first grade may review the concepts by using selected work pages: S-43, 44, 45, 46, 47, 48, 53, 54, 57, 58, 59, and 60.

The importance of the concept of place value must not be minimized. All children should have a clear understanding of the place-value meaning of numbers zero through ninety-nine by the time they reach second grade. Work pages S-69 through S-74 provide a review of fundamental ideas related to this significant phase of the set of whole numbers.

Two blocks of work pages related to the concept of liquid measurement and the concept of fractional numbers are interspersed in the development of the basic mathematical concepts related to addition and subtraction combinations through nineteen. All children will benefit from a complete coverage of work pages S-31 through S-36 and S-61 through S-68 as review work proceeds.

The core of the second-grade program begins with work page S-75. It is here that work with harder combinations begins and out of this concept and the concept of place value grows the presentation of carrying and borrowing—two of the major ideas to be presented in second-grade GCMP. Although some children may have been exposed to harder combinations in first grade, their understanding of these number facts cannot be assumed. Each child should have the opportunity to receive a complete and thorough background in this area of work with numbers before advancing to the next level in the learning sequence.

The development of multiplication and division combinations related to products through twenty-five constitutes the final step in the development of work with whole numbers in the second grade. If not all children are able to undertake with understanding this phase of the work, the teacher may be assured that a complete presentation of these essential concepts will be offered in grade three.

The growth of mathematical concepts is an important part of the total growth of the second-grade child. The teacher will find that much of the understanding, skills, and enthusiasm developed in the GCMP will serve the child well in other areas of the curriculum. As the search for new ideas and relationships becomes part of the child's thinking pattern, learning will indeed become an exciting adventure. The child's sense of wonder will have been sharpened, he will have had the opportunity to develop a sense of confidence which comes from knowing why ideas work.

Addition

SUMS

ZERO THROUGH NINE

Work Pages S-1 and S-2

Objectives

To review the concept of addition.

To review the concept of placeholder equations.

To review addition combinations whose sums are less than ten.

Fundamentals

The teacher will want to plan carefully to meet the needs and ability levels of all children in her class. She will want to determine the strengths and weaknesses of each child in order to begin the arithmetic program at the level of understanding which will insure maximum progress for the child.

By the time the child is ready for second grade, he has had many experiences with sets, or collections of objects. He has come to think of certain collections as an entity. He uses the terms "set," "collection," and "group" when referring to elements in the physical world which he considers as belonging together or as belonging to a unit.

At the second-grade level, the teacher will want to be sure the child knows that two sets may contain different objects and still have a common property. She will want to be sure he understands the property common to the two sets: that the objects, or elements, in the two sets can be paired, or matched one to one. She will want to be sure he knows that this property common to the two sets is called a number.

It is important for the second-grade child not to confuse the idea of numeral with that of number. He must understand that the numeral is only the name or the symbol of the number that is written to represent a number. He must become accustomed to using the word "numeral" only when making a direct reference to the written symbol or the name. In all other cases he should say "number." The child must understand the concepts of set, number, and numeral before he goes on to operations on numbers.

A quick review of addition and subtraction combinations related to sums of ten or less will suffice for some groups. Others will need a more detailed presentation of the beginning phase of work with these two operations on whole numbers.

In order to have a solid foundation for progress, the teacher will want to reassure herself that the child has a clear understanding of union of sets and addition of numbers. An understanding of the union of sets is the foundation on which the first-grade development of the concept of addition of numbers is based. Only disjoint pairs of sets—sets which have no elements in common—are considered in this development.

The second-grade child should be able to determine the cardinal number of each of two given sets. He must be able to join one of the sets to the other to form a new set and then determine the number of this union of the two sets. He must know that this number is called the *sum* of the numbers of the two given sets.

As at the first-grade level, care should be taken to use precise language so that the child is not confused by the concepts of union of sets and addition of numbers. When dealing with the union of sets, the child should learn to use the term "join" and to reserve "add" and "plus" for work with numbers. He should avoid speaking about "adding sets" and should never use a plus sign between pictures of sets.

The second-grader needs to be familiar with the important concepts of *equation* and *placeholder*. An understanding of placeholders and their role in equations is of fundamental importance in the elementary arithmetic program. An equation containing a placeholder is an open sentence which is neither true nor false. The equation becomes either true or false after the placeholder is replaced with a numeral for a number. When the child encounters an equation containing a placeholder, he must realize that his job is to search for a number which makes the equation a true statement. He must be aware of the fact that finding such a number solves the equation.

The child's satisfactory grasp of all these concepts will determine his readiness for second-grade work. The wise teacher knows that time spent in meaningful review is time well spent.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Sets and One-To-One Correspondence; Numbers and Numerals; Addition of Whole Numbers.

Readiness for Understanding

Ability to join two sets to make a new set.

Ability to add a pair of numbers.

Ability to work with placeholder equations and vertical notation.

Developmental Experiences

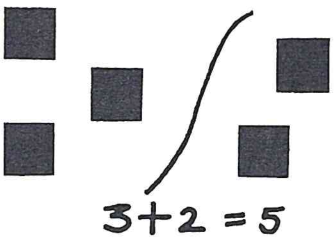
MATERIALS

flannel board
felt cutouts
felt numerals
  0 through 9
felt plus and
  equal signs
felt placeholder
yarn
large piece of
  cardboard
clothespins
Countingman

VOCABULARY

set
join
add
plus
equation
equal sign
equals
sum
addend
plus sign
number
numeral
placeholder
addition
empty set
vertical notation

▶ Place a *set* of three and a set of two felt cutouts in that order on the flannel board. Have a child *join*

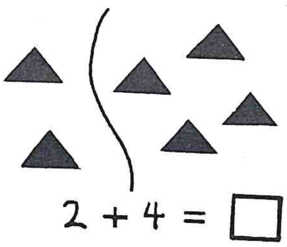


the set of two to the set of three and explain what he is doing. He should say that when he puts the set of two with the set of three he gets a set of five. Help the children visualize the original sets as subsets of the new set. Place a heavy piece of yarn between the two joined sets. Say, "A set of five contains a set of three and a set of two." Ask whether anyone thinks he can tell the *addition* combination the picture suggests. Place the *equation* 3 + 2 = 5 under the joined sets.

Review the concept of *equation*. Have the children recall that this is a special kind of sentence used in arithmetic. Point to each part of the equation and review what each symbol means. Tell the children that the *plus sign* means to *add* the numbers of the sets. Say that the numeral on the left of the *equal sign* represents the same *number* as the *numeral* on the right of the equal sign. Say that the sentence below the sets on the flannel board tells them that 3 + 2 names the same number as 5. Tell them that the sentence is read "Three *plus* two *equals* five."

Remind the children that certain words belong with sets and other words belong with equations. Tell them that "join" belongs with sets, that "add" and "plus" belong with equations. Set the example for correct use of this terminology.

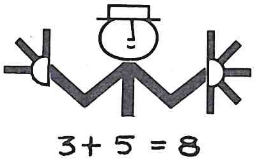
Remove the sets and the equation from the flannel board and begin again. Join a set of four to a set of two. Put a placeholder equation under the sets. Review the concept of *placeholder*. Tell the children that they are to find the missing number in this sentence. Remind them that the placeholder holds a place for the numeral for the *sum* —the number which results when two numbers are added, the number which represents the cardinal number of the set formed when two sets are joined. Remind them that each of the other numbers represented in the sentence is called an *addend*. They are the numbers which are being added. Have a child put the correct numeral in the placeholder, remove the placeholder from the flannel board, and have the completed equation read.



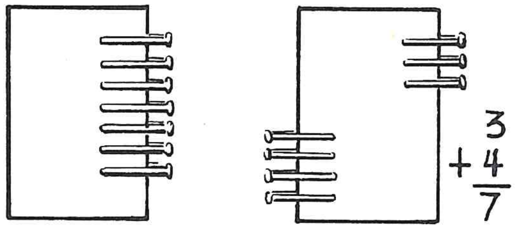
Continue the development of the concepts of union and addition. Use sets to represent addition combinations with sums less than ten. After the manipulation of the objects, have one child describe the activity in terms of the joining of sets. Ask a second child to tell which two sets he sees in the new set. Have a third child place the related addition equation below the joined sets.

▶ Use the Countingman to review addition combinations. Place three fingers on the left hand of the Countingman, and have the children tell how many are in this set. Call on a child to put five more fingers on the Countingman (two more on his left hand and three on his right hand). Ask how many are in this new set. Have a child build on the flannel board the addition equation representing this activity, 3 + 5 = 8.

Continue in this way, developing other addition combinations whose sums are less than ten. Include zero in this development. Begin with a set of six fingers on the Countingman and tell someone to put zero more fingers on the Countingman. Have someone place the related equation, 6 + 0 = 6, on the flannel board. Begin with zero fingers on the Countingman. Join a set of five to the *empty set*. Have the related equation, 0 + 5 = 5, placed on the flannel board. Remind the children that zero is the number of the empty set, the number that tells how many objects are in the set that contains no objects.

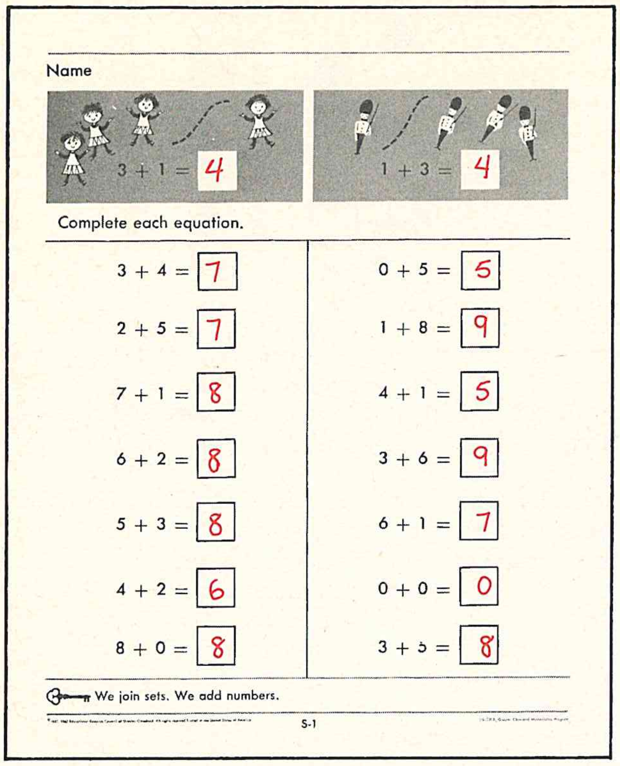


▶ Attach seven clothespins to one side of a strip of heavy cardboard. Ask the children to tell how many there are in the set they see. Ask someone to show a pair of sets he sees in the set of seven by moving one of the sets to the other side of the card. Have the class read from top to bottom and give the related set story. Have the child bring the card to the chalkboard. While he holds the card up to the board, write the *vertical notation* for this addition combination next to the card. Remind the children that this is just another way to write addition combinations. Tell them that since they read the set picture from top to bottom, they can also read this number story from top to bottom: "Three plus four equals seven." Continue in this way, developing the vertical notation for other addition combinations with sums through nine.



▶ Work page S-1 will provide the children with an opportunity to complete placeholder equations related to sums less than ten. Work the first two problems with the children, and then have them complete the page independently.

Work page S-2 provides a review of addition combinations whose sums are not greater than nine. It gives the children an opportunity to work with the vertical form. Work the first problem with the children. Then tell them to write the sums for each addition combination on the remainder of the work page.


Name

3 + 1 = 4

1 + 3 = 4

Complete each equation.

3 + 4 = 7
2 + 5 = 7
7 + 1 = 8
6 + 2 = 8
5 + 3 = 8
4 + 2 = 6
8 + 0 = 8

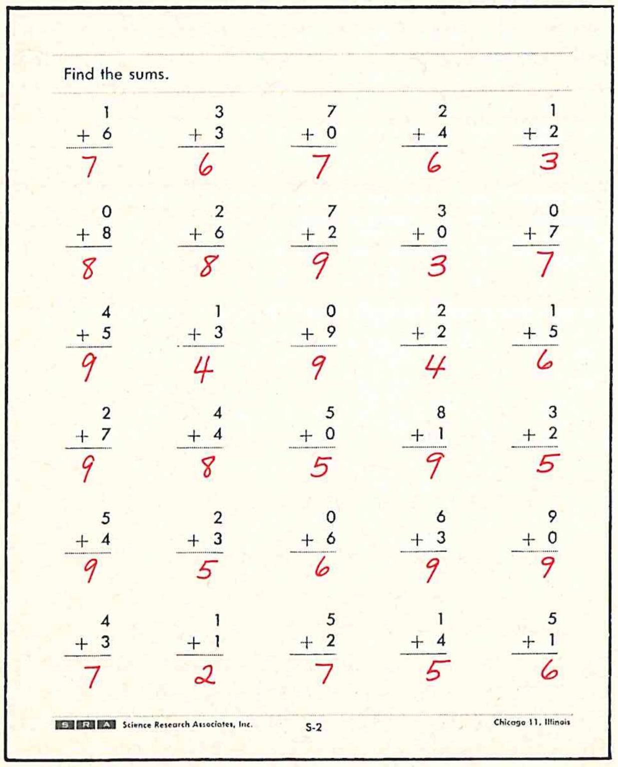
0 + 5 = 5
1 + 8 = 9
4 + 1 = 5
3 + 6 = 9
6 + 1 = 7
0 + 0 = 0
3 + 5 = 8

We join sets. We add numbers.

S-1

Supplemental Experiences

On a bead frame, place sets representing three different addition combinations whose sums are less than ten. Use beads of two different colors, so that the subsets are evident. Have the children study the sets of beads from top to bottom. Then have them use their plastic numerals and plus signs to show the vertical notation for the related number combinations. Have these combinations read aloud. Continue in this way to have the children work with vertical notation as a means of showing addition combinations of less than ten.


Find the sums.

1 + 6 7	3 + 3 6	7 + 0 7	2 + 4 6	1 + 2 3
0 + 8 8	2 + 6 8	7 + 2 9	3 + 0 3	0 + 7 7
4 + 5 9	1 + 3 4	0 + 9 9	2 + 2 4	1 + 5 6
2 + 7 9	4 + 4 8	5 + 0 5	8 + 1 9	3 + 2 5
5 + 4 9	2 + 3 5	0 + 6 6	6 + 3 9	9 + 0 9
4 + 3 7	1 + 1 2	5 + 2 7	1 + 4 5	5 + 1 6

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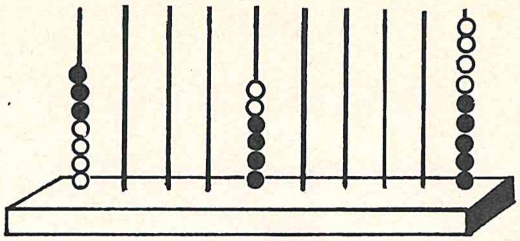
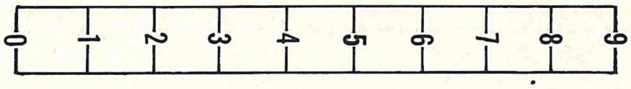


Write on the chalkboard placeholder equations related to addition combinations of nine. Call on a child to come forward and write in one of the placeholders a numeral that will make a true statement. When all of the equations have been completed, ask the children whether they noticed anything about the pairs of numbers. Discuss the pattern of the two addends in the equation, helping the children to see that when the sum remains the same, each time one addend increases by one the other addend decreases by one. Proceed in this way, having the children complete equations and observe addition patterns of other sums less than ten.

0 + 9 = ☐
1 + 8 = ☐
2 + 7 = ☐
3 + 6 = ☐
4 + 5 = ☐
5 + 4 = ☐
6 + 3 = ☐
7 + 2 = ☐
8 + 1 = ☐
9 + 0 = ☐

Mark off nine nine-inch squares on the floor. Label the squares with the numerals 0 through 9, as shown in the illustration. Have the class sit in a circle around the strip of nine blocks. Choose one child to be "it." Ask the rest of the children to close their eyes during the game. Have the child who is "it" start at zero and walk forward, stepping on each of the numbered lines in turn, and tell the rest of the class where he stops. He may say, "I've walked forward five steps. I am standing on 5." Then let him walk forward again and tell the class how many more steps he has moved: for example, "I walked forward two more steps." He should then call on someone to tell where he is now. Have another child first move two steps to 2 and then five more steps. Ask the children to give the addition combinations related to the two experiences: 5 + 2 = 7 and 2 + 5 = 7.



Addition

SUMS LESS THAN TEN

Work Pages S-3 and S-4

Objectives

To review the concept of the missing addend.

To help children learn to think flexibly when working with number concepts.

Fundamentals

The teacher will note a different feature in the approach to combinations on work pages S-3 and S-4. On the preceding work pages, the child was asked to put two sets together—to join these sets. He was asked to find the sum of two numbers—to add these numbers.

Now the child will have to find the missing addend when the sum and one addend are known. For example, he will be asked to decide how many more objects to put with a set of two to form a set of three, and to determine what number must be added to two to equal three. In another type of problem he will have to decide to what set a set of two must be joined to make a set of three—what number plus two equals three.

In these new types of problems, the child will use some of the facts he already knows and re-examine them from different angles. He will explore more than one approach to number combinations less than ten and see new relations between ideas.

Although the chief emphasis is still on addition, the child will get an important secondary benefit from working with problems of this type: he will be exposed indirectly to the concept of subtraction.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Ability to join two sets to form a new set.

Ability to determine the cardinal number of the union of sets.

Ability to add a pair of numbers.

Ability to work with placeholder equations and vertical notation.

Developmental Experiences

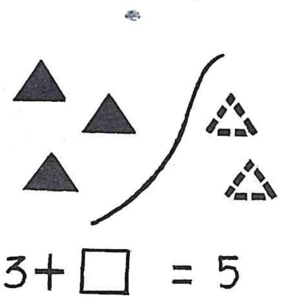
MATERIALS

flannel board
felt counters
plus and equal signs
placeholders
felt numerals 0 through 9
Countingman

VOCABULARY

missing addend
reconstruction

▶ Have a child draw a set of three objects on the chalkboard. Ask another child to tell how many objects must be joined to this set of three to make a set of five. Have the child draw the set of two to the right of the set of three. Write the equation 3 + □ = 5 below the set illustration and read it with the children: "Three plus placeholder equals five." Tell the children that one of the addends is missing. Encourage them to think, when they see an equation with a *missing addend,* "What number must be added to three to give five?" or "Three plus some number equals five. What is that number?"



When the children determine that two is the number they have been looking for, have a child write the numeral 2 in the placeholder. Then erase the placeholder and have a child read the completed equation. Work in this same way with other equations whose sums are less than ten and in which the placeholder appears in the second-addend position.

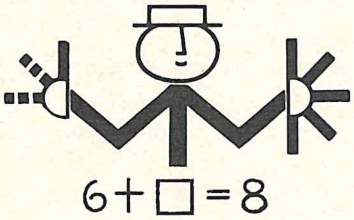
▶ Write the equation □ + 4 = 7 below a set of four on the chalkboard. Help the children read this equation: "Placeholder plus four equals seven." Explain that when they see the placeholder in this position they should think, "What number plus four equals seven?" Have a child draw the needed number of objects to the left of the wavy line, and have another child write the correct numeral in the placeholder. Then erase the placeholder and have the completed equation read aloud. Repeat the activity with other addition combinations whose sums are less than ten. Have the placeholder appear in the first-addend position.

▶ Read the following number sentences and have the children build the related placeholder equations

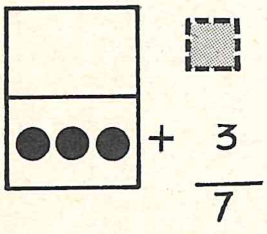
Some number plus two equals five.
(□ + 2 = 5)
Four plus some number equals eight.
(4 + □ = 8)
Two plus some number equals six.
(2 + □ = 6)
Some number plus one equals nine.
(□ + 1 = 9)
Three plus some number equals seven.
(3 + □ = 7)
Some number plus one equals four.
(□ + 1 = 4)

on the flannel board. Tell the children that in each equation one of the addends is missing. Have different children come to the flannel board and replace each placeholder with the correct missing addend. Have the completed equations read.

▶ Place six fingers on the Countingman. Put the equation 6 + □ = 8 on the chalkboard. Have a child explain what the equation requires: the number which must be added to six to give eight. Have a child place enough more fingers on the Countingman to make eight, and have the placeholder equation completed. Continue the activity, using other placeholder equations of this type with sums less than ten. Include zero in the activity.



▶ Draw on the chalkboard a partially completed domino picture in vertical position. Next to it, write the related *reconstruction* problem as illustrated below. Have a child read the number combination: "Some number plus three equals seven." Have another child draw the required number of dots on the domino. Ask a third child to write the numeral for the missing addend in the shaded box and read the number combination. Use other combinations with sums less than ten.



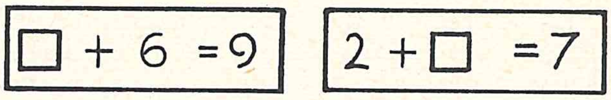
Vary the activity by placing the dots in the top section of the domino picture, and write the vertical-notation form for the number combination next to the picture. Have a child read the addition combination: "Six plus some number equals nine." Have another child draw the required number of dots on the domino. Ask a third child to fill in the numeral for the missing addend.

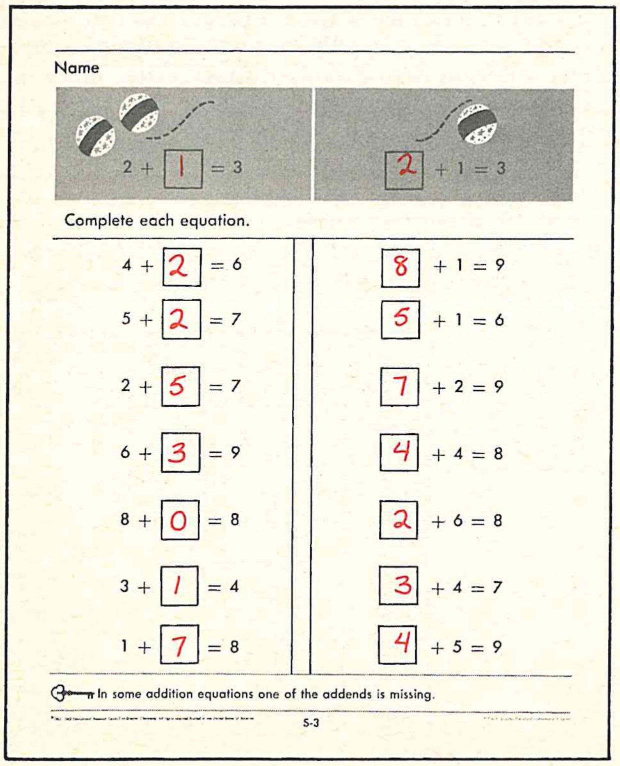
▶ Use work page S-3 to give children additional practice in finding missing addends. Work the first two problems with the children and then have them work the rest of the problems on the page independently. Tell them to complete each equation by writing the numeral for the missing addend in the placeholder.

Work page S-4 gives the children an opportunity to find the missing addend in a vertical notation. After working the first problem with the children, tell them to see how quickly they can find the correct numeral for the missing addends on the remainder of the page.

Supplemental Experiences

▶ Prepare missing-addend equation cards for addition combinations with sums less than ten. Use the cards to build up quick recognition of numbers needed to complete equations by having a child read the completed equation as each card is exposed to view.




Name

2 + $\boxed{1}$ = 3

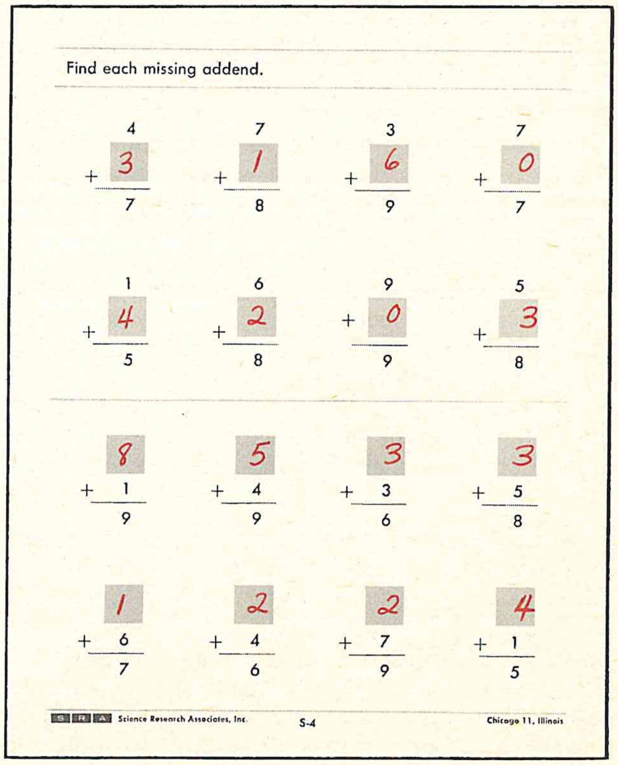
$\boxed{2}$ + 1 = 3

Complete each equation.

4 + $\boxed{2}$ = 6	$\boxed{8}$ + 1 = 9
5 + $\boxed{2}$ = 7	$\boxed{5}$ + 1 = 6
2 + $\boxed{5}$ = 7	$\boxed{7}$ + 2 = 9
6 + $\boxed{3}$ = 9	$\boxed{4}$ + 4 = 8
8 + $\boxed{0}$ = 8	$\boxed{2}$ + 6 = 8
3 + $\boxed{1}$ = 4	$\boxed{3}$ + 4 = 7
1 + $\boxed{7}$ = 8	$\boxed{4}$ + 5 = 9

In some addition equations one of the addends is missing.

S-3


Find each missing addend.

$$\begin{array}{r}4\\+\boxed{3}\\\hline 7\end{array}\quad\begin{array}{r}7\\+\boxed{1}\\\hline 8\end{array}\quad\begin{array}{r}3\\+\boxed{6}\\\hline 9\end{array}\quad\begin{array}{r}7\\+\boxed{0}\\\hline 7\end{array}$$

$$\begin{array}{r}1\\+\boxed{4}\\\hline 5\end{array}\quad\begin{array}{r}6\\+\boxed{2}\\\hline 8\end{array}\quad\begin{array}{r}9\\+\boxed{0}\\\hline 9\end{array}\quad\begin{array}{r}5\\+\boxed{3}\\\hline 8\end{array}$$

$$\begin{array}{r}\boxed{8}\\+1\\\hline 9\end{array}\quad\begin{array}{r}\boxed{5}\\+4\\\hline 9\end{array}\quad\begin{array}{r}\boxed{3}\\+3\\\hline 6\end{array}\quad\begin{array}{r}\boxed{3}\\+5\\\hline 8\end{array}$$

$$\begin{array}{r}\boxed{1}\\+6\\\hline 7\end{array}\quad\begin{array}{r}\boxed{2}\\+4\\\hline 6\end{array}\quad\begin{array}{r}\boxed{2}\\+7\\\hline 9\end{array}\quad\begin{array}{r}\boxed{4}\\+1\\\hline 5\end{array}$$

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Addition

SUMS LESS THAN TEN

Work Pages S-5 and S-6

Objectives

To help children record known addition combinations in the addition table.

To re-emphasize the concept of equality in addition equations.

Fundamentals

The most important use of the addition table is for the tabulation of the addition combinations which the child has learned. After these combinations have been recorded, some interesting patterns can be observed.

The table can be used as an occasional reference for looking up forgotten facts, but it should not be allowed to become a crutch on which the child leans every time he does not remember a fact. The teacher should encourage the child to work out the sums by manipulating objects on his desk or by using other helpful devices.

The teacher will note that the heading of the addition table includes the numeral 0. The inclusion of 0 emphasizes two facts: (1) zero is one of the whole numbers; (2) zero is the identity element of addition—in other words, any number plus zero is that same number, and zero plus any number is that same number. The inclusion of 0 simply causes the headings to be repeated in the first row and column in the body of the table.

The teacher will want to review the important concept, stressed throughout the program, that many different numerals can represent the same number. In the first grade the child is encouraged to think "nine" as readily when he sees the combinations $6 + 3$, $7 + 2$, or $0 + 9$, as when he sees the numeral 9. The teacher will want to encourage the second-grade child to maintain this flexibility in his thinking. It will prove especially helpful when the child works with sums greater than ten. When he solves a problem such as $7 + 9 = \square$, he will be shown that by thinking of the number nine as three plus six, he can rewrite the problem as $7 + (3 + 6) = \square$, then as $(7 + 3) + 6 = \square$, and finally as $10 + 6 = \square$. The final equation, $10 + 6 = \square$, is easier to solve than the original equation, $7 + 9 = \square$.

At this point the teacher must begin to discourage the habit of thinking of the equation as a stereotyped form, with the operation to be performed recorded on the left of the equality sign and the result of the operation recorded on the right side. In an equation such as $2 + 5 = 7$ the mathematical sentence is stating that $2 + 5$ represents exactly the same number as 7, or that $2 + 5$ and 7 are different numerals for the number seven. Since the numeral on the right of the equality sign represents the same number as the numeral on the left, these numerals can be interchanged without changing the meaning of the sentence and without violating the meaning of equality. The teacher will want to remind the child that he can replace the numeral on the right with the numeral on the left and state the mathematical sentence as $7 = 2 + 5$.

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers.

Readiness for Understanding

An understanding of the operation of addition.

An understanding of the meaning of the word "equals."

Ability to work with addition combinations whose sums are less than ten.

Developmental Experiences

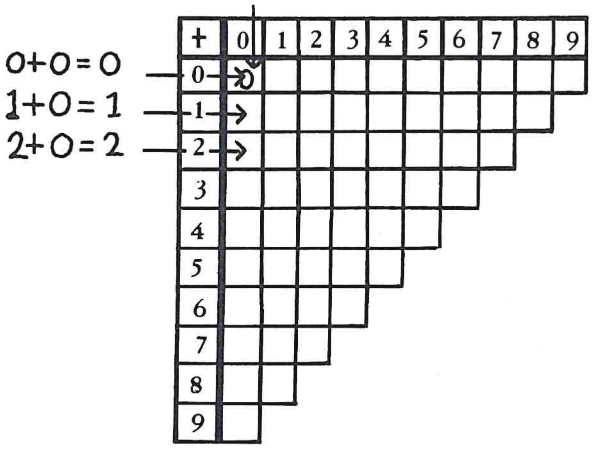
MATERIALS

chart paper
easel
black crayon
tagboard cards
2 pointers
felt numerals 0 through 9
felt symbols for operations and relations
felt cutout counters
yarn

VOCABULARY

addition table
row
column
order
commute

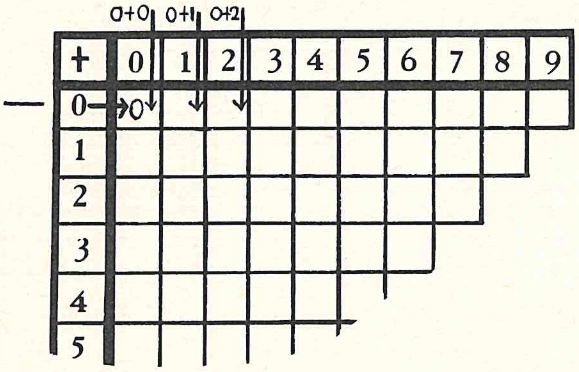
▶ Construct a large *addition table* on a piece of chart paper and mount it on an easel. Tell the class that they are going to help you write down the addition combinations for sums through nine. Point to the box with the plus sign in it and review the meaning of the symbol. Point to the numerals across the top of the chart, and say that this is the first *row* of the table. Now go back to the box with the plus sign and point to the numerals which run down the left side of the chart. Tell the children that this is the first *column* of the table.



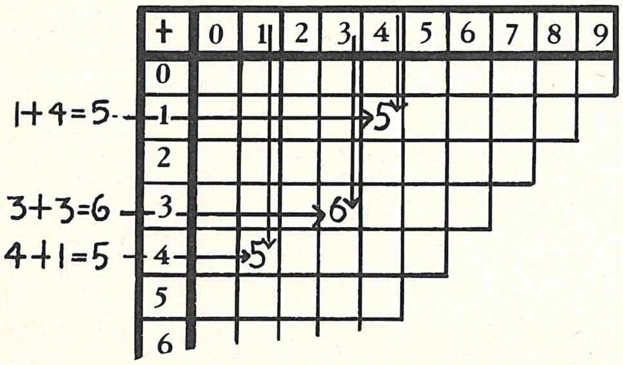
Tell the class that they are going to record sums in the boxes of the table. Have someone tell what the sum of zero and zero is. Place a pointer on the 0 in the first column and another pointer on the 0 in the first row. Move the two pointers together until they meet in the first empty box. Tell the children that this is where they are to record the sum of zero and zero. Call on a child to write the numeral 0 in this box with a black crayon.

Have the sums from one plus zero through nine plus zero recorded in the table in this way. Next have the sums from zero plus one through eight plus one placed in the table. Continue down the columns of the table until the children have filled in all the sums.

▶ Fill in a second table by rows instead of columns. Begin again with zero plus zero. Have children write the sums of zero plus one through zero plus nine. Move on to one plus zero through one plus eight. Have the rest of the sums placed in the table a row at a time.



▶ Fill in a third table by having children choose any number combination with sums zero through nine, find the box for the sum, and write the correct numeral in this space. One child may record the sum of four and one; call on a second child to record the sum represented by the commuted form of 4 + 1. Remind the children that they learned that the *order* of the addends can be changed, or commuted, without changing the sum.



A third child may wish to record three plus three. Ask a child to give the commuted form for this combination. Help the children see that for pairs of like numbers, the commuted forms are the same. Lead them to note the position in the table of the sums of these pairs.

If some child comments on the fact that the first row and the first column are the same as the headings, help him see that this is because zero is one of the addends in each case. Remind him that any number plus zero is that number, and that zero plus any number is that number.

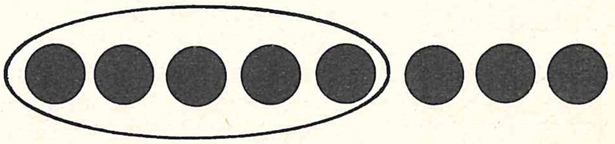
The addition patterns made evident by filling in the first two tables (in which one addend is held constant and the other addend increases by one) are missing from this third table. However, children should often be given the freedom to choose and try out their own ideas.

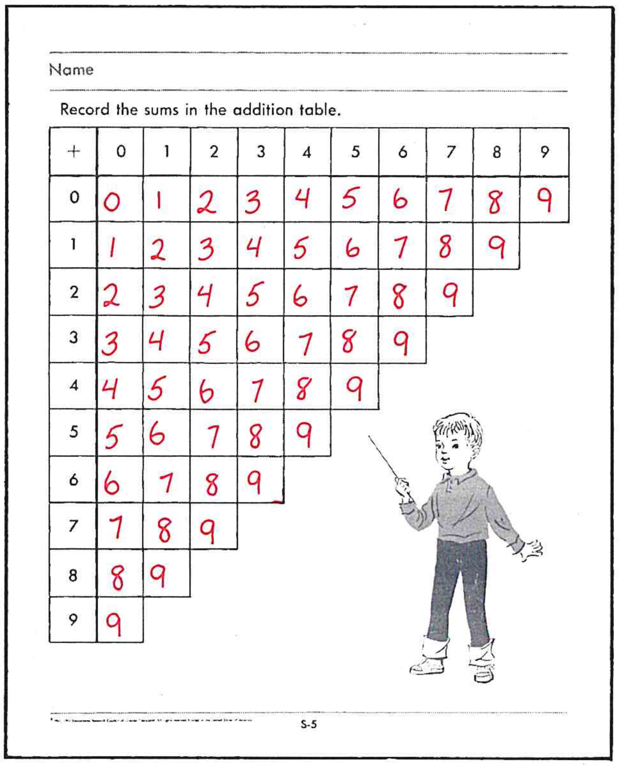
▶ Have two children choose number combinations and find the sums in the table. Tell them to place one pointer on the first addend (in the first column) and another pointer on the second addend (in the top row). Then have them move the pointers, one down and one across, until they meet at the box that contains the sum. Direct them to read the combinations and verify their accuracy.

▶ Distribute cards on which several different numerals for the numbers six through nine are written. Place the felt numeral 8 on the flannel board. Tell the children that there are many ways to think about the number eight. Have children holding cards that illustrate different ways of representing eight come forward in turn and place their cards on the flannel board. One child may have the card 4 + 4. Place the equal sign between the felt numeral 8 and the card 4 + 4 to show the equation 8 = 4 + 4. Say, "Eight names the same number as four plus four; eight is another name for four plus four; eight equals four plus four."

Explain that it is not always necessary to think of the addition of the two numbers first and then the sum, as we usually do when we see an equation such as 4 + 4 = 8. Place this equation below 8 = 4 + 4 on the flannel board. Tell the children that sometimes we may wish to think of the sum first and then ask ourselves how many different ways we can think of that number. Follow the same procedure with combinations of six, seven, eight, and nine.

▶ Place eight felt objects in a row on the flannel board. Call on a child to show how he thinks of eight. Have him place a piece of yarn around one of the subsets in the set of eight and then tell the class how he thinks about the sets. He may say, "I think of the set of eight as a set of five and a set of three." Have him place the related addition equation under the set picture and read it to the class. Continue in this manner to have children describe numbers in many different ways.





Name

Record the sums in the addition table.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	
2	2	3	4	5	6	7	8	9		
3	3	4	5	6	7	8	9			
4	4	5	6	7	8	9				
5	5	6	7	8	9					
6	6	7	8	9						
7	7	8	9							
8	8	9								
9	9									

S-5

▶ Use work page S-5 to review addition combinations with sums not greater than nine. Place a duplicate table on the chalkboard, and work with the children as they fill in the addition table. Point to each addend as it is given, first the one in the column on the left and then the one in the top row. Point to the box which is to hold the answer. Have the children fill in this box on their own paper with the sum of the first two specified addends. Continue to guide the children as they work down the columns or across the rows.

Use work page S-6 to emphasize the meaning of the word "equals." Tell the children that they must think of different numerals for the number represented. Tell them that they must complete each equation and that the plus signs are there to guide them. Work the first problem with the children, and then have them complete on their own the other equations related to six. Follow the same procedure with equations related to seven, eight, and nine.

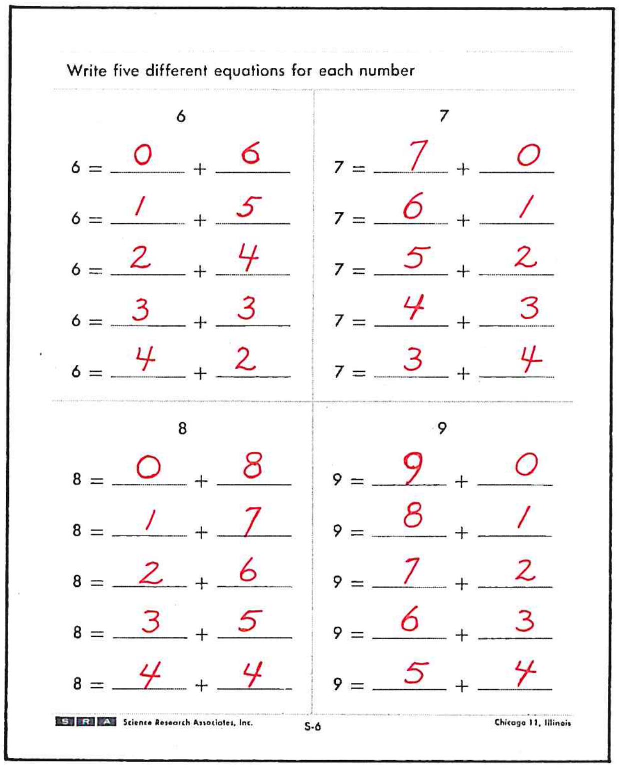
Supplemental Experiences

+	3
1	4
2	5
3	6
4	7
5	8
6	9

▶ Have the children construct miniature addition tables to show the "one-more" concept, the identity element for addition, or other patterns. Review with the children the fact that each number in the natural order of whole numbers is one more than the preceding number. Remind them that zero is the identity element for addition. Zero plus any number is that same number; any number plus zero is that same number. In addition, the sum of any pair of numbers (excluding zero) results in a number different from either addend. When zero is one of the pair of addends, the sum is the same as the other addend.

▶ Help the children construct an addition combination chart to summarize different numerals for the numbers one through nine. Have the children observe the patterns in the table as it is being constructed. Encourage them to anticipate what number combination will be next in each column. Have them tell what they observe about each succeeding addend and each neighboring pair of addends in the column.

1	2	3	4	5	6	7	8	9
1 + 0	2 + 0	3 + 0	4 + 0	5 + 0	6 + 0	7 + 0	8 + 0	9 + 0
0 + 1	1 + 1	2 + 1	3 + 1	4 + 1	5 + 1	6 + 1	7 + 1	8 + 1
	0 + 2	1 + 2	2 + 2	3 + 2	4 + 2	5 + 2	6 + 2	7 + 2
		0 + 3	1 + 3	2 + 3	3 + 3	4 + 3	5 + 3	6 + 3
			0 + 4	1 + 4	2 + 4	3 + 4	4 + 4	5 + 4
				0 + 5	1 + 5	2 + 5	3 + 5	4 + 5
					0 + 6	1 + 6	2 + 6	3 + 6
						0 + 7	1 + 7	2 + 7
							0 + 8	1 + 8
								0 + 9



Write five different equations for each number

6

6 = 0 + 6
6 = 1 + 5
6 = 2 + 4
6 = 3 + 3
6 = 4 + 2

7

7 = 7 + 0
7 = 6 + 1
7 = 5 + 2
7 = 4 + 3
7 = 3 + 4

8

8 = 0 + 8
8 = 1 + 7
8 = 2 + 6
8 = 3 + 5
8 = 4 + 4

9

9 = 9 + 0
9 = 8 + 1
9 = 7 + 2
9 = 6 + 3
9 = 5 + 4

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★ More than one correct answer is possible for each problem

Subtraction

COMBINATIONS LESS THAN TEN

Work Pages S-7 and S-8

Objectives

To review the concept of subtraction.

To help the children understand the relation between subtraction and addition.

To help the children learn to write the equation which relates to the set representation of subtraction.

Fundamentals

The child has reviewed the idea of relating addition to the union of sets. He will now review the idea of relating subtraction to the separation of sets. The teacher will want to start with a set and illustrate the joining of another set to it. Using this union of sets, she will then want to illustrate the separation or removal of a subset from the given set and associate a subtraction equation with this idea. From the beginning, the child should be encouraged to relate subtraction to addition. He should learn to think of subtracting a number as the inverse of adding the number, and to understand that what addition "does," subtraction "undoes."

The teacher will want to remind the child to use the words "take away" or "remove" when referring to set separation, and "minus" or "subtract" when discussing the subtraction of numbers. An expression such as 5 − 3 should be read "five minus three" and not "five take away three." The idea of taking away should be reserved for the physical operation of removing objects from a set.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concept of addition.

Ability to read, write, and solve addition equations.

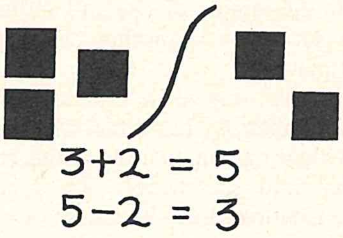
Developmental Experiences

MATERIALS
- flannel board
- felt cutouts
- plus signs
- minus signs
- equal signs
- felt placeholders
- felt numerals 0 through 9
- yarn
- Countingman
- plastic numerals and other symbols

VOCABULARY
- take away
- remove
- minus
- subtract
- subtraction
- difference

▶ In this exercise the child will move from the known concept of addition into the related concept of subtraction. Put three felt cutouts on the flannel board. Have a child join a set of two cutouts to the set of three on the board. Have another child put the addition equation for this activity, 3 + 2 = 5, below the joined sets. Have someone tell what was done with the sets: "A set of two was joined to a set of three to make a set of five." Have the equation read aloud: "Three plus two equals five."

Call on a child to *remove* two objects from the set. Place the related *subtraction* equation under the addition equation on the flannel board. Point out that this is a different kind of equation. Explain that in arithmetic when we *take away* an object from a set, we begin to think about a new operation: subtraction. Point to the subtraction symbol in this equation. Review the meaning of the *minus* sign. Tell the children that the minus sign indicates that two is *subtracted* from five. Ask them to follow along with their eyes as you read the subtraction equation for them: "Five minus two equals three." Tell them that this is the sentence that belongs with what they saw happen on the flannel board when two objects were removed from a set of five. Say that the numeral 3 in this equation represents the *difference* between five and two.

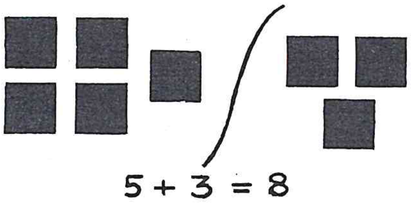


Remove these sets and equations from the flannel board and begin again. Place a set of four objects on the flannel board, and have a child join a set of three to the set of four. Ask another child to put the addition equation for this acivity below the joined sets. Have the joining of the sets described and the equation read. Have another child take three objects from the set. Place the related subtraction placeholder equation below the addition equation. Have a child tell what numeral belongs in the placeholder in order to make the sentence, or equation, relate to the activity of removing three objects from a set of seven. Perhaps some child may remember, or another child may discover, that the number of the set with which the story on the flannel board began is the number of the set with which the story ends. These children may remember, or perhaps discover, that the addition equation began with number four and the subtraction equation ends with the same number. Have the numeral 4 put in the placeholder and have the number sentence read.

Repeat this activity for other equations related to sums less than ten.

▶ Place a set of five objects and a set of three objects on the flannel board to make a set of eight.

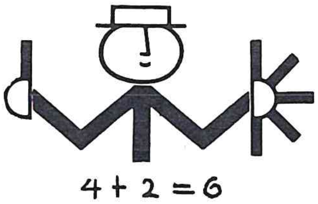
Place a piece of yarn between the set of five and the set of three to help children visualize the two sets that are contained in the set of eight. Have a child place the related addition equation below the joined sets. Have him tell about the joining of the sets: "A set of three was joined to a set of five to make a set of eight." Have the addition equation read.



Ask the children to tell what number of objects must be removed from the set of eight in order to have five, the number of objects with which you started. Have a child come forward and cover up the three objects on the right that must be removed in order to have only five objects left. Guide another child in placing the related subtraction equation below the addition equation. Have the set removal described and the subtraction equation read. Ask someone to tell about both activities with the sets, beginning with the joining of sets and ending with the removal of a subset. Ask another child to read the two related equations.

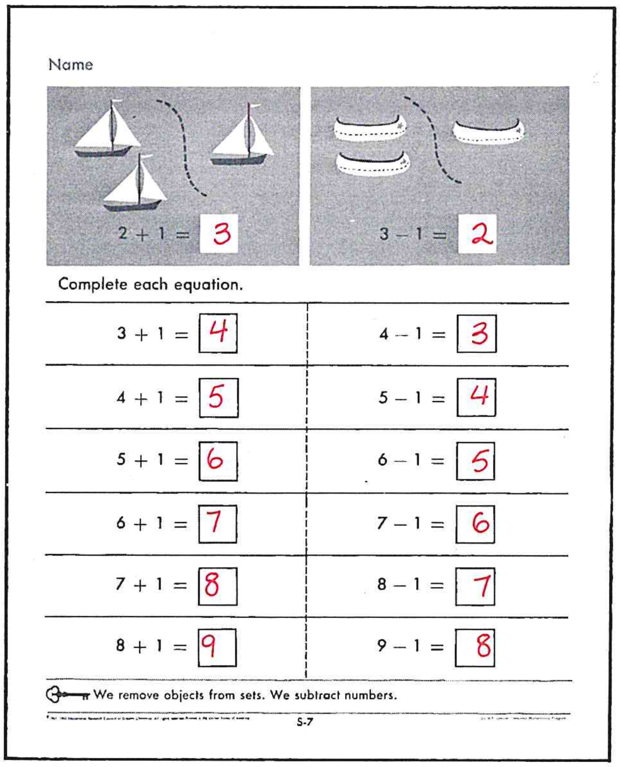
Continue in this way with related addition and subtraction combinations less than ten.

▶ Use the Countingman to review the relation between addition and subtraction. To a set of four fingers on the Countingman, join a set of two fingers. Tell the children to use their plastic numerals and their operation and relation symbols at their desks to build an equation representing the activity with the Countingman. Have the addition combination read.



Remove two fingers from the Countingman. Have children build, directly under the addition equation, an equation representing this activity, 6 − 2 = 4. Have the subtraction combination read. To emphasize the inverse relation between addition and subtraction, ask a child to read both equations.

Continue to join and take away sets of fingers on the Countingman to illustrate other addition and subtraction combinations less than ten. Include zero in the activity. Have children build the related equations each time.



Name

2 + 1 = 3

3 − 1 = 2

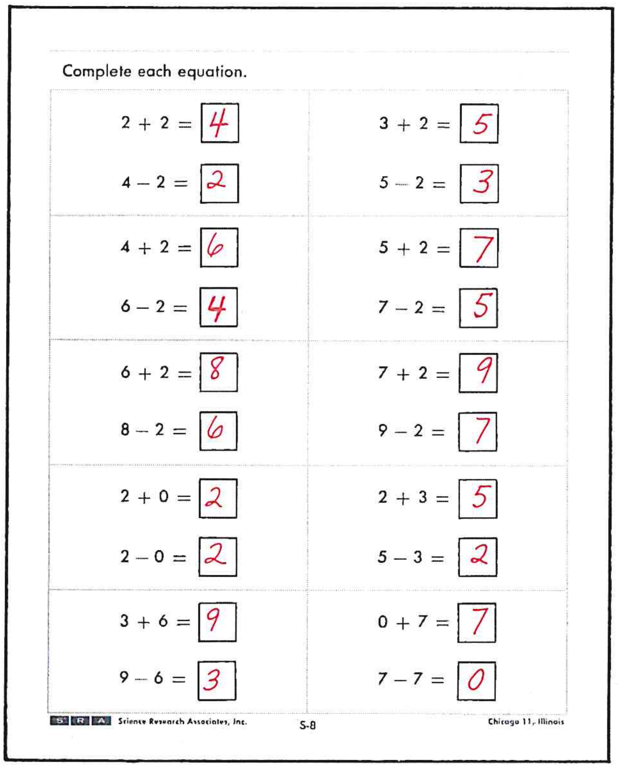
Complete each equation.

3 + 1 = 4	4 − 1 = 3
4 + 1 = 5	5 − 1 = 4
5 + 1 = 6	6 − 1 = 5
6 + 1 = 7	7 − 1 = 6
7 + 1 = 8	8 − 1 = 7
8 + 1 = 9	9 − 1 = 8

We remove objects from sets. We subtract numbers.

S-7

▶ Use work pages S-7 and S-8 to re-emphasize that what addition "does," subtraction "undoes." Work the first two problems on work page S-7 with the children. Then have them work independently to complete each placeholder equation on S-7 and S-8.



Complete each equation.

2 + 2 = 4 4 − 2 = 2	3 + 2 = 5 5 − 2 = 3
4 + 2 = 6 6 − 2 = 4	5 + 2 = 7 7 − 2 = 5
6 + 2 = 8 8 − 2 = 6	7 + 2 = 9 9 − 2 = 7
2 + 0 = 2 2 − 0 = 2	2 + 3 = 5 5 − 3 = 2
3 + 6 = 9 9 − 6 = 3	0 + 7 = 7 7 − 7 = 0

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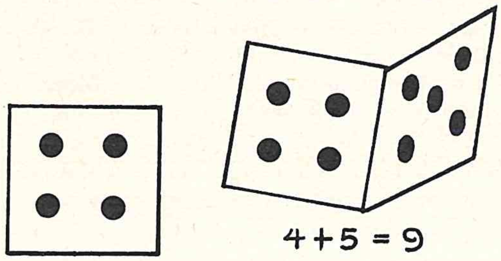
9	8	7	6	5	4	3	2	1
9 — 0	9 — 1	9 — 2	9 — 3	9 — 4	9 — 5	9 — 6	9 — 7	9 — 8
	8 — 0	8 — 1	8 — 2	8 — 3	8 — 4	8 — 5	8 — 6	8 — 7
		7 — 0	7 — 1	7 — 2	7 — 3	7 — 4	7 — 5	7 — 6
			6 — 0	6 — 1	6 — 2	6 — 3	6 — 4	6 — 5
				5 — 0	5 — 1	5 — 2	5 — 3	5 — 4
					4 — 0	4 — 1	4 — 2	4 — 3
						3 — 0	3 — 1	3 — 2
							2 — 0	2 — 1
								1 — 0

Supplemental Experiences

▶ Construct a table to show subtraction combinations which result in the numbers one through nine.

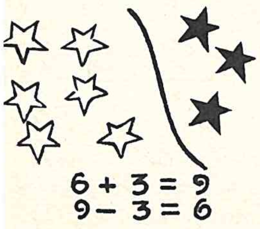
▶ Draw four dots on one side of a domino card and five dots on the other side. Fold the card in half so that only four of the dots are in view.

Show to the class the side of the card with four dots. Have someone tell how many dots he sees. Unfold the card so that the children can see the five dots joined to the four dots. Have the card read from left to right and the joining of the sets described. Ask someone to give the addition equation corresponding to this activity.



Now tell the children to watch as you fold the side of the card with five dots back out of sight. Ask how many dots they saw when both sides of the card were in view. Ask how many dots were taken away and how many dots remain. Have the action described in terms of sets and have a child give the corresponding subtraction equation. Continue in this way with other combinations less than ten.

▶ Play the game "Stars at Night." Place six white felt cutout stars on the flannel board. Have the children tell how many stars they see "in the sky." Tell them to close their eyes. Put three red stars on the board with the six white stars. Have the children open their eyes and tell how many more stars are now "in the sky." Direct one child to tell about the joined sets: "A set of three was joined to a set of six to make a set of nine." Someone else should place the related addition equation on the flannel board.



Say that a cloud came by and covered some of the stars. Place your hand over the three red stars. Ask a child to tell about the removal of objects in terms of sets: "From a set of nine a set of three was taken away; a set of six remains." Have someone place the related subtraction equation under the addition equation on the flannel board.

Continue in this way to develop other subtraction combinations related to sums less than ten.

Subtraction

COMBINATIONS

LESS THAN TEN

Work Pages S-9 and S-10

Objectives

To extend the concept of subtraction.

To strengthen the children's ability to write subtraction equations.

To emphasize the relation between addition and subtraction.

To provide practice in use of vertical notation.

Fundamentals

In his early work with sets at the kindergarten and first-grade levels, the child learns that one way to describe a set is to identify its elements. He also learns that it is possible to single out certain elements of the set. These sets within sets are called subsets of the given set. A subset that is different from the set itself and from the empty set is called a proper subset.

The child has been learning to think of subsets as parts of a given set.

The subset idea will now be used to strengthen the understanding of the relation between addition and subtraction. The teacher should help the child understand that when two sets are joined, they still exist as subsets of the union. She should also help him understand that when a set is separated into two disjoint sets, these sets must have been thought of first as subsets of the given set.

Two interesting properties of zero will be explored. On preceding work pages the child worked with zero in addition equations. Now the teacher will want to review what happens when zero is subtracted from a given number; she will also want to encourage the child to think about the solution to problems such as 5 − 5 = ☐.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concept of subtraction.

An awareness of the inverse relation between addition and subtraction.

Ability to read, write, and solve subtraction placeholder equations.

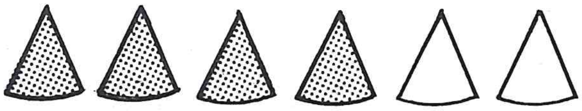
Ability to work with vertical notation.

Developmental Experiences

MATERIALS

- bead frame
- heavy wire
- wooden beads
- individual sets of plastic numerals
- symbols for operations and relations
- cardboard cones

▶ Place six cardboard cones on a table in front of the class. Have the children tell how many they see. Tell the class to close their eyes, and then remove two of the cones. Ask the children to open their eyes and tell if there are more cones than before or fewer. After the children say that there are fewer cones, ask them to tell how many fewer. Have children use their plastic numerals and symbols to build an equation related to the activity that must have taken place when their eyes were closed (6 − 2 = 4).



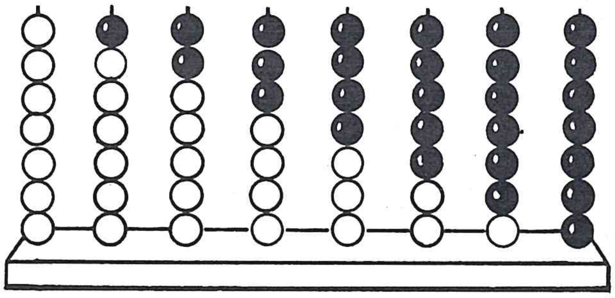
Tell the children to look again at the number of cones on the table. Direct them to close their eyes once more. Put the two cones back on the table. After the children open their eyes, ask them to tell whether there are more cones than before or fewer. Ask them to tell how many more there are. Have the children build an equation related to the activity that must have taken place when their eyes were closed (4 + 2 = 6).

Have both equations read aloud. Continue in this way to develop other subtraction combinations with sums less than ten.

▶ Place five beads on a wire. Ask a child to place as many more beads on the wire as are needed to make a set of five. Have children use their plastic numerals and symbols to build an equation that corresponds to the set-joining activity (5 + 0 = 5). Have the equation read aloud. Once more hold up the wire with the five beads. Ask a child to remove beads from the wire so that five beads remain. Have the children build, under the addition equation, an equation that corresponds to the set-removal activity (5 − 0 = 5). Have this equation read aloud.

Continue in this way to have children join the empty set with, and remove the empty set from, sets of zero through nine. Have them build the related equation for each activity. Lead the children to conclude that the sum of any number and zero is that number, and that any number minus zero is that number.

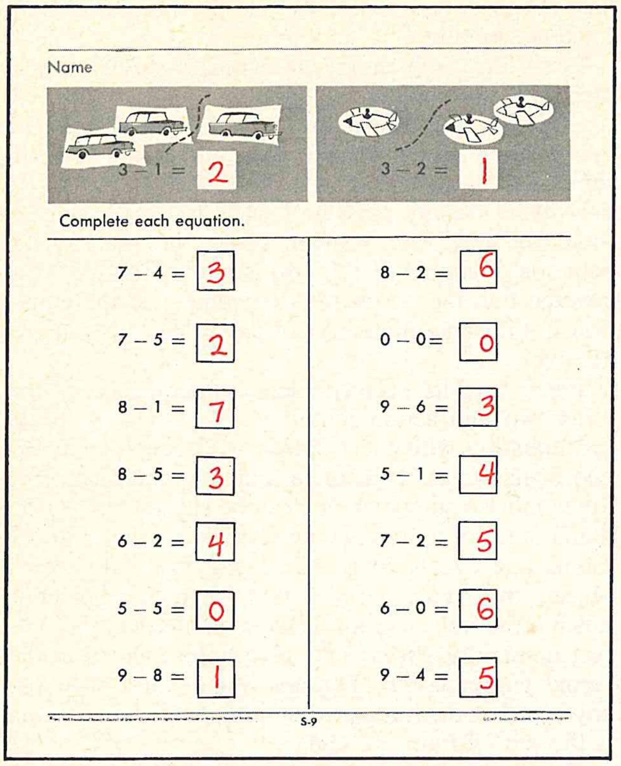
Now place four beads on the wire. Ask, "How many beads must be removed so that no beads remain on the wire?" Write the related equation (4 − 4 = 0) on the chalkboard. Have the children build this equation on their desks with their plastic numerals. Continue in this way to develop zero minus zero through nine minus nine. Have children conclude that any number subtracted from that same number gives the result of zero.



▶ Place sets representing combinations of seven on the bead frame. Use beads of two different colors so that the subsets are evident. Write subtraction combinations of seven, in vertical form, on the chalkboard.

$$\begin{array}{cccccccc} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\ \underline{-0} & \underline{-1} & \underline{-2} & \underline{-3} & \underline{-4} & \underline{-5} & \underline{-6} & \underline{-7} \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{array}$$

Point to one of the problems and ask a child to select the set on the bead frame which will help him solve this problem. Lead him to remove the necessary



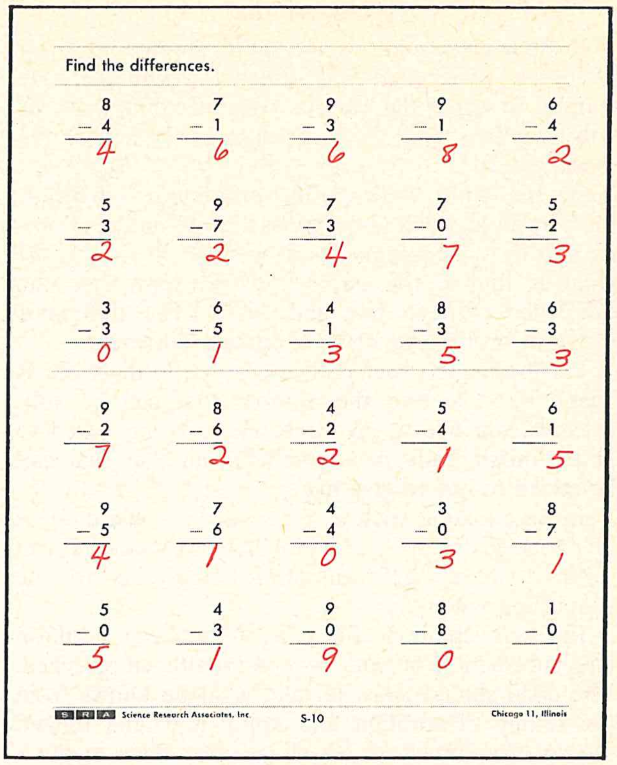
Name

$3 - 1 = \boxed{2}$
$3 - 2 = \boxed{1}$

Complete each equation.

$7 - 4 = \boxed{3}$	$8 - 2 = \boxed{6}$
$7 - 5 = \boxed{2}$	$0 - 0 = \boxed{0}$
$8 - 1 = \boxed{7}$	$9 - 6 = \boxed{3}$
$8 - 5 = \boxed{3}$	$5 - 1 = \boxed{4}$
$6 - 2 = \boxed{4}$	$7 - 2 = \boxed{5}$
$5 - 5 = \boxed{0}$	$6 - 0 = \boxed{6}$
$9 - 8 = \boxed{1}$	$9 - 4 = \boxed{5}$

S-9



Find the differences.

$\begin{array}{r}8\\-4\\\hline 4\end{array}$	$\begin{array}{r}7\\-1\\\hline 6\end{array}$	$\begin{array}{r}9\\-3\\\hline 6\end{array}$	$\begin{array}{r}9\\-1\\\hline 8\end{array}$	$\begin{array}{r}6\\-4\\\hline 2\end{array}$
$\begin{array}{r}5\\-3\\\hline 2\end{array}$	$\begin{array}{r}9\\-7\\\hline 2\end{array}$	$\begin{array}{r}7\\-3\\\hline 4\end{array}$	$\begin{array}{r}7\\-0\\\hline 7\end{array}$	$\begin{array}{r}5\\-2\\\hline 3\end{array}$
$\begin{array}{r}3\\-3\\\hline 0\end{array}$	$\begin{array}{r}6\\-5\\\hline 1\end{array}$	$\begin{array}{r}4\\-1\\\hline 3\end{array}$	$\begin{array}{r}8\\-3\\\hline 5\end{array}$	$\begin{array}{r}6\\-3\\\hline 3\end{array}$
$\begin{array}{r}9\\-2\\\hline 7\end{array}$	$\begin{array}{r}8\\-6\\\hline 2\end{array}$	$\begin{array}{r}4\\-2\\\hline 2\end{array}$	$\begin{array}{r}5\\-4\\\hline 1\end{array}$	$\begin{array}{r}6\\-1\\\hline 5\end{array}$
$\begin{array}{r}9\\-5\\\hline 4\end{array}$	$\begin{array}{r}7\\-6\\\hline 1\end{array}$	$\begin{array}{r}4\\-4\\\hline 0\end{array}$	$\begin{array}{r}3\\-0\\\hline 3\end{array}$	$\begin{array}{r}8\\-7\\\hline 1\end{array}$
$\begin{array}{r}5\\-0\\\hline 5\end{array}$	$\begin{array}{r}4\\-3\\\hline 1\end{array}$	$\begin{array}{r}9\\-0\\\hline 9\end{array}$	$\begin{array}{r}8\\-8\\\hline 0\end{array}$	$\begin{array}{r}1\\-0\\\hline 1\end{array}$

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number of beads from this set of seven and tell the number of the set which remains. Then have him write the numeral for the difference on the chalkboard and read his completed problem.

When asked to find the difference in the equation $7 - 2 = \square$, some child might select a set of beads other than the anticipated set. For example, he might choose to remove two beads from the first rod. Help him see that he can think of the removal of the two beads in two steps: take away one, take away one. He can think of the operation as subtracting "in pieces." If such an observation is made, the teacher may wish to point out that there are two equations related to this child's set removal: $7 - 1 = 6$ and $6 - 1 = 5$. A shorter way to write this idea is $(7 - 1) - 1 = 5$. The child will have to be reminded that the parentheses in the problem help him think of $7 - 1$ as one number—six.

Follow the same procedure to develop other subtraction combinations related to sums less than ten.

▶ Work page S-9 provides practice with subtraction combinations related to sums less than ten. Work the first two problems with the children, then tell them to complete each equation by finding the difference. Remind them that the number which results when one number is subtracted from another is called the "difference."

Work page S-10 provides practice with subtraction combinations, in vertical form, related to sums less than ten. Direct the children to write the difference below the line for each problem.

Supplemental Experiences

▶ Mark off ten nine-inch squares on the floor. Label the squares with the numerals 0 through 9. Have the class sit in a circle around this line of ten squares. Have one child be "it." Ask the rest of the children to close their eyes during this game.

The child who is "it" may choose any numeral he wishes as his starting point; he then tells the rest of the class where he is. He might say, "I am standing on the 8." Now have him walk backward, stepping on one numeral with each step, and tell the class how many steps he has taken. For example, "I have walked backward five steps."

Call upon someone to tell where the child who is "it" is now standing. Have the related subtraction equation written on the chalkboard $(8 - 5 = 3)$.

As a variation of this activity, a child may combine forward and backward steps. In each instance have the children determine whether an addition or subtraction idea is involved.

Subtraction

COMBINATIONS LESS THAN TEN

Work Pages S-11 and S-12

Objectives

To provide experience with the placeholder in different positions in subtraction equations.

To emphasize the relation between addition and subtraction.

To help children find the numeral for the missing number in vertical notation for subtraction combinations.

Fundamentals

In addition equations the child learned to work with the placeholder in different positions. He learned to apply the various concepts that were involved as he searched for the numeral to replace the placeholder.

As the child works with subtraction equations, the placeholder will also appear in different positions. In solving an equation such as $5 - 4 = \square$, the child is finding the number which represents the difference between five and four. (The difference can also be thought of as a missing addend.)

In an equation such as $5 - \square = 1$, the child is being asked to find the number that can be subtracted from five to give one. He is being asked to find a missing addend, to find the number that can be added to one to give five.

In an equation such as $\square - 2 = 4$, the child is being asked to find that number from which he can subtract two and get four. He is being asked for the sum of four and two.

In each situation the relation between addition and subtraction should be continually emphasized. The child should learn to take what he knows from the setting of addition and apply it to the related setting of subtraction. He should not learn to think of addition and subtraction as isolated operations.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Awareness of the relation between addition and subtraction.

Ability to read, write, and solve subtraction equations.

Ability to work with vertical notation.

Developmental Experiences

MATERIALS

flannel board
felt counters
felt numerals 0 through 9
felt symbols for operations and relations
felt placeholders
yarn
individual sets of plastic numerals
plus and minus signs
placeholders

▶ Quickly review a known situation by placing a set of five objects on the flannel board. Put the subtraction equation $5 - 1 = \square$ beside the objects. Have a child take a piece of yarn and separate the set of five into two sets, one of which will eventually be removed. Tell him that the equation will help him think about the number of objects that will be removed. Cover up the one object on the right. Have the set story read: "From a set of five objects, one object was taken away." Have the placeholder equation completed. When a child puts the numeral 4 in the placeholder, remove the placeholder from the flannel board. Ask a child to read the completed subtraction equation.

Leave this problem on the flannel board. Below it place the placeholder equation $5 - \square = 4$. Tell the children that when they see the placeholder in this position they must find the number that can be subtracted from five to give four. Ask them to tell how many objects must be taken away from five in order to have four remain. Have the placeholder equation completed. Perhaps some child has discovered that the number which completed the equation is the same number that was added to four to get five.

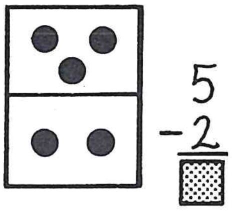
Place a third placeholder equation below the other two subtraction equations: $\square - 1 = 4$. Read this equation with the children: "Placeholder minus one equals four." Tell them that they must think of the number from which one can be subtracted to give a difference of four. They must think of what number minus one equals four. Ask them what number of objects they must have so that when one object is taken away, four remain. Have a child select the correct numeral to replace the placeholder and read the completed equation. Perhaps some child has discovered that the number that completes the equation is the sum of four and one.

Continue in this way with other sets and subtraction equations related to sums less than ten.

▶ Have the children use their plastic numerals, plus and minus signs, and placeholders at their desks to build subtraction equations related to the following number sentences.

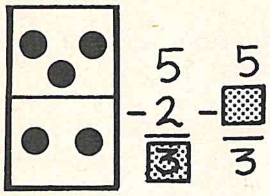
Three minus some number equals two.
($3 - \square = 2$)
Some number minus one equals seven.
($\square - 1 = 7$)
Six minus some number equals four.
($6 - \square = 4$)
Some number minus three equals five.
($\square - 3 = 5$)
Two minus some number equals zero.
($2 - \square = 0$)
Some number minus three equals six.
($\square - 3 = 6$)

After each placeholder equation is constructed, have the children select the numeral for the missing number and use it to complete the equation. Have each number sentence read aloud.



▶ Draw a domino picture in vertical position on the chalkboard. Next to the picture place the vertical form for the related number combination.

Tell the class that the number combination will help them think about the number of the set that will be removed. Cover up the two dots in the lower section of the domino picture. Have the set story read: "From a set of five objects, two objects were taken away." Have a child write the correct numeral for the difference in the shaded box below the line. Ask a child to read the completed subtraction combination.



Leave this problem on the chalkboard. Next to it place the same number combination, but this time put the shaded box in place of the numeral two. Tell the children that when they see a numeral missing in this position they must find the number which can be subtracted from five to give three. Ask them to tell how many objects must be taken away from five to have three remain. Have the correct numeral placed in the shaded box and have the number combination read. Perhaps some child has discovered that the number which belonged in the vertical notation is the same number which when added to three gives five.

Place a third subtraction combination next to the other two. Read this number combination with the children: "Some number minus two equals three." Tell them that they must think of the number from which two can be subtracted to give a difference of three. They must think of what number, minus two, equals three. Ask them what number of objects they must have so that when two are taken away three objects remain. Have a child write the correct numeral in the shaded box and read the completed number combination. Perhaps some child has discovered that the number that belongs in the problem is the sum of three and two.

Continue in this way with other domino pictures and subtraction combinations related to sums less than ten.

▶ Use work page S-11 to give children the opportunity to work with the placeholder in different positions in subtraction equations. Work the problems at the top of the page with the children. Tell them to complete the remaining equations on the page by writing the correct numeral in each placeholder.

Use work page S-12 to give children the opportunity to find the correct numeral for a missing number in the vertical form for subtraction combinations. Work the first problem in the first row and the first problem in the third row with the children. Have them complete the rest of the page on their own. Give help only when needed.

Supplemental Experiences

▶ Play a riddle game.

If I subtract one from my number, I get eight. What is my number?

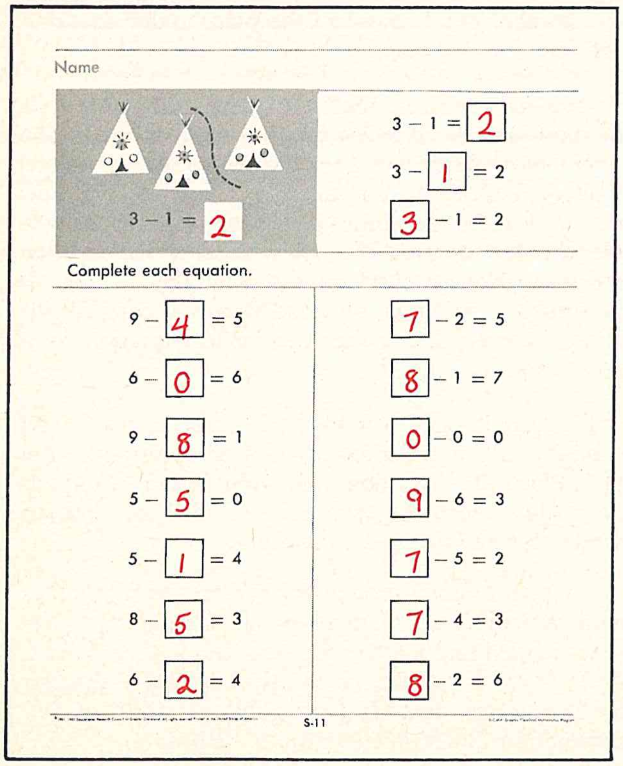
If I subtract two from seven, what number do I have?

If you subtract three from me, you will have six. What number am I?

One part of a set of three is a set of two. I am the other part. I am a set of how many objects?

▶ Show children a box and tell them that it contains six objects. Take out two objects in such a way that the class cannot see how many are being removed. Have one child come forward and count the number of objects remaining in the box. Have him tell how many objects were taken away. Have the corresponding subtraction equation written on the chalkboard.

Continue the activity by placing in the box sets whose numbers range from four through nine.



Name

$3 - 1 = \boxed{2}$

$3 - 1 = \boxed{2}$

$3 - \boxed{1} = 2$

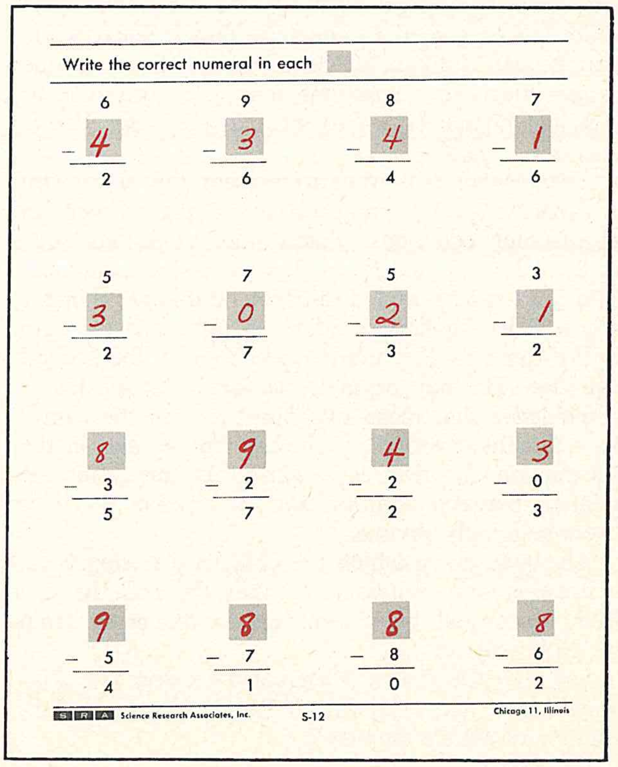
$\boxed{3} - 1 = 2$

Complete each equation.

$9 - \boxed{4} = 5$
$6 - \boxed{0} = 6$
$9 - \boxed{8} = 1$
$5 - \boxed{5} = 0$
$5 - \boxed{1} = 4$
$8 - \boxed{5} = 3$
$6 - \boxed{2} = 4$

$\boxed{7} - 2 = 5$
$\boxed{8} - 1 = 7$
$\boxed{0} - 0 = 0$
$\boxed{9} - 6 = 3$
$\boxed{7} - 5 = 2$
$\boxed{7} - 4 = 3$
$\boxed{8} - 2 = 6$

S-11



Write the correct numeral in each ▨

$\begin{array}{r}6\\ -\boxed{4}\\ \hline 2\end{array}$ $\begin{array}{r}9\\ -\boxed{3}\\ \hline 6\end{array}$ $\begin{array}{r}8\\ -\boxed{4}\\ \hline 4\end{array}$ $\begin{array}{r}7\\ -\boxed{1}\\ \hline 6\end{array}$

$\begin{array}{r}5\\ -\boxed{3}\\ \hline 2\end{array}$ $\begin{array}{r}7\\ -\boxed{0}\\ \hline 7\end{array}$ $\begin{array}{r}5\\ -\boxed{2}\\ \hline 3\end{array}$ $\begin{array}{r}3\\ -\boxed{1}\\ \hline 2\end{array}$

$\begin{array}{r}\boxed{8}\\ -3\\ \hline 5\end{array}$ $\begin{array}{r}\boxed{9}\\ -2\\ \hline 7\end{array}$ $\begin{array}{r}\boxed{4}\\ -2\\ \hline 2\end{array}$ $\begin{array}{r}\boxed{3}\\ -0\\ \hline 3\end{array}$

$\begin{array}{r}\boxed{9}\\ -5\\ \hline 4\end{array}$ $\begin{array}{r}\boxed{8}\\ -7\\ \hline 1\end{array}$ $\begin{array}{r}\boxed{8}\\ -8\\ \hline 0\end{array}$ $\begin{array}{r}\boxed{8}\\ -6\\ \hline 2\end{array}$

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Place eight objects in a box, but do not let the children see how many objects there are. Reach into the box and remove two of the objects; show them to the children. Have one child come forward and count the number of objects that remain in the box. Have him tell how many were in the box at the start of the game. Ask another child to write the appropriate subtraction equation on the chalkboard.

Addition—Subtraction

COMBINATIONS

LESS THAN TEN

Work Pages S-13 and S-14

Objectives

To provide experience with the placeholder in different positions in addition and subtraction equations.

To emphasize the relation between addition and subtraction.

Fundamentals

The child will discover that in placeholder equations such as $1 + \square = 4$ and $4 - \square = 1$, the numeral that replaces the placeholder in each instance is the same. If he is guided to think about this situation, he will conclude that the number added to one to give four is the same as the number subtracted from four to give one. In each instance, the same missing addend results in the same sum. The child can be guided to conclude that in each equation he can think in terms of adding three to one to get four. The equations may look different to him, but he can be led to see that the same idea is inherent in both.

The teacher will want to present the child with a variation in this approach to finding answers in placeholder equations. Placeholder equations such as $3 + 2 = \square$ and $\square - 2 = 3$ can be examined. The child can be guided to conclude that the number required in the first equation to represent the sum is the same as the number required in the second equation. He may begin to wonder: "Is this just a coincidence that these two numbers are the same? Is it possible to think of the number needed in the subtraction equation as a sum?" At this point the relation between addition and subtraction becomes overwhelmingly obvious.

The road down which the child is traveling is an interesting one. When he reaches the end, he can shift gears, and back right up to the point from which he started.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Awareness of the inverse relation between addition and subtraction.

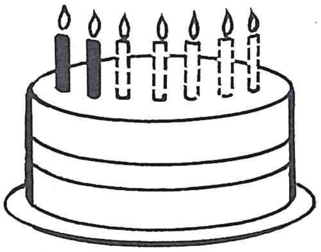
Ability to read, write, and solve addition and subtraction placeholder equations.

Developmental Experiences

MATERIALS

flannel board
felt numerals 0 through 9
felt symbols for operations and relations
felt placeholders
colored chalk
individual Ten-frames
number-strips

▶ Draw a large birthday cake on the chalkboard. Draw two candles on the cake, with colored chalk.



Tell the children that this cake is for a little girl seven years old. Ask if the cake has enough candles. Write the placeholder equation $2 + \square = 7$ below the cake. Tell the children that the problem to be solved is now recorded; they must find the solution. Ask a child to draw, with different-colored chalk, as many more candles as are needed to make a set of seven. Have another child tell the set story; have still another child complete the placeholder equation and read it to the class.

Tell the children that when the little girl went to blow out the candles on the birthday cake, she didn't blow hard enough and two candles remained lighted. Tell the class that they must decide how many candles the little girl blew out. Write the placeholder equation $7 - \square = 2$ below the addition equation. Have a child erase the number of candles he thinks were blown out, and have the class decide if he is correct. Have the placeholder equation completed and read aloud.

Proceed in this way to build other addition and subtraction combinations related to sums less than ten. Place the addition and subtraction equations one below the other each time so that the children are exposed to the inverse concept.

To help children conclude that the same number is always involved in these pairs of equations, present the idea that it would be possible, in the problem $5 - \square = 4$, to think in terms of adding a number to four to get five. It is possible to think of this subtraction equation in terms of addition.

▶ Place the equation 4 + 3 = 7 on the flannel board. Below this equation write 7 − 3 = 4. Ask the children to look carefully at the two equations and compare the position of the numerals in each equation. Point out that the same numbers are involved in both equations.

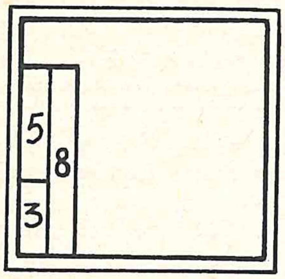
Tell the children that you would like to try an experiment. Say that you wonder what would happen if they would read the second equation from right to left and think of addition instead of subtraction.

When the children get the answer by this "backward" way of doing things, have them subtract and see if they get the same number they started with. Children will be surprised to discover that this "different" way of looking at a subtraction problem gives them the answer in placeholder equations such as ☐ − 1 = 7. Give children many opportunities to see if this idea works with other subtraction combinations related to sums less than ten.

▶ Have children use their Ten-frames to illustrate that subtracting a number is the inverse of adding that number. Use number-strips on the flannel board to guide the activity. These number-strips can be made from heavy construction paper or tagboard. Make them twice the size of the bars in the Ten-frame and fix them so that they adhere to the flannel board.

Have the three-bar and the five-bar placed in the Ten-frame. Ask children to find a single bar that has the same number of blocks as these two bars together. Have them temporarily place the eight-bar next to the three-bar and the five-bar to observe that it contains the same number of squares. Have the addition combination given.

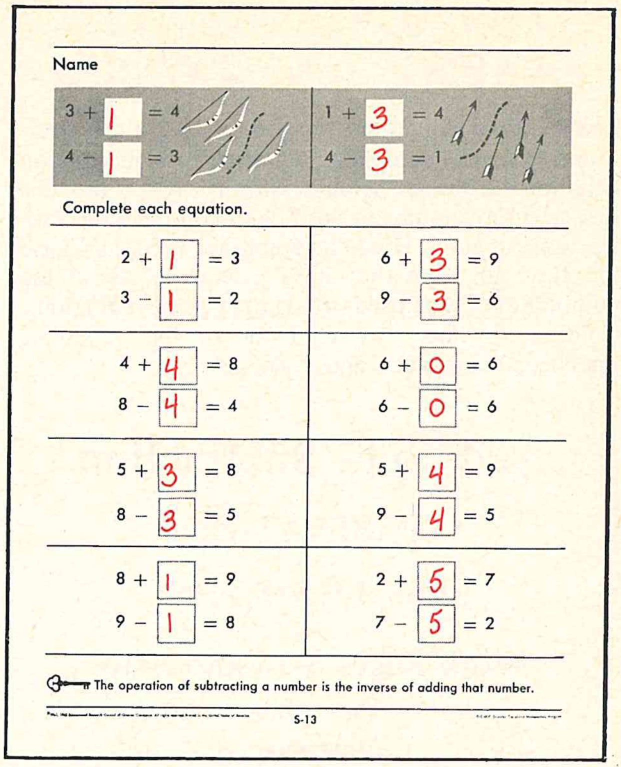
Tell the children that you now want them to remove five squares from the set of eight squares. In order to do this they must again think of the subsets contained within the set of eight: the set of three and the set of five. (Have the children push the eight-bar out of the way.) After they take a set of five squares from the set of eight squares, have them tell how many squares remain. Have the related subtraction combination stated.



3+5=8
8-5=3

Continue in this way with other addition and subtraction combinations related to sums less than ten. Include zero in the activity.

▶ Use work pages S-13 and S-14 to emphasize the inverse relation between addition and subtraction. Work the first two pairs of equations on work page S-13 with the children. Have them complete the rest of this page and all of work page S-14 independently.



Name

$3 + \boxed{1} = 4$ $4 - \boxed{1} = 3$	$1 + \boxed{3} = 4$ $4 - \boxed{3} = 1$

Complete each equation.

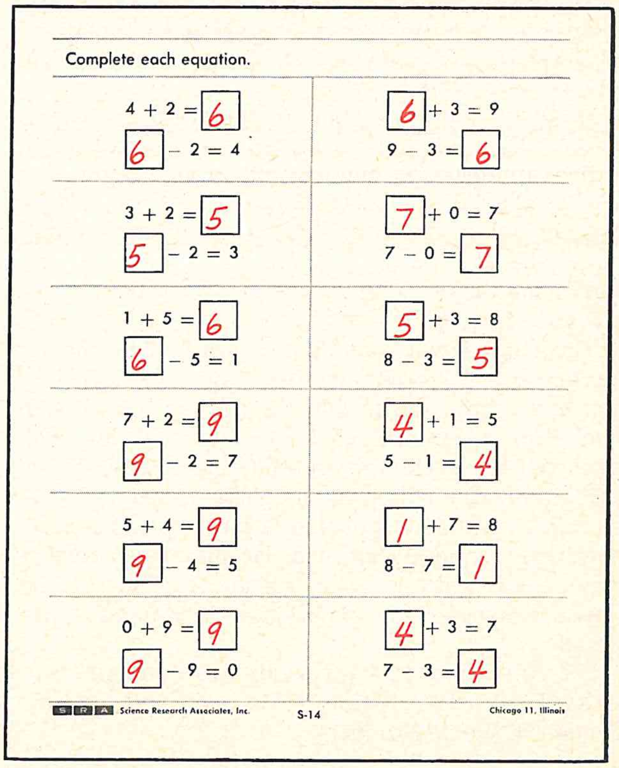
$2 + \boxed{1} = 3$ $3 - \boxed{1} = 2$	$6 + \boxed{3} = 9$ $9 - \boxed{3} = 6$
$4 + \boxed{4} = 8$ $8 - \boxed{4} = 4$	$6 + \boxed{0} = 6$ $6 - \boxed{0} = 6$
$5 + \boxed{3} = 8$ $8 - \boxed{3} = 5$	$5 + \boxed{4} = 9$ $9 - \boxed{4} = 5$
$8 + \boxed{1} = 9$ $9 - \boxed{1} = 8$	$2 + \boxed{5} = 7$ $7 - \boxed{5} = 2$

The operation of subtracting a number is the inverse of adding that number.

S-13

Supplemental Experiences

▶ Place two columns of placeholder equations on the chalkboard. Put addition equations in the column on the left and subtraction equations in the column on the right. Call upon a child to complete the



Complete each equation.

$4 + 2 = \boxed{6}$ $\boxed{6} - 2 = 4$	$\boxed{6} + 3 = 9$ $9 - 3 = \boxed{6}$
$3 + 2 = \boxed{5}$ $\boxed{5} - 2 = 3$	$\boxed{7} + 0 = 7$ $7 - 0 = \boxed{7}$
$1 + 5 = \boxed{6}$ $\boxed{6} - 5 = 1$	$\boxed{5} + 3 = 8$ $8 - 3 = \boxed{5}$
$7 + 2 = \boxed{9}$ $\boxed{9} - 2 = 7$	$\boxed{4} + 1 = 5$ $5 - 1 = \boxed{4}$
$5 + 4 = \boxed{9}$ $\boxed{9} - 4 = 5$	$\boxed{1} + 7 = 8$ $8 - 7 = \boxed{1}$
$0 + 9 = \boxed{9}$ $\boxed{9} - 9 = 0$	$\boxed{4} + 3 = 7$ $7 - 3 = \boxed{4}$

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$2 + \square = 8$
$\square + 1 = 7$
$3 + \square = 9$
$2 + 4 = \square$

$7 - 1 = \square$
$8 - \square = 2$
$\square - 4 = 2$
$9 - \square = 3$

first addition equation. Have a second child draw a line from this completed equation to a subtraction equation he thinks belongs with it. When the line has been drawn, have a third child complete the subtraction equation. Have the equations read, and have the class tell what they have discovered about the number that was used to complete the equations. Continue in this way to have all the equations matched, completed, and read.

Addition—Subtraction

COMBINATIONS LESS THAN TEN

Work Pages S-15 and S-16

Objectives

To help the children use their understanding of addition and subtraction in a new setting.

To introduce the triangle as a placeholder for an operational sign in an equation.

To review addition and subtraction sums less than ten.

Fundamentals

The skillful teacher strives to offer the child many and varied opportunities to be creative in his attack on numbers. She encourages him to re-examine known ideas continually.

When the child is taught to think only in terms of one aspect of a problem, he is severely limited in his approach to number concepts. When the teacher confines the child's understanding to known sums or differences in addition or subtraction equations such as $5 + 2 = \square$ or $7 - 2 = \square$, she denies him the opportunity to view ideas about numbers from many sides.

The teacher will want to encourage the child to use the facts he knows about the operations of addition and subtraction to find the missing operational symbol in equations such as $4 \triangle 2 = 6$. She will want to have the child try out different possibilities in each problem situation. In an equation such as $5 \triangle 0 = 5$, the child may substitute a plus sign and, since it works, he may be satisfied. He may never realize that a minus sign can also be substituted. The teacher should emphasize the importance of trying all possibilities.

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Understanding of the concepts of addition and subtraction.

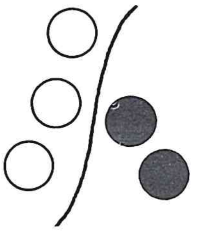
Ability to work with addition and subtraction combinations in vertical form.

Developmental Experiences

MATERIALS

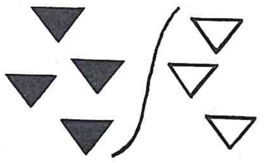
flannel board
felt counters
felt numerals 0 through 9
felt symbols for operations and relations
felt triangles
yarn
individual sets of plastic numerals
plastic triangles
plastic symbols for operations and relations

▶ Place on the flannel board a set of two objects and a set of three objects, separated by a piece of yarn. Have the children tell how many objects are in each set. Remove the piece of yarn and ask the children to tell what they think was done with these two sets in order to make a set of five. Lead the children to say that the sets have been joined. Place the numerals 2 and 3 on the flannel board about three inches apart. Ask the children to tell what they think will have to be done to the numbers in order to get the result of five. After a child says that the numbers must be added, place a plus sign between the numerals: $2 + 3$.



Put the equation $2 \triangle 3 = 5$ on the flannel board. Explain to the children that the triangle in the equation means that they are to put a plus sign or a minus sign in the equation in order to make the number sentence a true sentence. Ask a child to come forward and place what he believes to be the correct operational sign, $+$ or $-$, in the triangle. Have the class read the completed equation and decide whether or not it is true.

Remove the joined sets and the completed equation from the flannel board. Place a set of seven objects on the board. Separate the set of seven into subsets of three and four with a piece of yarn. Have the children tell what they will have to do with the set in order to have only three objects. They should say that a set of four must be taken away, or removed. Place the numerals 7 and 4 on the board



about three inches apart. Ask the children to tell what they think they will have to do with the numbers in order to get the result of three. After they say that four must be subtracted from seven, place a minus sign between the numerals: 7 − 4.

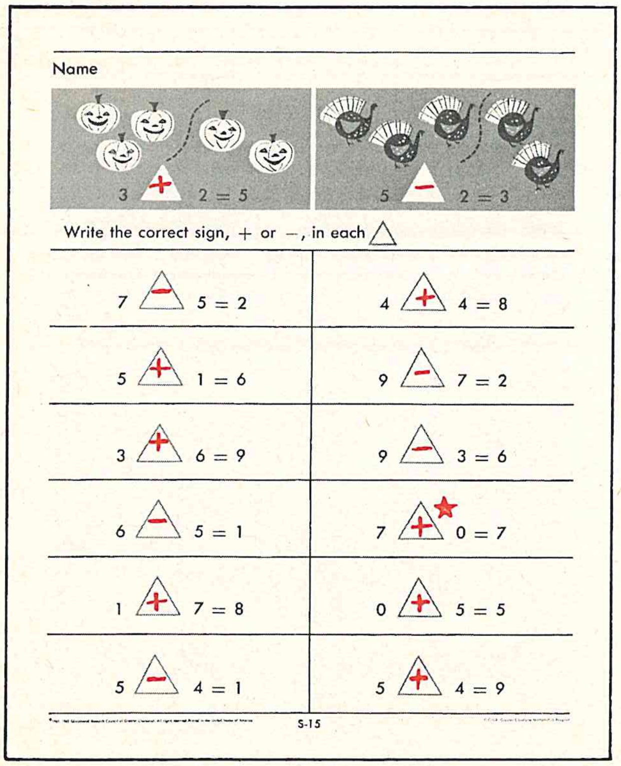
Put the equation 7 △ 4 = 3 on the flannel board. Ask a child to come forward and place what he believes to be the correct operational sign, + or −, in the triangle. Have the class read the completed equation and decide whether or not it is true.

Continue in this way to work with sets and with equations in which the operational sign is missing. Use other addition and subtraction combinations related to sums less than ten.

▶ Have the children use their plastic numerals, symbols for operations and relations, and plastic triangles to build equations requiring operational signs. Give directions on the equations you wish built: 1 △ 6 = 7; or 8 △ 3 = 5. After the children have replaced the triangle with the correct operational sign, have the equation read. Have the children tell the name of the operational sign they chose and the name of the operation the symbol represents.

▶ Read the following story problems to the class. Then have children tell what operation is needed to get the answer to the problem. Finally, have the children work the problems.

> Sally had seven pennies. On the way to school she lost two pennies. How many pennies does Sally have now?
>
> Three birds flew down from a tree and sat next to six birds on the grass. How many birds are now on the grass?



Name

3 + 2 = 5

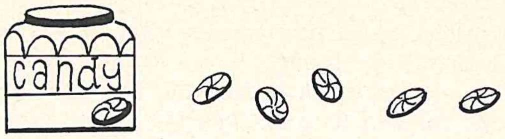
5 − 2 = 3

Write the correct sign, + or −, in each △

7 − 5 = 2	4 + 4 = 8
5 + 1 = 6	9 − 7 = 2
3 + 6 = 9	9 − 3 = 6
6 − 5 = 1	7 +★ 0 = 7
1 + 7 = 8	0 + 5 = 5
5 − 4 = 1	5 + 4 = 9

S-15

★ More than one correct answer is possible for each problem.



> Ellen gave two of her pieces of candy to Tom. Then she had six pieces of candy. How many pieces did she have to begin with?

▶ Use work page S-15 to give the children additional opportunities to complete equations with the appropriate operational signs. Work the first two problems with the group, and then have them complete the remainder of the work page independently.

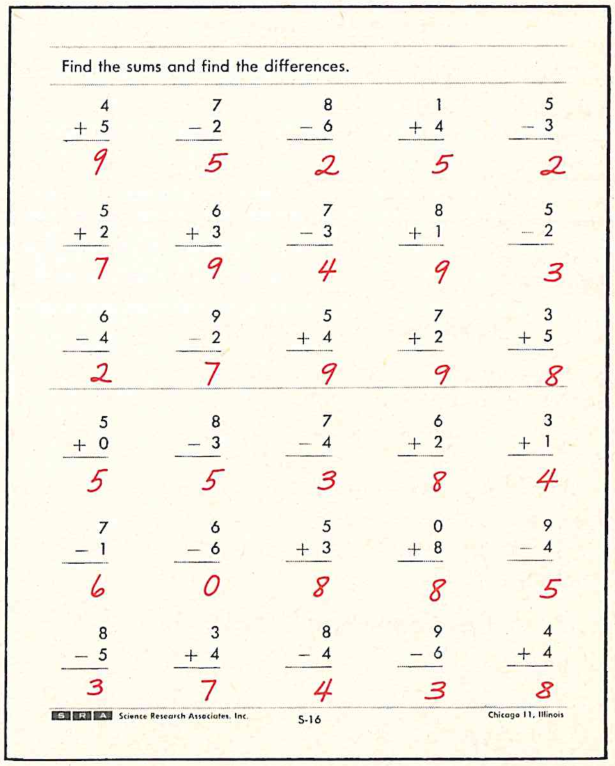
Work page S-16 provides a quick review of known addition and subtraction combinations related to sums less than ten. Tell the children to complete each problem by writing the numeral for the sum or difference.

Supplemental Experiences

▶ Prepare a set of flash cards showing equations requiring an operational sign. Expose the cards one at a time to the class, and call upon a child to give the answer. If he is not able to answer correctly, give him the card and ask him to see if he can work out the answer with counters at the flannel board. Give him additional help only if it is needed. Continue the activity with the next card.

7△5 = 2

2△6 = 8



Find the sums and find the differences.

4 + 5 9	7 − 2 5	8 − 6 2	1 + 4 5	5 − 3 2
5 + 2 7	6 + 3 9	7 − 3 4	8 + 1 9	5 − 2 3
6 − 4 2	9 − 2 7	5 + 4 9	7 + 2 9	3 + 5 8
5 + 0 5	8 − 3 5	7 − 4 3	6 + 2 8	3 + 1 4
7 − 1 6	6 − 6 0	5 + 3 8	0 + 8 8	9 − 4 5
8 − 5 3	3 + 4 7	8 − 4 4	9 − 6 3	4 + 4 8

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▶ Distribute cards bearing plus signs, minus signs, and numerals 0 through 9, as below. Write two partially completed equations on the chalkboard. Ask the children to come forward if they have the card that will make the number sentence true. Have the cards placed in the proper position to complete the statements and have the completed equations read. Have the children tell whether these statements are true. Continue in this way with other combinations related to sums less than ten.

$3 \quad = 8 \qquad 9 \quad =6$

$3\boxed{+}\boxed{5}\,8 \qquad 9\boxed{-}\boxed{3}=6$

Story Problems

Work Pages S-17 and S-18

Objectives

To develop children's ability to analyze and solve story problems.

To relate children's experiences with concrete objects and with semi-concrete story problems to addition and subtraction combinations.

Fundamentals

The introduction of story problems should lead the children from individual experiences to the stories on the work page. Help children find the question or problem in the story, then analyze the facts given and decide on the operation to be used in solving the problem. When the answer is found, the children should check it to make sure it is a reasonable solution.

Readiness for Understanding

Ability to put ideas and experiences into sentences involving numbers.

Ability to recognize the problem to be solved and to follow through to the solution.

Knowledge of addition and subtraction combinations up to nine.

Ability to complete equations and to solve simple column additions.

Developmental Experiences

MATERIALS	VOCABULARY
wooden blocks or paper squares plastic numerals	story problem

▶ Ask those children who like to build with blocks to raise their hands. Have one child tell some things he has built with blocks. Display a set of wooden blocks or paper squares on which appear the numerals and symbols 8, 5, 3, +, −, =. Ask one child to arrange the blocks or paper squares to show an equation. Have three other children show different equations.

▶ Now have the children work at their desks with plastic numerals to show the four possible equations:

$5 + 3 = 8 \qquad 8 - 3 = 5$

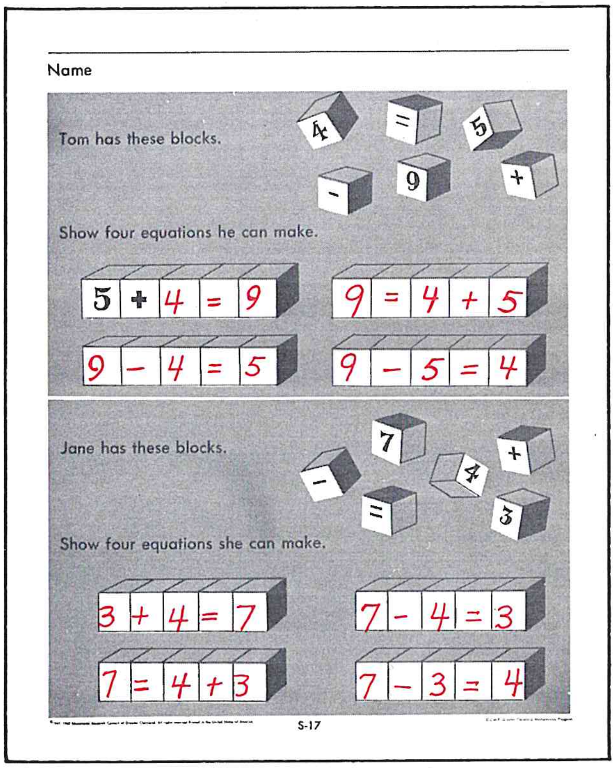
$3 + 5 = 8 \qquad 8 - 5 = 3$

Reinforce the concept that subtracting a number is the inverse of adding that number by discussing the relationship between pairs of equations.

Write the numerals 6, 3, and 9 on the chalkboard and have children make four equations with their plastic numerals. Have them tell which subtraction equation is related to each addition equation.

▶ Pass out work page S-17. Lead into the *story problem* on the work page by asking: "What operation symbols are on the blocks in the picture?" "What numerals are on the blocks?" Tell the children to write four equations that could be built with Tom's and Jane's blocks. Have the children complete the equations independently.

Use work page S-18 to give the children an opportunity to work with addition and subtraction combinations in a functional setting. Have one child read the first problem aloud. Lead the children to the solution step by step by asking questions such as the following:


Name

Tom has these blocks.

Show four equations he can make.

5 + 4 = 9

9 = 4 + 5

9 − 4 = 5

9 − 5 = 4

Jane has these blocks.

Show four equations she can make.

3 + 4 = 7

7 − 4 = 3

7 = 4 + 3

7 − 3 = 4

S-17

★ More than one correct answer is possible for each problem.

1. "What must we find out?"
2. "What facts does the story tell?"
3. "How can we use the facts to find the answer?"
4. "Is the answer reasonable?"

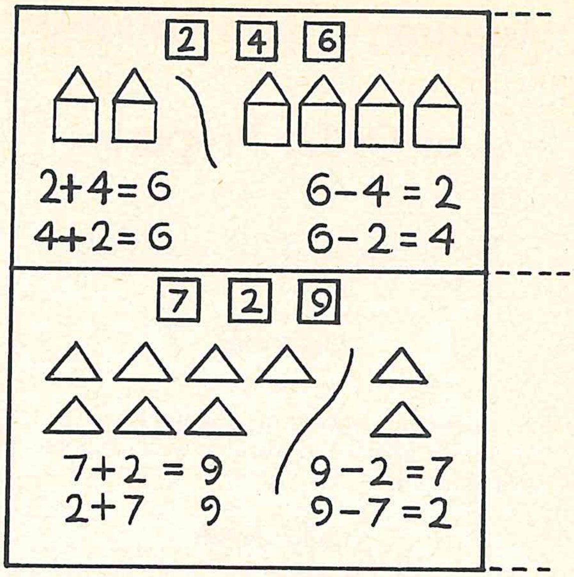
When helping the children decide which operation must be used, ask these questions:

"To find how many there are all together, what do you do?" (Add)

"To find how much greater one number is than another, or how many things are left if you take some away, what do you do?" (Subtract)

Show the children how to write the answers in the blanks provided.

Have each of the remaining problems read aloud. Encourage the children to fill in the answers independently. Help those who do not understand the problem. Check the answers together. Have the children discuss and judge the reasonableness of their answers.



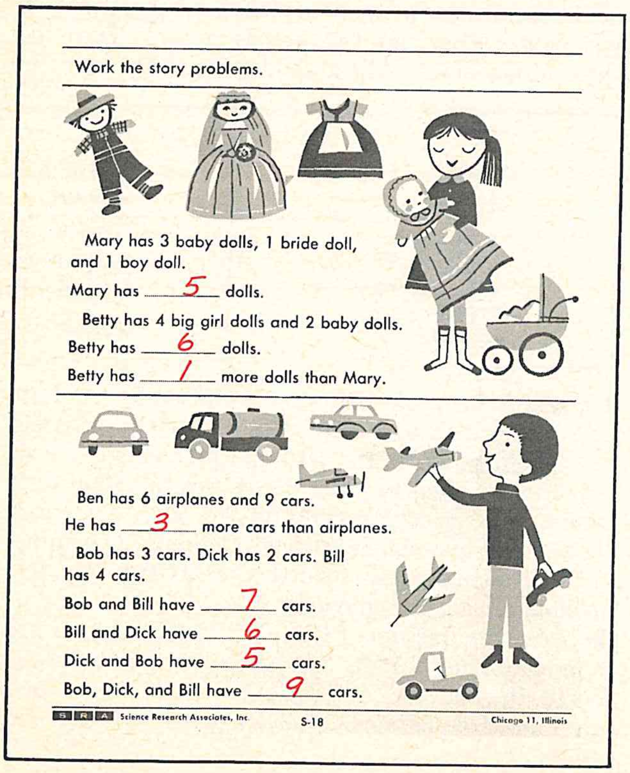
Supplemental Experiences

▶ Furnish each child with a 12″ x 18″ sheet of Manila paper. Have them fold the sheets in half and again into fourths. Write three numerals in each section of the paper. Direct the children to think of the numbers represented by these numerals and to write four equations for each set of numbers. Let them draw pictures to illustrate some of their equations if they wish.

▶ Give the children second-grade writing paper and have them write and solve their own story problems. Then give each child a 12″ x 18″ sheet of Manila paper. Have the children fold the Manila paper in half and draw a picture in one half about the story problem or about the solution. Then have them paste the story problem on the other half of the Manila paper.

Let the children take turns reading their stories and calling on others to work the problems at the board.

Encourage additional creativeness in writing and illustrating story problems by arranging a bulletin board display of the children's completed stories.



Work the story problems.

Mary has 3 baby dolls, 1 bride doll, and 1 boy doll.
Mary has 5 dolls.
Betty has 4 big girl dolls and 2 baby dolls.
Betty has 6 dolls.
Betty has 1 more dolls than Mary.

Ben has 6 airplanes and 9 cars.
He has 3 more cars than airplanes.
Bob has 3 cars. Dick has 2 cars. Bill has 4 cars.
Bob and Bill have 7 cars.
Bill and Dick have 6 cars.
Dick and Bob have 5 cars.
Bob, Dick, and Bill have 9 cars.

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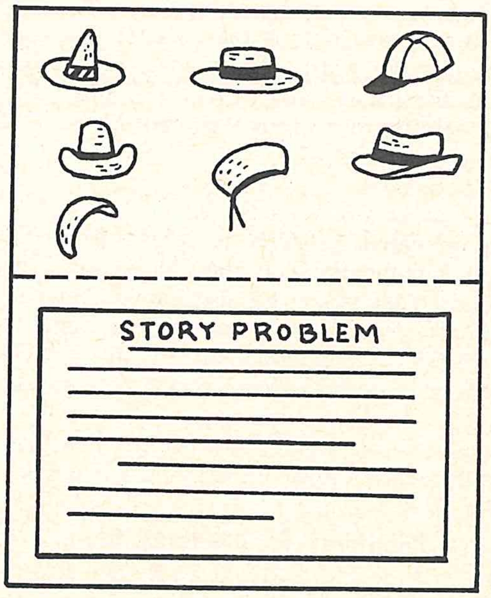


Addition—Commutative or Order Property

Work Pages S-19 and S-20

Objectives

To review the concept of commutativity.

To provide practice in working with equations and vertical notation.

Fundamentals

At the first-grade level, the child discovers that the order in which two sets are joined does not alter the cardinal number of the union of the sets. He finds that when a set of five is joined to a set of one, a new set of six is formed. And if a set of one is joined to a set of five, a new set of six is formed. The child learns that the order in which two numbers are added does not alter the sum. If one is added to five, the sum is six. If five is added to one, again the sum is six.

The child wonders if this happens with other numbers and he is provided with many opportunities to find out. He soon becomes aware of one of the fundamental properties of addition—the commutative or order property.

The teacher will want to review the concept of commutativity at the second-grade level. The child need not learn this term. The use of the words "commute," "commutative property," or "commutativity" is optional. It is the concept which is important, not the words. The teacher may substitute "change the order" and "order property" for the more sophisticated words "commute" and "commutative property."

The teacher will want the child to be aware that the commutative property is helpful in work with addition combinations. If the child knows that six plus one equals seven, he also knows that one plus six equals seven. The value of the commutative property will become apparent when the child begins to work with combinations related to sums greater than ten.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Ability to join two sets to make a new set.

Ability to add a pair of numbers.

Ability to work with placeholder equations and vertical notation.

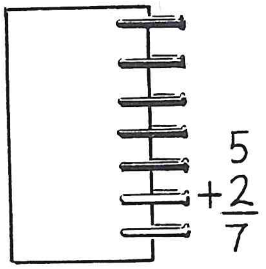
Developmental Experiences

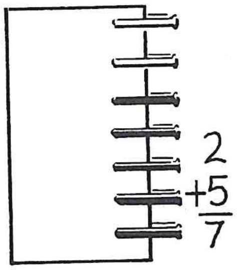
MATERIALS	VOCABULARY
flannel board felt numerals 0 through 9 felt operation and relation symbols felt cutout counters clothespins cardboard	order

▶ Place four red disks on the flannel board. Join three green disks to the set of four and ask how many disks are in the new set. Have the corresponding equation, 4 + 3 = 7, placed under the joined sets. Tell the class that you wonder how many would be in the new set if you joined the four red disks to the three green disks. Then join the sets in this *order*. Guide the class to make the observation that the cardinal number of the union of sets was not changed when the order in which the sets were joined was changed. Place the equation 3 + 4 = 7 under the first equation on the flannel board.

In order that the children can see whether or not the order in which the sets are joined makes a difference in the cardinal number of the union of sets, have them name other pairs of sets. Then have the sets joined and the related equation placed on the flannel board.

▶ Clip five red clothespins on a heavy strip of cardboard, then clip two yellow pins under the five red pins. Ask the children to tell the story of the joined sets. Have the class read the card from top to bottom: "A set of two was joined to a set of five to make a set of seven." Hold the card up to the flannel board, and have the vertical-notation form of the number combination placed on the board as you read, "Five plus two equals seven." Turn the card upside down. Ask a child to read the story of the sets from top to bottom. He will read, "A set of five was joined to a set of two to make a set of seven." As you hold the card up to the flannel board, ask another child to place the vertical-notation form of the number combination on the board and read, "Two plus five equals seven." Continue with other number combinations related to sums less than ten; include zero in the activity.





▶ Pose the question, "Does the order matter when removing sets?" Place six objects on the flannel board, then have a child remove three of them. Have the related equation 6 − 3 = 3 placed under the set that remains.

Now place three objects on the board, and ask a child to remove six objects. The child will observe that this is an impossible situation, but let him try this procedure with other sets. Guide the children to conclude that there is no answer to the problem when they change the order in the set-removal situation and that subtraction is not commutative.

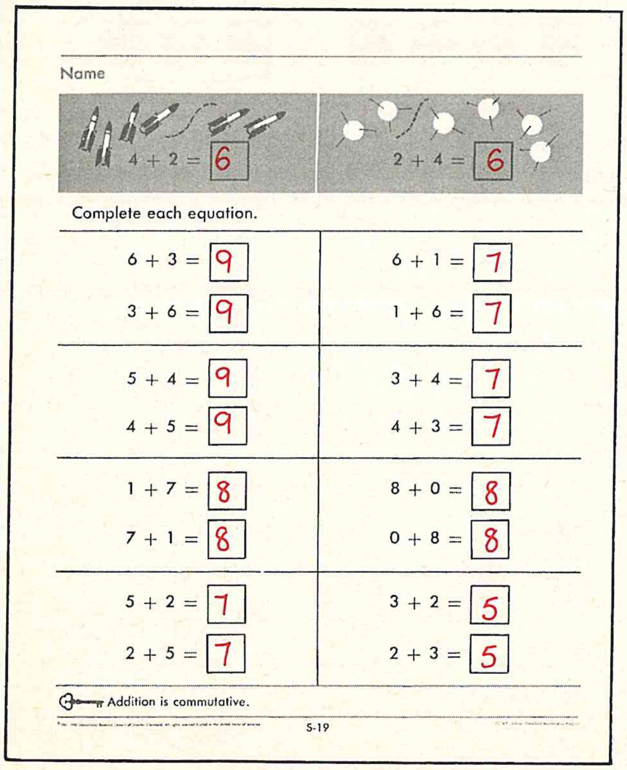
▶ Use work page S-19 to reinforce the children's understanding of the commutative, or order, property of addition. Work the first two problems at the top of the page with them, then have the class work the remainder of the page by themselves.

Work page S-20 provides a review of known addition combinations. Encourage the children to take advantage of the order property of addition to get the answers to these problems quickly. Tell them that they must find the sums in the problems in the first three rows.

In the last two rows on the page, tell the children that they must find the missing addend in each problem and write the correct numeral for this addend in the colored box.

Supplemental Experiences

▶ Write all the possible addition combinations with sums zero through nine on the chalkboard. Write a set of these combinations on cards and place them in a box. Call on a child to reach into the box and choose a card. Have him hold his card next to the commuted form of the addition equation on the board and read both equations to the class. Have the child give you his card and continue the game until all cards have been taken from the box and matched to equations on the chalkboard.



Name

4 + 2 = 6

2 + 4 = 6

Complete each equation.

6 + 3 = 9
3 + 6 = 9

6 + 1 = 7
1 + 6 = 7

5 + 4 = 9
4 + 5 = 9

3 + 4 = 7
4 + 3 = 7

1 + 7 = 8
7 + 1 = 8

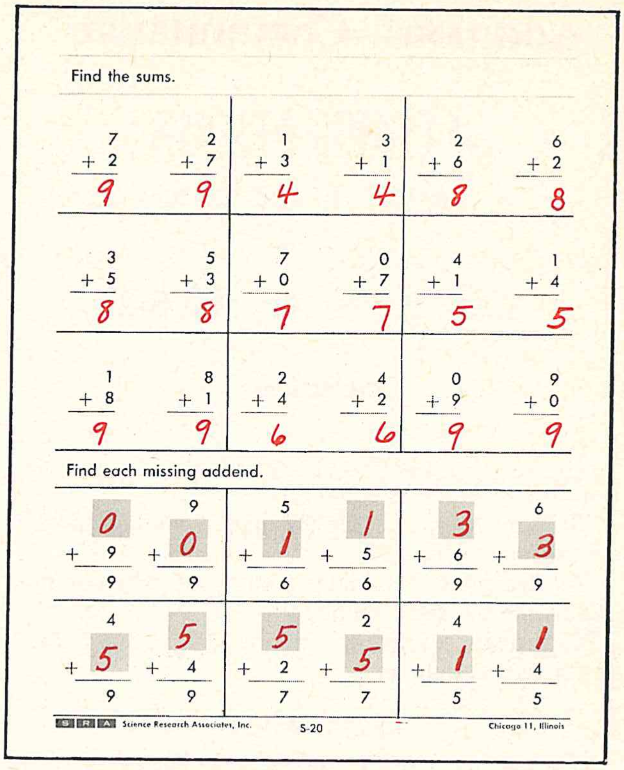
8 + 0 = 8
0 + 8 = 8

5 + 2 = 7
2 + 5 = 7

3 + 2 = 5
2 + 3 = 5

Addition is commutative.

S-19



Find the sums.

7	2	1	3	2	6
+ 2	+ 7	+ 3	+ 1	+ 6	+ 2
9	9	4	4	8	8
3	5	7	0	4	1
+ 5	+ 3	+ 0	+ 7	+ 1	+ 4
8	8	7	7	5	5
1	8	2	4	0	9
+ 8	+ 1	+ 4	+ 2	+ 9	+ 0
9	9	6	6	9	9

Find each missing addend.

0	9	5	1	3	6
+ 9	+ 0	+ 1	+ 5	+ 6	+ 3
9	9	6	6	9	9
4	5	5	2	4	1
+ 5	+ 4	+ 2	+ 5	+ 1	+ 4
9	9	7	7	5	5

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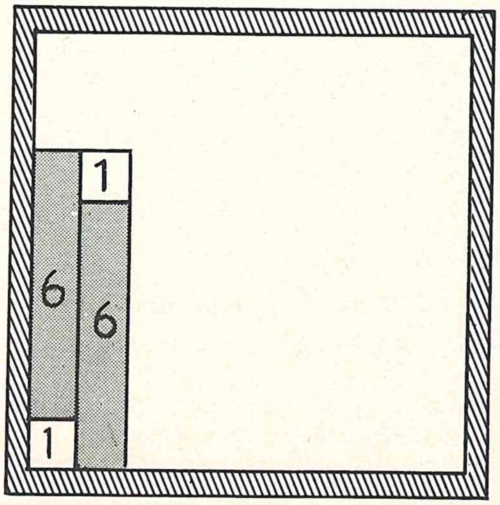
The children will observe that the commuted forms of some equations, those in which both addends are the same, have the same numerals.

▶ The class can use their Ten-frames to represent the commuted forms of number combinations. Have them place the one-bar in the lower left corner of the frame, and then place the six-bar above it. Have children see how quickly they can find the bars to represent the commuted form of this combination. Ask them to place these two bars in the Ten-frame and name the combinations which they represent. The activity can be repeated with other pairs of numbers related to sums less than ten.



Addition—Subtraction

COMBINATIONS

LESS THAN TEN

Work Pages S-21 and S-22

Objectives

To re-emphasize the concept of equality in addition and subtraction equations.

To strengthen the child's understanding of the commutative property.

To strengthen his understanding of different numerals for the same number.

To provide practice in working with number combinations less than ten.

Fundamentals

The teacher will want to reassure herself that the child has a good understanding of the concepts relating to numbers, operations, and relations. If the child answers readily the many kinds of questions pertaining to these concepts, then the teacher will know that these learnings have meaning for him, that he has not learned by rote.

Does the child only respond readily to such questions as, "How much is five plus three?" Or does he answer just as readily the question, "What number must be added to five to give eight?" or "What number plus three gives eight?" Can he explain how he got his answers to these last two queries? Is the child beginning to recognize that $8 = 5 + 3$ says the same thing as $5 + 3 = 8$? Does he realize that $5 + 3$ and $8 - 0$ are just different numerals for the same number, eight? Is he beginning to feel secure in the realization that because he knows three plus five, he also knows five plus three?

One of the main objectives of the carefully planned arithmetic program is to encourage the child to be flexible in his thinking; to relate to the known and unknown the ideas to which he has been exposed. The teacher plays her role well when she accepts the responsibility for guiding the child to use his powers of reasoning.

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Understanding of the concepts of addition and subtraction.

Understanding of the concept of commutativity.

Understanding of the concept of equality.

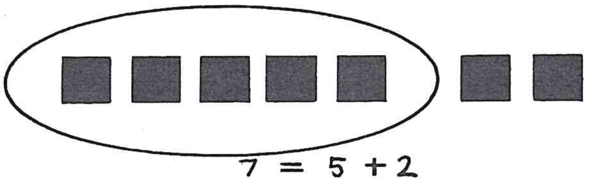
Ability to work with placeholder equations and vertical notation.

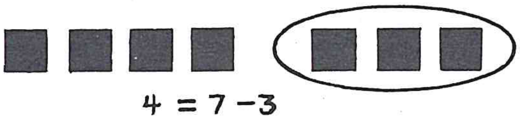
Developmental Experiences

MATERIALS

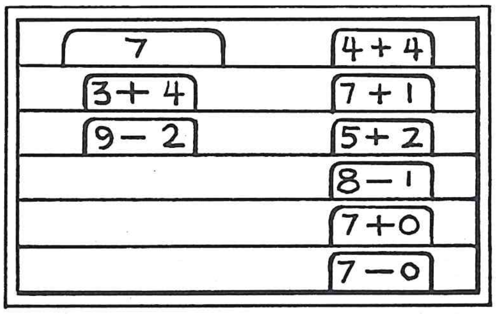
pocket chart
numeral cards
flannel board
felt numerals 0 through 9
felt operation and relation symbols
felt cutouts
yarn

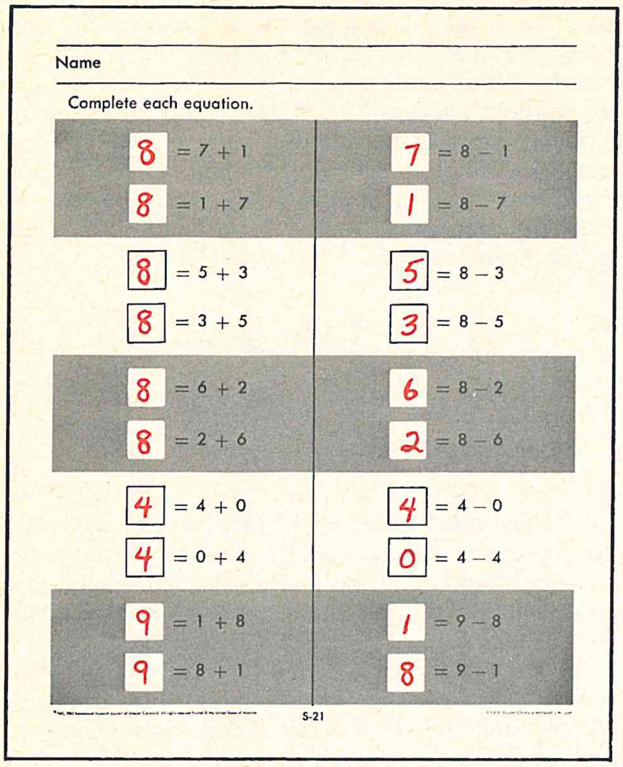
▶ Place seven felt cutouts in a row on the flannel board. Ask a child to show how he thinks of the set of seven. Let him place a piece of yarn around one of the subsets and then present his interpretation to the class. He may say, "I think of seven as a set of five and a set of two." Ask him to place the related addition equation under the set picture and read it to the class. Call on another child to use the same set of seven cutouts to show how he can think of the number four. He may wish to place the piece of yarn around a subset of three within the set of seven. He may cover up a set of three with his hand and tell the class that he thinks of four when he removes a set of three from a set of seven. Then have the child place the related subtraction equation under the set picture and read it to the class. In this manner the children can describe numbers in many different ways as the activity continues.





▶ Place the numeral-card seven on the left side of the pocket chart, and several other numerals for seven and eight in the pockets on the right side of the chart. Let the children take their turn in choosing other numerals for seven. Have them place the selected numeral-cards under the standard numeral seven and name the numeral: Three plus four is another name for seven, or nine minus two is another name for seven. This is a fine way to have the children review and become completely familiar with other names for the numbers zero through nine.





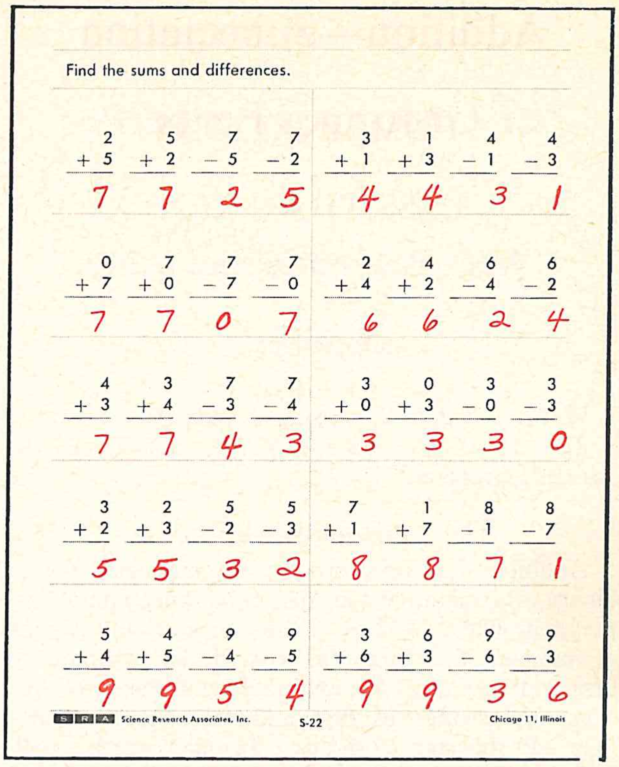
Name

Complete each equation.

8 = 7 + 1
8 = 1 + 7
8 = 5 + 3
8 = 3 + 5
8 = 6 + 2
8 = 2 + 6
4 = 4 + 0
4 = 0 + 4
9 = 1 + 8
9 = 8 + 1

7 = 8 − 1
1 = 8 − 7
5 = 8 − 3
3 = 8 − 5
6 = 8 − 2
2 = 8 − 6
4 = 4 − 0
0 = 4 − 4
1 = 9 − 8
8 = 9 − 1

S-21



Find the sums and differences.

$\begin{array}{r}2\\+5\\\hline 7\end{array}$	$\begin{array}{r}5\\+2\\\hline 7\end{array}$	$\begin{array}{r}7\\-5\\\hline 2\end{array}$	$\begin{array}{r}7\\-2\\\hline 5\end{array}$	$\begin{array}{r}3\\+1\\\hline 4\end{array}$	$\begin{array}{r}1\\+3\\\hline 4\end{array}$	$\begin{array}{r}4\\-1\\\hline 3\end{array}$	$\begin{array}{r}4\\-3\\\hline 1\end{array}$
$\begin{array}{r}0\\+7\\\hline 7\end{array}$	$\begin{array}{r}7\\+0\\\hline 7\end{array}$	$\begin{array}{r}7\\-7\\\hline 0\end{array}$	$\begin{array}{r}7\\-0\\\hline 7\end{array}$	$\begin{array}{r}2\\+4\\\hline 6\end{array}$	$\begin{array}{r}4\\+2\\\hline 6\end{array}$	$\begin{array}{r}6\\-4\\\hline 2\end{array}$	$\begin{array}{r}6\\-2\\\hline 4\end{array}$
$\begin{array}{r}4\\+3\\\hline 7\end{array}$	$\begin{array}{r}3\\+4\\\hline 7\end{array}$	$\begin{array}{r}7\\-3\\\hline 4\end{array}$	$\begin{array}{r}7\\-4\\\hline 3\end{array}$	$\begin{array}{r}3\\+0\\\hline 3\end{array}$	$\begin{array}{r}0\\+3\\\hline 3\end{array}$	$\begin{array}{r}3\\-0\\\hline 3\end{array}$	$\begin{array}{r}3\\-3\\\hline 0\end{array}$
$\begin{array}{r}3\\+2\\\hline 5\end{array}$	$\begin{array}{r}2\\+3\\\hline 5\end{array}$	$\begin{array}{r}5\\-2\\\hline 3\end{array}$	$\begin{array}{r}5\\-3\\\hline 2\end{array}$	$\begin{array}{r}7\\+1\\\hline 8\end{array}$	$\begin{array}{r}1\\+7\\\hline 8\end{array}$	$\begin{array}{r}8\\-1\\\hline 7\end{array}$	$\begin{array}{r}8\\-7\\\hline 1\end{array}$
$\begin{array}{r}5\\+4\\\hline 9\end{array}$	$\begin{array}{r}4\\+5\\\hline 9\end{array}$	$\begin{array}{r}9\\-4\\\hline 5\end{array}$	$\begin{array}{r}9\\-5\\\hline 4\end{array}$	$\begin{array}{r}3\\+6\\\hline 9\end{array}$	$\begin{array}{r}6\\+3\\\hline 9\end{array}$	$\begin{array}{r}9\\-6\\\hline 3\end{array}$	$\begin{array}{r}9\\-3\\\hline 6\end{array}$

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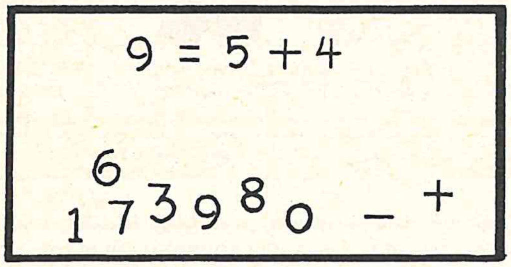
▶ Place the numeral 9 followed by the equal sign in the center of the flannel board. Under these symbols scatter a plus sign, a minus sign, and the numerals 0 through 9. Ask a child to use them to construct a numeral representing the number nine.



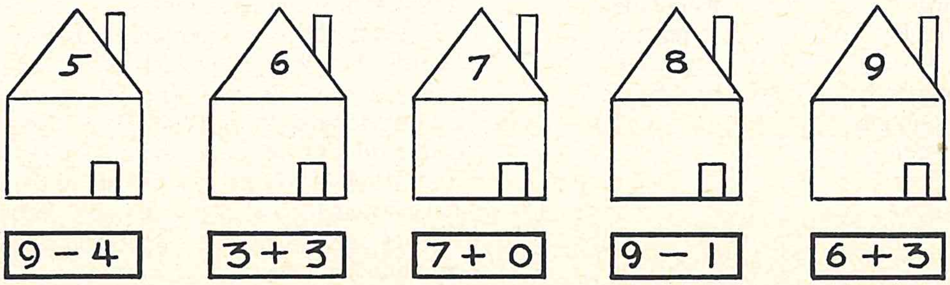
He is to place it to the right of the equal sign and read his equation. The activity allows the children to construct equations using different numerals for the numbers one through ten.

▶ Use work page S-21 to reinforce for children the meaning of the word "equals." Tell the children that they are to complete all the equations on the page. The equations on the left are designed to help the children take advantage of the concept of commutativity. If they are aware of this concept, then computation will not be necessary for the second equation in each pair in the left column of problems.

Work page S-22 provides practice with addition and subtraction combinations in vertical form. Again, the child's awareness of the commutative property will assist him in arriving at answers rapidly.

Supplemental Experiences

▶ To play a "Mailman" game, place five paper houses on the flannel board. Place one of the numerals 5 through 9 on the roof of each house. Distribute cards representing all possible addition and subtraction combinations related to sums of five through nine. Tell the children that they are to act as mailmen, and when their turn comes, they must deliver their letter to the correct address. The number represented on the house must match the number represented on their card. Tell them that the numerals may look different but the numbers are the same. The mailman must place his letter under the corresponding house.



Addition—Associative or Grouping Property

Work Pages S-23 and S-24

Objectives

To review addition with three addends.

To review the concept of the associative or grouping property.

Fundamentals

Addition is an operation on two numbers; it is a binary operation. When the child encounters expressions like 2 + 1 + 3, he can add only a pair of numbers at a time. He must decide whether to first add one and two and then add three to this sum, or whether to first add three to one and then add this sum to the two. In other words, shall 2 + 1 + 3 be interpreted as (2 + 1) + 3 or as 2 + (1 + 3)? The child will learn from experience that the sum is the same in either case.

The child must have a clear understanding of the associative property in order to work easily with sums greater than ten. The use of this property is important when working with such problems as 12 + 3 and 7 + 5.

In the first problem the child will be asked to think of 12 + 3 as (10 + 2) + 3, as 10 + (2 + 3), as 10 + 5, and finally as 15. In the second problem he will be asked to think of 7 + 5 as 7 + (3 + 2), as (7 + 3) + 2, as 10 + 2, and finally as 12. In preparing the child to work with sums greater than ten, the associative property will be reviewed and used in the simplest setting.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Ability to join two sets to make a new set.
Ability to add a pair of numbers.
Ability to work with placeholder equations.

Developmental Experiences

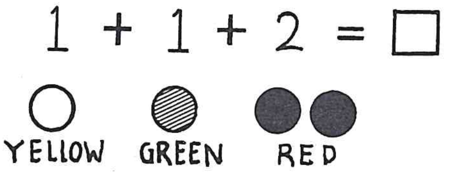
MATERIALS

flannel board
felt numerals 1 through 9
felt operation and relation symbols
felt placeholder
felt discs
tagboard cards
cardboard
clothespins

VOCABULARY

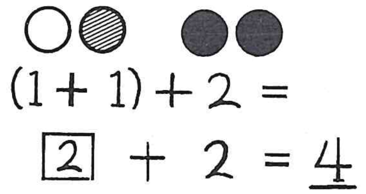
associative property
grouping
parentheses

▶ Place a set of one yellow disk, a set of one green disk, and a set of two red disks on the flannel board. Tell the children that they are to join the sets without changing the order of any of the sets. Explain that only neighboring pairs of sets can be joined according to the rules that will be used in this game, and that they must join sets from right to left. Place the equation 1 + 1 + 2 = ☐ above the sets on the flannel board.



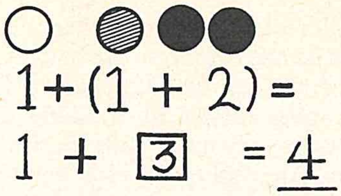
The first child called on to join pairs of sets may put the green with the yellow set. Ask the class to tell how many are in the joined set, then remind the child that he still has another pair of sets to join. As he puts the red set with the combined yellow-green set, ask the class to tell how many are in the new set formed in the second joining. Have the class tell the story of set: "A set of one was joined to a set of one to make a set of two; the set of two was then joined to a set of two to make a set of four."

Place a card reading (1 + 1) + 2 = ☐ under the union of sets. Enclose the 1 + 1 set in red *parentheses,* telling the children that the parentheses will help them think which sets were joined first. Read the card for the children, pointing to the 1 + 1 inside the parentheses and explaining that the numbers these numerals represent will be added first and then two will be added to their sum. Say, "One plus one equals two; two plus two equals four."



Place a card with the equation ☐ + 2 = _____ beneath the other card, and write the placeholder on this card in red. Talk about the equations so the children come to see that the numeral which they must put in the placeholder represents the same number as the numeral inside the red parentheses. Ask a child to write the correct numeral in the placeholder; have another child write the sum on the line at the end of the equation.

Moving the two equation cards to the left of the board, place the yellow disk, the green disk, and the two red disks in their original positions. Now ask a child to join the sets a different way. According to the rules of the game, there is only one possibility: first the child will join the red set to the green set and tell how many are in this new set. Then he will join the set of three to the set of one, stating that four are in this joined set.

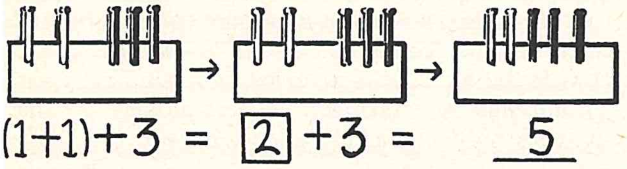


After the complete story is told, place a numeral card with the equation 1 + (1 + 2) = ☐ under the set of four. This time the numeral 1 + 2 is inside red parentheses. Read the card for the children, and tell them that first they are to think of the addition of the two numbers represented by the numerals inside the red parentheses: 1 + 2 = 3. Then the number three will be added to one to get the sum of four. Now place a second card with the equation 1 + ☐ = ______ under the first numeral card. (The placeholder is in color.) Emphasize that the steps involved in thinking this problem through are on these two cards. Tell the children that they must think of the number represented by the numerals inside the red parentheses on the first card. Have a child put the numeral for this number in the red placeholder, and ask another child to write the sum on the line at the end of the equation.

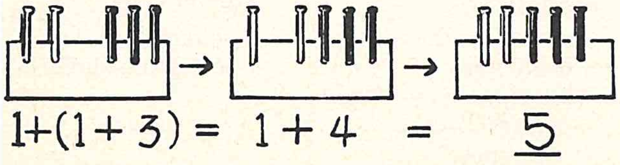
Ask the class to give the answer to the equation 1 + 1 + 2 = ☐. Have one of the children place the numeral 4 in the placeholder. You will want to remind the group that although they grouped the sets and added the numbers in different order, the answer was still four.

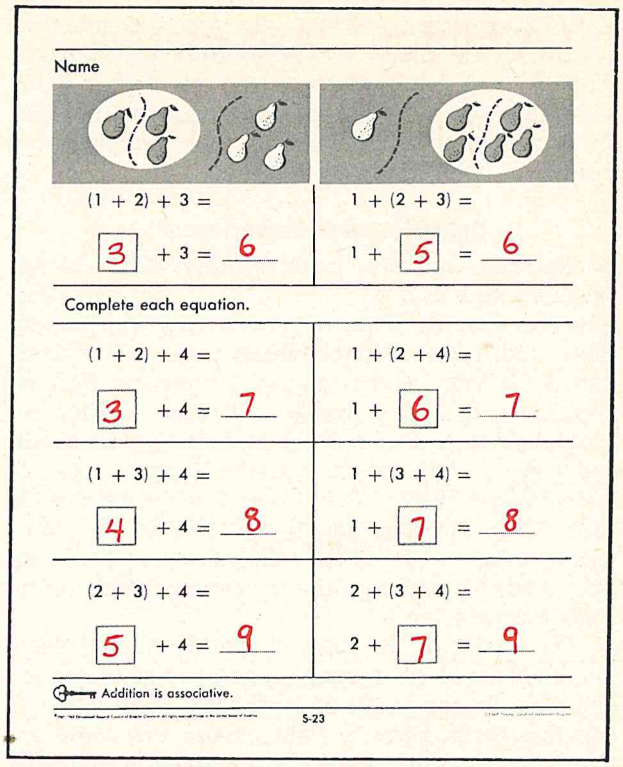
Repeat the procedure using sets of one, two, and three. The children fill in the placeholders and complete each equation by writing the sum on the line at the end of the equation.

▶ Place one yellow clothespin, one green, and three red clothespins on a strip of cardboard. Inform the children that they are to join the sets without changing the order of any of the sets. Call on a child to show the class how he would join the sets. He may decide to group the green clothespin with the yellow one, and then join the set of red clothespins to this joined set. Have him tell the other children the number of pins he has in the final union of sets.



Write the related number sentence on the chalkboard, using red parentheses to show the numerals which were added first. Have children fill in the placeholder, complete the equation by writing the sum on the line at the end of the equation, and read the sentence aloud.





Name

(1 + 2) + 3 =	1 + (2 + 3) =
$\boxed{3}$ + 3 = 6	1 + $\boxed{5}$ = 6

Complete each equation.

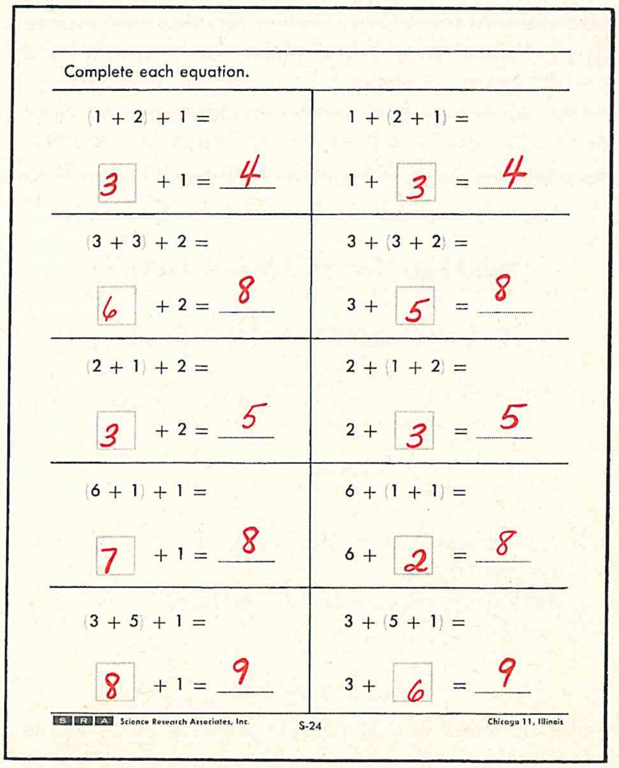
(1 + 2) + 4 =	1 + (2 + 4) =
$\boxed{3}$ + 4 = 7	1 + $\boxed{6}$ = 7
(1 + 3) + 4 =	1 + (3 + 4) =
$\boxed{4}$ + 4 = 8	1 + $\boxed{7}$ = 8
(2 + 3) + 4 =	2 + (3 + 4) =
$\boxed{5}$ + 4 = 9	2 + $\boxed{7}$ = 9

Addition is associative.

S-23

Have another child group the sets differently to see what happens to the final result. Write the related number sentence on the chalkboard, again using red parentheses to show which numbers were added first. Ask the children to fill in the placeholder and complete the equation by writing the sum.

Lead the class to conclude that the way the sets are grouped does not affect the final result. They should also discover that the numbers can be associated in different ways without changing the sum.



Complete each equation.

(1 + 2) + 1 =	1 + (2 + 1) =
$\boxed{3}$ + 1 = 4	1 + $\boxed{3}$ = 4
(3 + 3) + 2 =	3 + (3 + 2) =
$\boxed{6}$ + 2 = 8	3 + $\boxed{5}$ = 8
(2 + 1) + 2 =	2 + (1 + 2) =
$\boxed{3}$ + 2 = 5	2 + $\boxed{3}$ = 5
(6 + 1) + 1 =	6 + (1 + 1) =
$\boxed{7}$ + 1 = 8	6 + $\boxed{2}$ = 8
(3 + 5) + 1 =	3 + (5 + 1) =
$\boxed{8}$ + 1 = 9	3 + $\boxed{6}$ = 9

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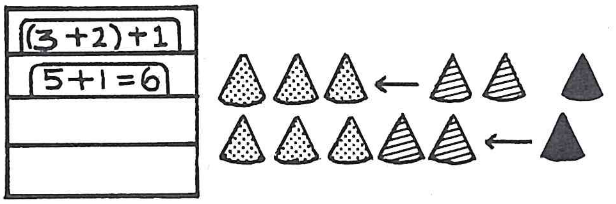
▶ Use work pages S-23 and S-24 to provide practice in working with the associative or grouping property. Work the first two problems at the top of page S-23 with the children, then encourage them to complete the remainder of the problems on these two pages by themselves. Give help only if needed.

Supplemental Experiences

▶ Distribute numeral cards bearing three-addend equations such as $2 + (3 + 4) = \square$ that represent sums less than ten. Place the numeral 7 in the pocket chart, and direct all the children with cards whose sum is the same as the number represented by this numeral to raise their hands. Ask these children to bring their cards forward one at a time, place them under the 7 and read the equation in two steps. If a child's card shows $(2 + 1) + 4 = \square$, he should read, "Two plus one equals three; three plus four equals seven." After all the sums of seven have been exhausted, the activity can be continued with other sums less than ten.

▶ On a table at the front of the classroom, place in order a set of three orange cones, two green cones, and a black cone, made of cardboard. From a set of equation cards, have a child choose the cards he thinks belong with a particular grouping of sets that he can envision, and place the cards in the pocket chart. Have him determine which number sentences correspond to the activity. When the alternate grouping of these cones takes place, have the related number sentences placed next to the first pair of sentences; in summary have the children read both pairs of number sentences aloud. Repeat the activity, this time placing in order sets of three, one, and two.



Addition—Associative or Grouping Property

Work Pages S-25 and S-26

Objectives

To re-emphasize the properties of commutativity and associativity.

To review the process of "scrambling."

To review column addition.

Fundamentals

The child will review column addition as he works with three addends whose sums are not greater than nine. He will recall that when doing column addition, he can start at the top or at the bottom. It is important for the teacher to help the child realize that getting the same answer when adding from the top, or the bottom, involves both commutativity and associativity. Analysis of a typical three-addend addition will illustrate this point. Since $(4 + 3) + 2$

$$\begin{aligned}(4+3)+2 &= 2+(4+3) && \text{interchanging } (4+3) \text{ and } 2 \text{ by the commutative property}\\ &= 2+(3+4) && \text{interchanging } 4 \text{ and } 3 \text{ by the commutative property}\\ &= (2+3)+4 && \text{regrouping by the associative property}\end{aligned}$$

$$\begin{array}{r}4\\3\\+2\\\hline\end{array}$$

corresponds to adding from the top in the problem and $(2 + 3) + 4$ corresponds to adding from the bottom in the same problem, it has been shown without computation and only by means of the commutative and associative property that adding from the top and adding from the bottom give the same answer.

By combining the properties of commutativity and associativity, the child learns that he may order and group the addends of an addition problem in any way he pleases. Some ways are better than others, especially when working with sums greater than ten, and the teacher will want to point this out to the child at the appropriate time. The process of combining commutativity and associativity is referred to as "scrambling."

At this stage, it is more important for the child to understand the commutative and associative properties as the basic ideas involved in scrambling, than it is for him to remember the terminology. As he continues his work in arithmetic, he will use these important properties again and again. Through constant use of and exposure to them, the child will gain respect for their power.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Ability to join two sets to make a new set.

Ability to add a pair of numbers.

Ability to work with placeholder equations and vertical notation.

Developmental Experiences

MATERIALS

strip of cardboard
clothespins
individual Ten-frames
numeral cards 0 through 9

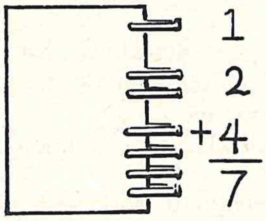
VOCABULARY

scrambling
column addition

To three children give numeral cards representing numbers whose sum is not greater than nine, and have them stand in a line in front of the class. Ask the class to add the numbers as the numerals are read from left to right. Place the related equation on the flannel board, reading it aloud, "Three plus five equals eight; eight plus one equals nine."

Ask the class to decide on a different arrangement of these numbers; the children with the cards change places accordingly. Have a child place the equation related to this new grouping on the flannel board and read it to the class. Continue the activity until the class discovers all the possible combinations for these three addends. [In this example, as with all other cases of three different addends, there are six possible combinations, exclusive of regroupings: (3 + 5) + 1, (3 + 1) + 5, (5 + 3) + 1, (5 + 1) + 3, (1 + 3) + 5, and (1 + 5) + 3. In cases such as (2 + 4) + 2 with only two different addends, there are only three possible combinations: (2 + 4) + 2, (2 + 2) + 4, and (4 + 2) + 2.] Then change the numerals and call on three other children to begin the activity again.

▶ Place one green, two yellow, and four red clothespins on a strip of cardboard. Have the children read the card from top to bottom and place the related *column addition* on the chalkboard. Then have the card and the column addition read from bottom to top. The children will discover that the sum in both cases is seven. Ask, "What will happen if we change the order of the four pins and the two pins?" Let the children try it and see; and have them give the number combinations involved in this situation.



Guide the class discussion to the point where the children conclude that it is possible to change the order of the addends and group the sets, in any way they like, without changing the sum. It is possible to *scramble* the addends in column addition problems.

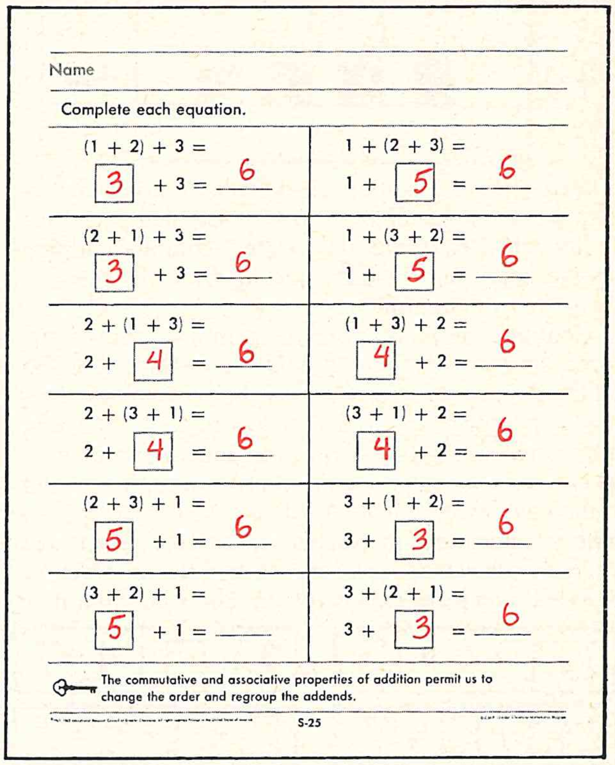
▶ Have the children use Ten-frames to reinforce this concept of scrambling. Have them place an eight-bar in the lower-left corner of the frame. Allow a child to choose any three bars that have the same total number of squares as the eight-bar and place them next to the eight-bar. He can direct the other children in reproducing this arrangement in their frames.

Perhaps he has chosen the three-bar, the four-bar, and the one-bar in that order from top to bottom. Call on another child to give the related number combinations and sums involved in a top-to-bottom and a bottom-to-top reading. Ask if someone else would like to scramble his three number bars and tell the class the number combinations and sums.

Continue until all possible addition combinations of eight are discovered and the related number combinations are read. Follow the same procedure beginning with the six-bar, seven-bar, or nine-bar.

▶ Use work page S-25 to provide practice for the children in working with the associative and commutative properties. Work the first two problems with the class and then encourage them to complete the remainder of the work on the page by themselves. Then discuss each equation in relation to the one above, the one below, and the one beside it.

Work page S-26 provides problems which stress column addition. Work the problems in the first row and discuss the different order of the addends with the children. Then ask them to work independently and complete the remainder of the column additions.



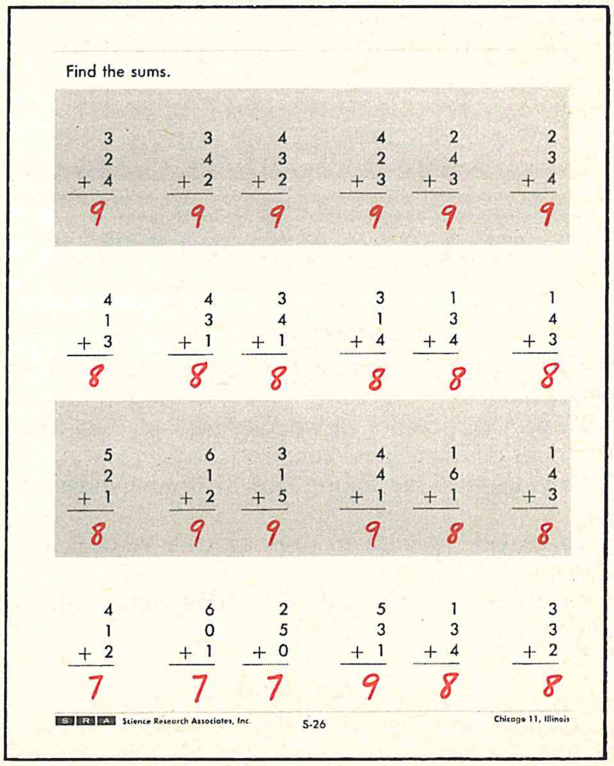
Name

Complete each equation.

(1 + 2) + 3 = $\boxed{3}$ + 3 = 6	1 + (2 + 3) = 1 + $\boxed{5}$ = 6
(2 + 1) + 3 = $\boxed{3}$ + 3 = 6	1 + (3 + 2) = 1 + $\boxed{5}$ = 6
2 + (1 + 3) = 2 + $\boxed{4}$ = 6	(1 + 3) + 2 = $\boxed{4}$ + 2 = 6
2 + (3 + 1) = 2 + $\boxed{4}$ = 6	(3 + 1) + 2 = $\boxed{4}$ + 2 = 6
(2 + 3) + 1 = $\boxed{5}$ + 1 = 6	3 + (1 + 2) = 3 + $\boxed{3}$ = 6
(3 + 2) + 1 = $\boxed{5}$ + 1 = ____	3 + (2 + 1) = 3 + $\boxed{3}$ = 6

The commutative and associative properties of addition permit us to change the order and regroup the addends.

S-25



Find the sums.

3 2 + 4 9	3 4 + 2 9	4 3 + 2 9	4 2 + 3 9	2 4 + 3 9	2 3 + 4 9
4 1 + 3 8	4 3 + 1 8	3 4 + 1 8	3 1 + 4 8	1 3 + 4 8	1 4 + 3 8
5 2 + 1 8	6 1 + 2 9	3 1 + 5 9	4 4 + 1 9	1 6 + 1 8	1 4 + 3 8
4 1 + 2 7	6 0 + 1 7	2 5 + 0 7	5 3 + 1 9	1 3 + 4 8	3 3 + 2 8

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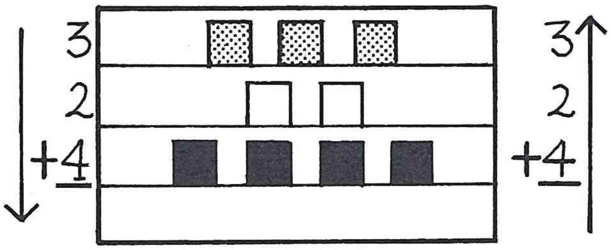
Supplemental Experiences

▶ Place three strips of green, two strips of yellow, and four strips of red paper in the pocket chart. Have children read the grouping of sets as seen from top to bottom. Have a child write the related column addition on the chalkboard. Have the chart and the related column addition read from bottom to top. Change the order of the top two sets and have one of the children write the related column addition on the board and read it, starting from the top and then from the bottom.



Continue until all possible combinations of the three sets are discovered and the related column additions have been written on the chalkboard. Follow the procedure with other sets and numbers, always restricting the sum of the addends to nine.

▶ Prepare number cards illustrating placeholder equations involving three addends with the placeholder appearing in various positions. Have the children view the cards one at a time in order to read and complete each equation. Some children may find it difficult to work with these equations; others will find a real challenge in them.

$\boxed{2+1+4=\square}$ $\boxed{2+\square+4=7}$

$\boxed{2+1+\square=7}$ $\boxed{\square+1+4=7}$

Addition—Subtraction

COMBINATIONS TO TEN

Work Pages S-27 and S-28

Objectives

To emphasize the concept of equality in addition equations.

To provide practice in working with placeholders in various positions in addition equations.

To emphasize the importance of combinations of ten.

To provide practice in working with vertical notation.

To review addition and subtraction combinations to ten.

Fundamentals

Addition combinations of ten are most important. As the teacher looks ahead, she becomes aware that in a problem such as $8 + 5 = \square$, the child must first think about the number needed to reach ten and then go on from there.

Ten is the base of our numeration system. In counting, whenever the child reaches ten (or a multiple of ten) he notices when he arrives at this point and begins his count all over again starting with one. However, the count will now be preceded by ten (or a multiple of ten) each time.

In the problem $8 + 5 = \square$, the child will count to ten and then go three steps beyond. He thinks 8, 9, 10, 10 + 1, 10 + 2, 10 + 3, or in rapid calculation 8 + 2, 10, 10 + 3, 13.

Combinations of ten are equally important when working with subtraction. In solving problems such as 13 − 5, the child will be learning to think (13 − 3) − 2 or 10 − 2. Knowledge of the subtraction combinations of ten is required in the second thinking step of this problem. Mastery of the combinations of ten will prepare the child to take this step.

These ideas emphasize one point: the child will need to use combinations of ten when working with single digit addends whose sums are greater than ten. Between now and the time these harder combinations are developed, the child should become familiar with the combinations of ten.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Understanding of the concepts of addition and subtraction.

Knowledge of addition and subtraction combinations.

Ability to work with placeholder equations and vertical notation.

Developmental Experiences

MATERIALS

chart paper
gummed stickers
crayon
numeral cards 0 through 9
tagboard cards

▶ Draw a grid of 1½-inch blocks on chart paper. Make eleven rows of ten blocks each. Have a child put a set of ten red gummed circles in the first row of the chart and have the set story told. With crayon write the related number combination on the chart to the right of the joined sets.

Have a second child place a set of nine red circles in the next row. Have him put one green circle next to the nine red circles and tell the set story. Have the related addition equation written to the right of the joined set. Continue in this way to develop other addition combinations of ten. When the chart is completed, have the commuted forms of each equation pointed out.

Ask the children if they notice anything about the pattern of the pairs of numbers. Discuss the pattern of the two addends in the equations: with the sum the same, each time one addend increases, the other decreases.

10 − 0 = 10	●	●	●	●	●	●	●	●	●	●	10 + 0 = 10
10 − 1 = 9	●	●	●	●	●	●	●	●	●	○	9 + 1 = 10
10 − 2 = 8	●	●	●	●	●	●	●	●	○	○	8 + 2 = 10
10 − 3 = 7	●	●	●	●	●	●	●	○	○	○	7 + 3 = 10
10 − 4 = 6	●	●	●	●	●	●	○	○	○	○	6 + 4 = 10
10 − 5 = 5	●	●	●	●	●	○	○	○	○	○	5 + 5 = 10
10 − 6 = 4	●	●	●	●	○	○	○	○	○	○	4 + 6 = 10
10 − 7 = 3	●	●	●	○	○	○	○	○	○	○	3 + 7 = 10
10 − 8 = 2	●	●	○	○	○	○	○	○	○	○	2 + 8 = 10
10 − 9 = 1	●	○	○	○	○	○	○	○	○	○	1 + 9 = 10
10 − 10 = 0	○	○	○	○	○	○	○	○	○	○	0 + 10 = 10

Return to the first addition combination, 10 + 0 = 10. Say that you will remove zero objects from the set of ten. Ask how many objects remain in the set. Write the related subtraction equation to the left of the first row of objects.

Move on to the second row of objects. Have the addition combination given. Cover the subset of one within the set and tell the children that they must pretend this part of the set is being removed. Have them tell the set story and have the related subtraction equation given. Write this equation on the chart.

Continue in this way to develop the other subtraction combinations related to the sum of ten. When the last combination has been written, ask the children if they can find the commuted form for each subtraction equation. Help them conclude that such an equation does not exist—subtraction is not commutative.

Discuss the subtraction patterns evident in the equations. When the sum remains constant and the number subtracted increases by one, the difference decreases by the same number.

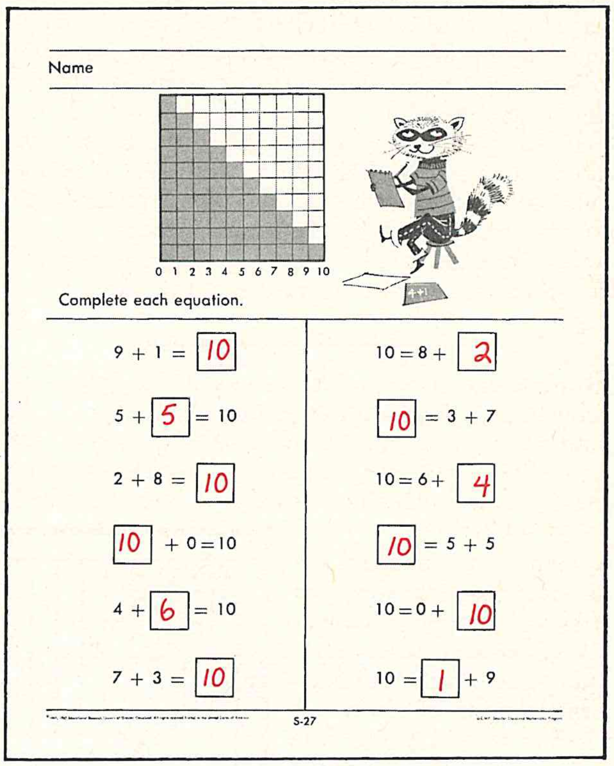
▶ Plan a quick oral review of addition combinations of ten. Tell the children that you are going to play a game with them. A child must answer when his name is called and give a different numeral for whatever number you mention. Use ten most of the time. Say, "We will play a game today, Jack must name number ten another way."

▶ Construct a set of flash cards to review all possible combinations of ten. Write a variety of placeholder equations on the cards. Display them, one at a time, and have the children read the completed equations.

▶ Write a series of true and untrue mathematical sentences on cards and place each sentence in a box. Call upon a child to reach into the box and choose a card. Have him show his card to the class, read the number sentence on the card, and tell whether or not the sentence is true. If it is not true, the child must choose from a set of numeral cards 0 through 9 one which can be placed in the equation to make it a true statement. Have him reread the equation.

▶ Work page S-27 provides an opportunity to complete placeholder equations related to sums of ten. Work the first problem in each column with the children. Encourage them to complete the rest of the page independently. When the work page is completed, have children discuss the picture of the Ten-frame at the top of the page to summarize the addition combinations of ten.

Work page S-28 provides practice with combina-



Name

Complete each equation.

9 + 1 = 10

5 + 5 = 10

2 + 8 = 10

10 + 0 = 10

4 + 6 = 10

7 + 3 = 10

10 = 8 + 2

10 = 3 + 7

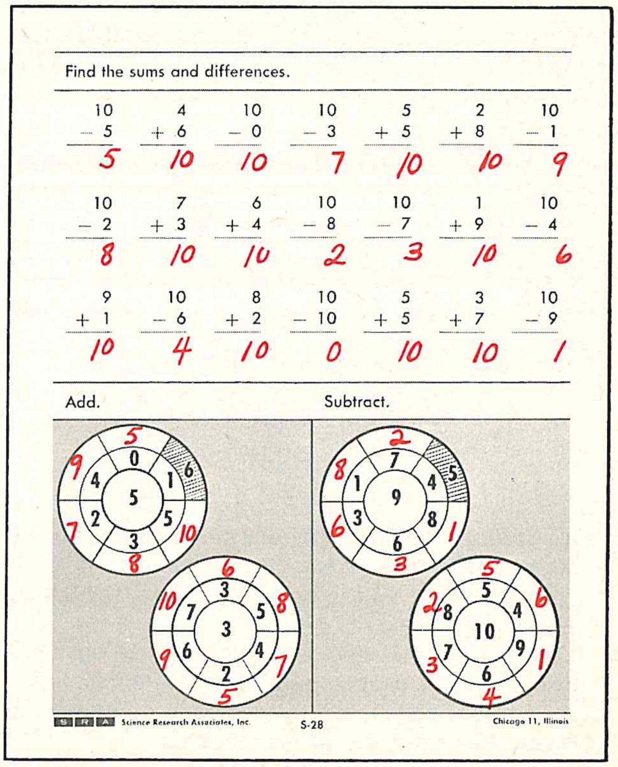
10 = 6 + 4

10 = 5 + 5

10 = 0 + 10

10 = 1 + 9

S-27



Find the sums and differences.

10	4	10	10	5	2	10
− 5	+ 6	− 0	− 3	+ 5	+ 8	− 1
5	10	10	7	10	10	9
10	7	6	10	10	1	10
− 2	+ 3	+ 4	− 8	− 7	+ 9	− 4
8	10	10	2	3	10	6
9	10	8	10	5	3	10
+ 1	− 6	+ 2	− 10	+ 5	+ 7	− 9
10	4	10	0	10	10	1

Add.

Subtract.

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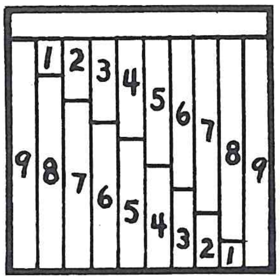
tions whose sums are not greater than ten. It gives children an opportunity to work with the vertical-notation form. Tell the children to write the sums or differences for each of the number combinations in the first three rows.

The number wheels in the lower section of the page can be used to review addition and subtraction combinations with sums not greater than ten. In the two wheels on the left of the page, have children add the number represented by the numeral in the center of the wheel to each of the numbers represented by the other numerals in the wheel. Tell them to write the answers in the spaces around the outer edge of the wheel.

In the two wheels on the right of the page, have the children subtract the number represented by the numeral in the center of the wheel minus each number represented by the other numerals in the wheel. Tell the children to write their answers in the spaces around the outer edge of the wheel.

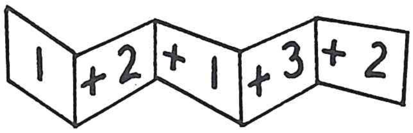
Supplemental Experiences

▶ Have the children use individual Ten-frames to show number combinations of nine. Have them begin with the nine-bar in the lower left corner of the frame and continue to build the nine-block by placing the bars in the frame in the pattern illustrated.

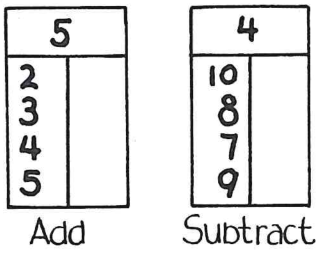


Perhaps some children will discover that there are ten possible addition combinations of nine, including zero, and eight possible addition combinations of nine, excluding zero. Try this idea out with other number blocks. Pose pertinent questions: Does the eight-block have nine possible combinations if zero is included, and seven possible addition combinations if zero is not included?

▶ Design a set of accordion cards. Place numerals and plus signs on the cards as illustrated. Fold the card in such a way that only one section at a time is exposed to view. Begin with the number represented by the numeral in the first section of the card. Expose it to view for a few seconds. Expose the other sections of the card, one at a time, until all additions have been completed. Call upon a child to give the sum. Continue in this way, presenting a series of additions.



Design a second set of accordion folding cards with numerals and operational signs. Combine addition and subtraction. Do not use any combinations that result in a number greater than ten.



▶ To provide additional review for children who are progressing more slowly in their mastery of combinations related to sums not greater than ten, design a series of miniature addition and subtraction tables on tagboard cards. Tell the children that they are to add (or subtract) the number at the top to (or from) each of the other numbers in the table. Have the children place their plastic numerals in the spaces for the answers.

Story Problems

Work Pages S-29 and S-30

Objectives

To extend the children's understanding of addition and subtraction through practice with story problems.

To develop the think-through procedure involved in comprehension of story problems.

Fundamentals

The children have had practice in putting the number concepts involved in personal experiences into number sentences. On work pages S-17 and S-18 they were introduced to a step-by-step approach to solving story problems. Continued practice will encourage the children to use these four steps in finding the correct answer. The teacher will want to use a variety of classroom and home experiences to help the children select the operation to fit the problem. Children need much practice in determining when to add and when to subtract.

Readiness for Understanding

Knowledge of addition and subtraction combinations through ten.

Ability to follow the four steps in the solution of story problems.

Developmental Experiences

▶ Ask children to make up a story problem about a classroom activity. Ask, for example, "Who can make up a story problem about the crayons we bought this morning?" Help the children verbalize the situation in different ways. Write their stories on the chalkboard.

We ordered 4 big boxes and 5 little boxes.
How many boxes did we buy?________
How many more little boxes than big boxes did we buy? ________

Help the children follow logical steps in solving each problem. Ask:

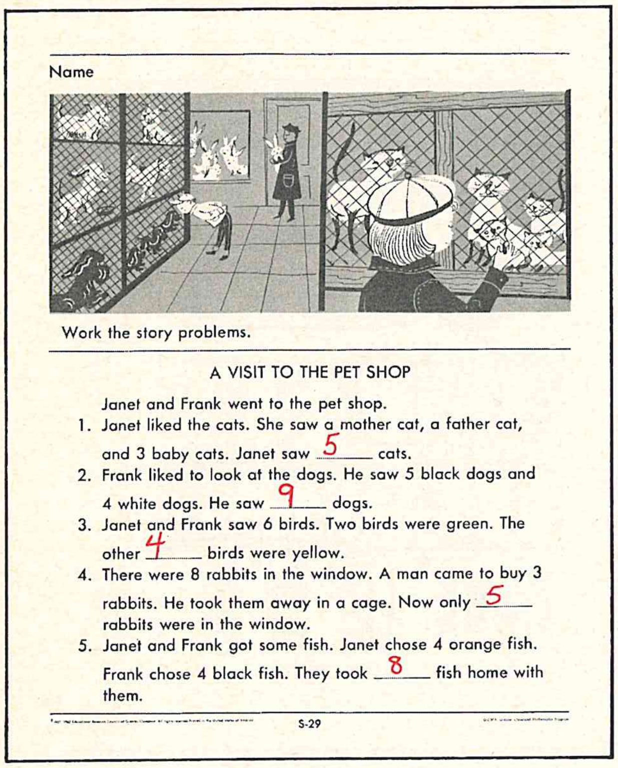
1. "What must we find out?"
2. "What facts do we know?"

3. "What operation do we use to find the sum? To find the difference?"

4. "Is that a reasonable answer?"

Have those children who bought big boxes of crayons hold them up. Have those who bought little boxes hold them up. Write related number sentences and have the children give the correct answers.

▶ Work pages S-29 and S-30 contain stories about Janet's and Frank's visit to a pet shop. Read through the stories with the children, but encourage them to find the answers independently. If any child is able to do these problems in his head, have him tell how he got the answers. Other children will be more successful if they learn to write number stories or mathematical sentences for each problem.



Name

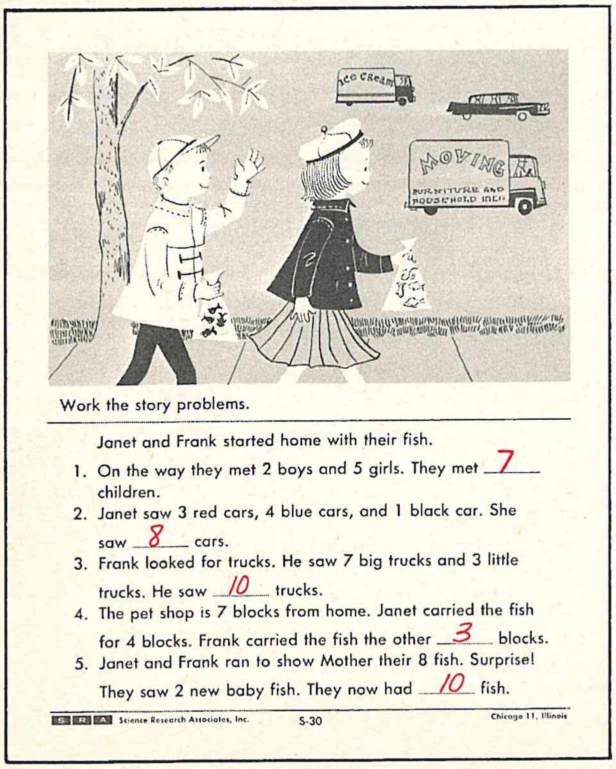
Work the story problems.

A VISIT TO THE PET SHOP

Janet and Frank went to the pet shop.

1. Janet liked the cats. She saw a mother cat, a father cat, and 3 baby cats. Janet saw 5 cats.
2. Frank liked to look at the dogs. He saw 5 black dogs and 4 white dogs. He saw 9 dogs.
3. Janet and Frank saw 6 birds. Two birds were green. The other 4 birds were yellow.
4. There were 8 rabbits in the window. A man came to buy 3 rabbits. He took them away in a cage. Now only 5 rabbits were in the window.
5. Janet and Frank got some fish. Janet chose 4 orange fish. Frank chose 4 black fish. They took 8 fish home with them.

S-29



Work the story problems.

Janet and Frank started home with their fish.

1. On the way they met 2 boys and 5 girls. They met 7 children.
2. Janet saw 3 red cars, 4 blue cars, and 1 black car. She saw 8 cars.
3. Frank looked for trucks. He saw 7 big trucks and 3 little trucks. He saw 10 trucks.
4. The pet shop is 7 blocks from home. Janet carried the fish for 4 blocks. Frank carried the fish the other 3 blocks.
5. Janet and Frank ran to show Mother their 8 fish. Surprise! They saw 2 new baby fish. They now had 10 fish.

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Supplemental Experiences

▶ Ask several children to dramatize a story problem and have the class work the problem on paper.

▶ For oral practice, place the equation $3 + \square = 7$ on the board. Have the children think of a story problem to match the equation. Repeat the activity using other equations: $\square + 7 = 9$, $\square - 7 = 2$, $3 + 4 = \square$, and so forth.

Liquid Measurement

Work Pages S-31 Through S-36

Objectives

To increase understanding of the concept of liquid measure.

To review pint and quart as standard units of liquid measure.

To introduce cup and gallon as standard units of liquid measure.

To emphasize that the symbol $\overset{m}{=}$ means "measures the same amount as."

Fundamentals

The child has had many informal experiences with the concept of liquid measure. In the first grade he was introduced to the formal concept of units of liquid measurement. He gained some understanding of pints and quarts and of their relation to each other. Now this understanding will be reinforced and extended. The cup and gallon will be introduced as units of liquid measurement, and the relation between these and other units will be investigated.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

Some understanding of liquid measure.

Awareness that pints, quarts, and gallons may be measured by containers having many different shapes.

Developmental Experiences

MATERIALS

measuring cups
pint containers
quart containers
  (different shapes)
milk bottles
Mason jars
gallon container

VOCABULARY

cup
pint
quart
half-gallon
gallon
liquid measure
standard units

▶ Ask the children to tell some of their experiences related to liquid measurement. Perhaps some of them had a lemonade stand. Others may remember how their fathers buy gasoline or oil for the car. Then lead them to think about how they can be sure, when buying ice cream, milk, and the like, that they really get the measure they want. Lead them to the conclusion that it is important to have *standard units* of *liquid measure.*

Review the *pint* and *quart* as standard units of liquid measure. Have the children bring in pint and quart containers for measuring liquid. Have them pour water from one pint container to another to see whether they both measure the same amount of water. Repeat with other pint containers and with quart containers.

▶ Provide opportunities for the children to review the relation between pint and quart. Ask them to show how many pints are needed to fill one quart container. Have them take turns pouring water from two pint bottles into one quart bottle. Write on the board "2 pints $\overset{m}{=}$ 1 quart." Remind the children that this is read "two pints measure the same amount as one quart."

Proceed to show that since 2 pints $\overset{m}{=}$ 1 quart, then 4 pints $\overset{m}{=}$ 2 quarts and 2 quarts $\overset{m}{=}$ 4 pints. Discuss the relation of pints to quarts by questions such as "Which is the greater amount, one pint or one quart? two quarts or three pints?"

▶ Use work page S-31 for further review of quart and pint. Discuss the picture at the top of the page. Direct the children's attention to the first box in the first row. Have the children tell how many quarts of ice cream are pictured here. Then tell them to mark the number of pints that measure the same amount. Finally, have them fill the blank with the correct numeral. Encourage them to complete the page independently.

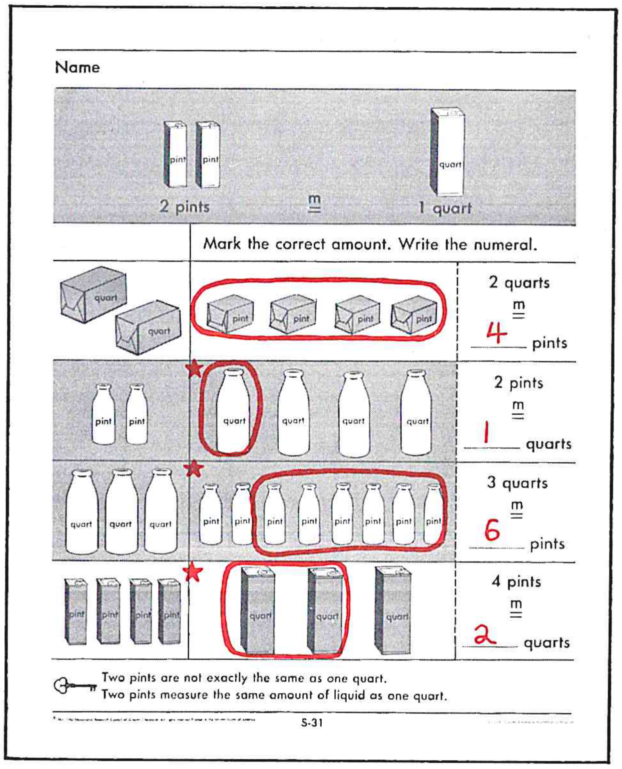
Read and discuss the story problems on work page S-32 with the children. It may help to dramatize the actual selling experiences. Then have them write the correct numerals and words in the blank spaces in each problem.

▶ Have the children use a standard eight-ounce measuring *cup* to show that it must be filled twice to fill one pint. A class demonstration in making gelatin may be helpful. Ask the children to tell of any experiences they have had in using cups, pints, and quarts. Give oral problems. Ask, for example, "How many cups of orange juice will be needed to make a pint of orange juice?" If a boy wants to share a quart of milk with three other children, how many cups will each child get?

▶ Use work page S-33 to reinforce the children's understanding of the relation between cups, pints, and quarts. Have the children circle the correct amount and write the correct numeral in the blank space in each row.

Discuss a possible arrangement for selling punch at a school picnic. Have the children tell, for example, what drinks they might want to sell, and what measuring containers would be needed. Now have the children read the story problems on work page S-34 Guide their thinking slowly through each step of the solution for the first few problems. Have them work the last few independently.

▶ Introduce the *gallon* and *half-gallon* as standard units of liquid measurement. Bring in a gallon container and a half-gallon container. Have some children fill the gallon container with quarts of water



Name

2 pints $\overset{m}{=}$ 1 quart

Mark the correct amount. Write the numeral.

2 quarts $\overset{m}{=}$ 4 pints

2 pints $\overset{m}{=}$ 1 quarts

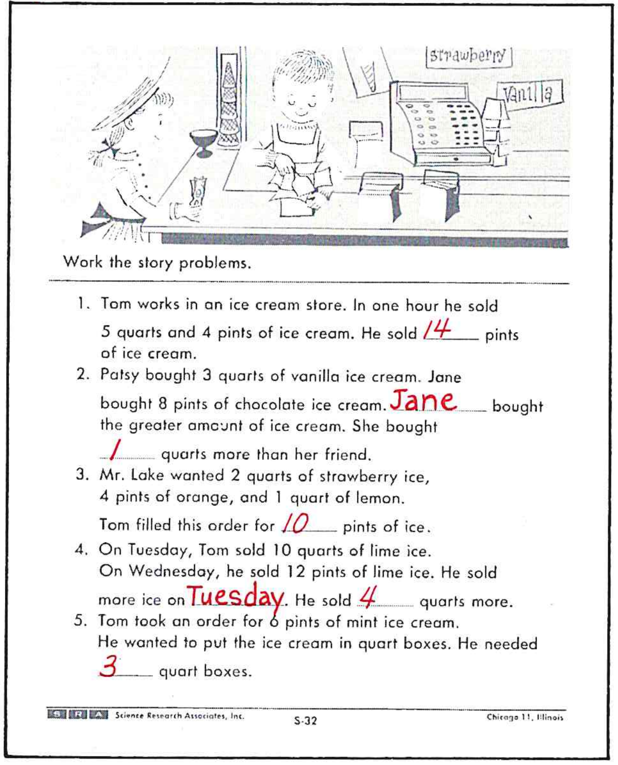
3 quarts $\overset{m}{=}$ 6 pints

4 pints $\overset{m}{=}$ 2 quarts

Two pints are not exactly the same as one quart.
Two pints measure the same amount of liquid as one quart.

S-31

★ More than one correct answer is possible for each problem.



Work the story problems.

1. Tom works in an ice cream store. In one hour he sold 5 quarts and 4 pints of ice cream. He sold 14 pints of ice cream.
2. Patsy bought 3 quarts of vanilla ice cream. Jane bought 8 pints of chocolate ice cream. Jane bought the greater amount of ice cream. She bought 1 quarts more than her friend.
3. Mr. Lake wanted 2 quarts of strawberry ice, 4 pints of orange, and 1 quart of lemon. Tom filled this order for 10 pints of ice.
4. On Tuesday, Tom sold 10 quarts of lime ice. On Wednesday, he sold 12 pints of lime ice. He sold more ice on Tuesday. He sold 4 quarts more.
5. Tom took an order for 6 pints of mint ice cream. He wanted to put the ice cream in quart boxes. He needed 3 quart boxes.

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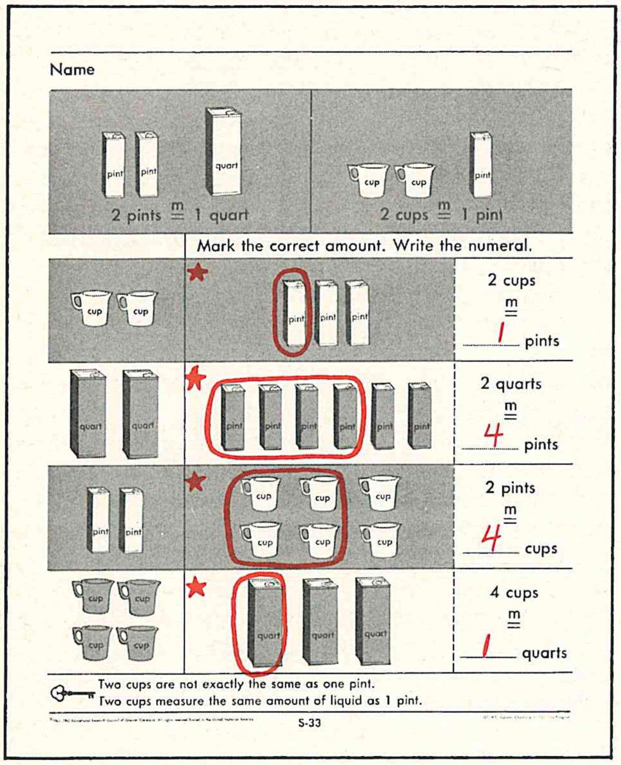
and count the number of quarts used. Write on the chalkboard, "4 quarts $\overset{m}{=}$ 1 gallon." Then pour the water from the gallon container into the two half-gallon containers. Write on the chalkboard, 1 gallon $\overset{m}{=}$ 2 half gallons. Provide many children with the opportunity to pour from two half-gallon containers into the gallon container, and so on. Lead them in the same way to conclude that 4 quarts $\overset{m}{=}$ 2 half gallons.


Name

2 pints $\overset{m}{=}$ 1 quart

2 cups $\overset{m}{=}$ 1 pint

Mark the correct amount. Write the numeral.

★ 2 cups $\overset{m}{=}$ 1 pints

★ 2 quarts $\overset{m}{=}$ 4 pints

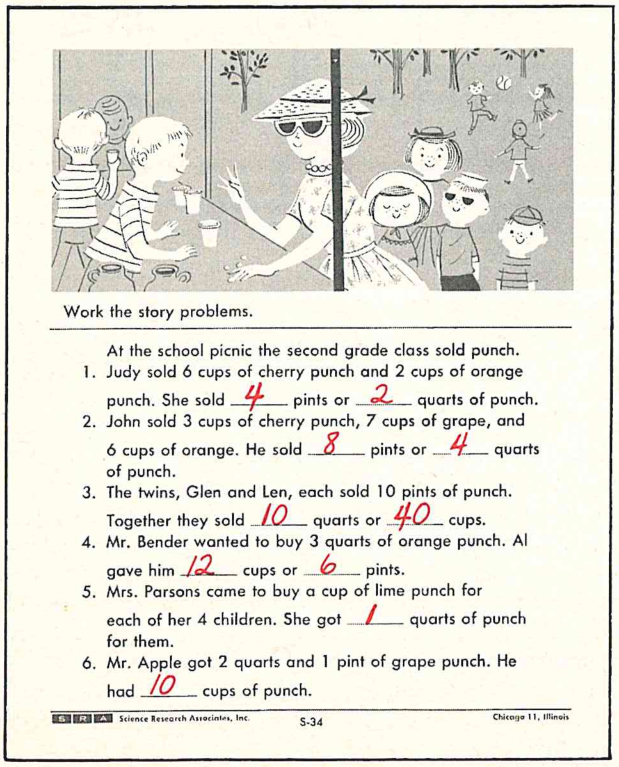
★ 2 pints $\overset{m}{=}$ 4 cups

★ 4 cups $\overset{m}{=}$ 1 quarts

Two cups are not exactly the same as one pint.
Two cups measure the same amount of liquid as 1 pint.

S-33
pint
quart
cup

★ **More than one correct answer is possible for each problem.**


Work the story problems.

At the school picnic the second grade class sold punch.

1. Judy sold 6 cups of cherry punch and 2 cups of orange punch. She sold 4 pints or 2 quarts of punch.
2. John sold 3 cups of cherry punch, 7 cups of grape, and 6 cups of orange. He sold 8 pints or 4 quarts of punch.
3. The twins, Glen and Len, each sold 10 pints of punch. Together they sold 10 quarts or 40 cups.
4. Mr. Bender wanted to buy 3 quarts of orange punch. Al gave him 12 cups or 6 pints.
5. Mrs. Parsons came to buy a cup of lime punch for each of her 4 children. She got 1 quarts of punch for them.
6. Mr. Apple got 2 quarts and 1 pint of grape punch. He had 10 cups of punch.

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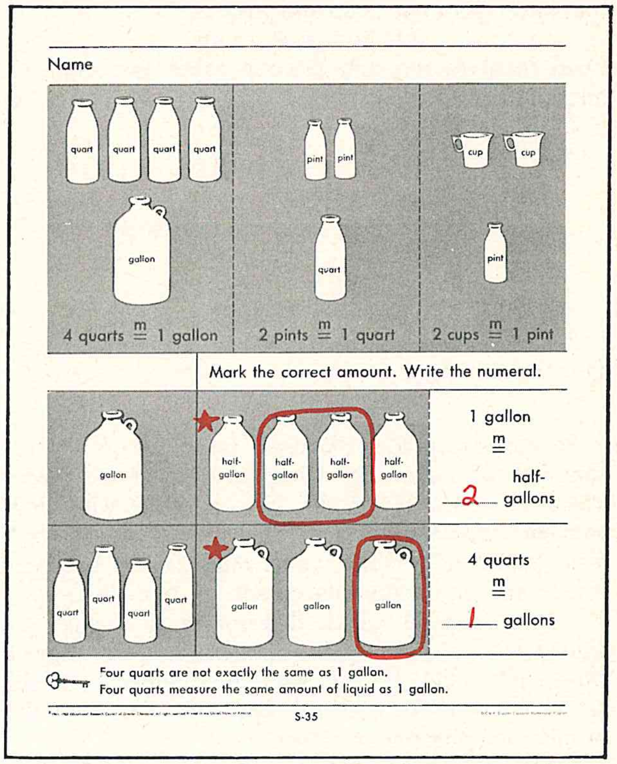
▶ Use work page S-35 to reinforce the children's understanding of gallon and half-gallon. Discuss the pictures at the top of the page and then have the children complete the page independently.

Work page S-36 gives an opportunity to evaluate the children's understanding of the relation between the various units of liquid measure. Tell the children to draw lines to match amounts of liquid. Have them work the page independently.


Name

4 quarts $\overset{m}{=}$ 1 gallon

2 pints $\overset{m}{=}$ 1 quart

2 cups $\overset{m}{=}$ 1 pint

Mark the correct amount. Write the numeral.

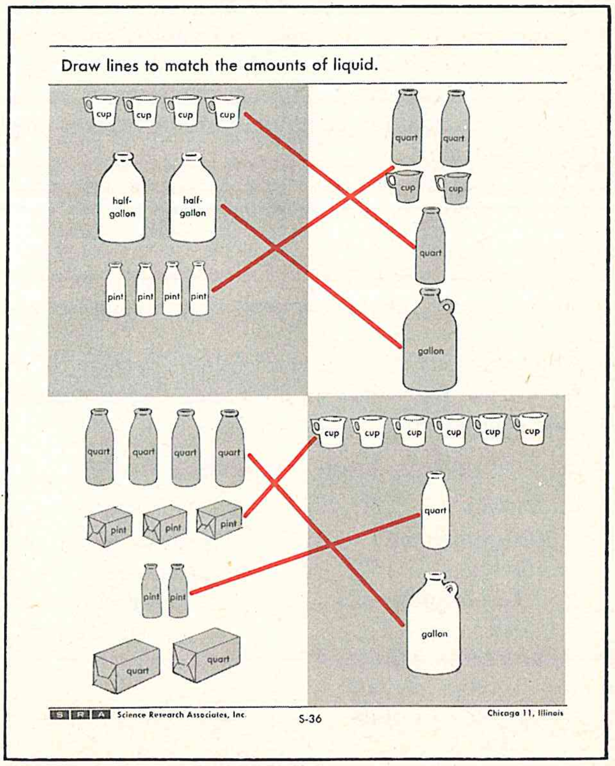
★ 1 gallon $\overset{m}{=}$ 2 half-gallons

★ 4 quarts $\overset{m}{=}$ 1 gallons

Four quarts are not exactly the same as 1 gallon.
Four quarts measure the same amount of liquid as 1 gallon.

S-35
quart
gallon
pint
cup
half-gallon

★ **More than one correct answer is possible for each problem.**


Draw lines to match the amounts of liquid.

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cup
half-gallon
pint
quart
gallon

Supplemental Experiences

▶ Have a party. Make a fruit drink to culminate learning about liquid measures. Let the children choose from among lemonade, orangeade, and various colored drinks. Help them decide on the number of cups they want to serve and help them determine how many pints, quarts, and gallons this would be.

▶ For the more able children, introduce the half-cup measure. Have them pour two half-cups into a standard eight-ounce cup and write:

2 half-cups $\overset{m}{=}$ 1 cup

Lead them to see that since it takes two cups to measure one pint, it would take four half-cups to measure one pint.

Place Value

Work Pages S-37 and S-38

Objectives

To review the concepts of place value.

To stress the concept of different numerals for the same number.

Fundamentals

In the first grade, the child is guided in the attainment of a thorough understanding of the place-value concepts. He learns it is possible with only the ten basic numerals or digits, 0 through 9, to write numerals for any whole number. He becomes aware that it is the placement of a digit in any standard numeral which determines the value assigned to this digit. In the numeral 11, the digit 1 on the left stands for ten, while the digit on the right stands for one. The numeral represents, therefore, the number ten plus one or eleven.

Often it is convenient to write a numeral for a number so as to emphasize the idea of place value. The numeral 10 + 1 is called "expanded notation" or the "expanded form" of the numeral 11. The ability to use expanded notation serves as a basis for work with addition combinations whose sums are greater than ten.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Systems of Numeration.

Readiness for Understanding

Understanding of the numbers zero through nineteen.

Ability to recognize the numerals 0 through 19.

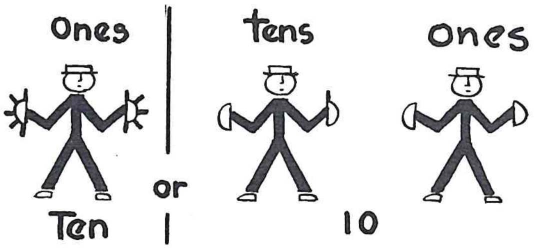
Awareness that different numerals can represent the same number.

Developmental Experiences

MATERIALS	VOCABULARY
2 Countingmen	place value
individual Ten-frames	expanded notation
books	expanded form
numeral cards, 11 through 19, and 10 + 1 through 10 + 9	

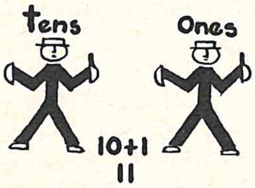
▶ Use two Countingmen to review the *place value* meaning of the numerals 10 through 19. Place a Countingman and nineteen books on a table. Tell the children that they are to count the books one by one, and that the Countingman will help them keep track of the count. Designate a child to be in charge of counting the books and another to operate the Countingman. As the one child counts each book and places it on a pile, the other child places one more finger on the Countingman.

When ten books are in the pile, the Countingman will have ten fingers. Remind the children that the Countingman can have only ten fingers, yet there are more books to count. Ask, "How can we keep track of these books and go on with our count?" With careful questioning you can lead the children to make the suggestion that they might start counting all over again. Then you will ask, "How can we keep track of the ten we already have?" Some child may suggest that he could remember the ten; another may say that another Countingman could be used to continue the count. Point out that both things can be done at the same time. Introduce the second Countingman, placing him to the left of the first Countingman. Explain that his job will be to remember or keep track of the ten that the first Countingman counted, and name him the Tens-man. Remind the class that the other man counted one at a time, so he might be called the Ones-man. Recall how many books are in the pile, then place one finger on the Tens-man and remove the ten fingers from the Ones-man, saying, "This is another way to show that we have counted ten books. The one finger on the Tens-man tells us that we have one set of ten. How many ones do we see on the Ones-man?" Write the numeral 10 and remind the children that the 1 now stands for one ten because it is in a special place; and that the numeral 0 tells that there are zero ones. Stress that it is possible to think of the books in the pile as ten sets of one, or as one set of ten, that 10 is a *place-value* numeral for the number ten.



Direct the book counter to start a new pile as he continues to count the remaining books. The child in charge of the Countingman should record each successive count on the Ones-man. Remind the children that eleven can be thought of as one ten and one one, twelve can be thought of as one ten and two ones. Have the children write the place-value numerals for the numbers eleven through nineteen on the chalkboard.

▶ Write the numeral 11 on the chalkboard; then have the children explain the meaning of the digit 1 in the ones place and the 1 in the tens place of this numeral. Ask the children to place fingers on the Countingmen to show the number which this numeral represents. Ask how many tens and how many ones there are in the number eleven, and if someone can think of a different numeral that might be used to write one ten and one one. Introduce the *expanded-notation* 10 + 1 as the numeral which shows what 11 really means.



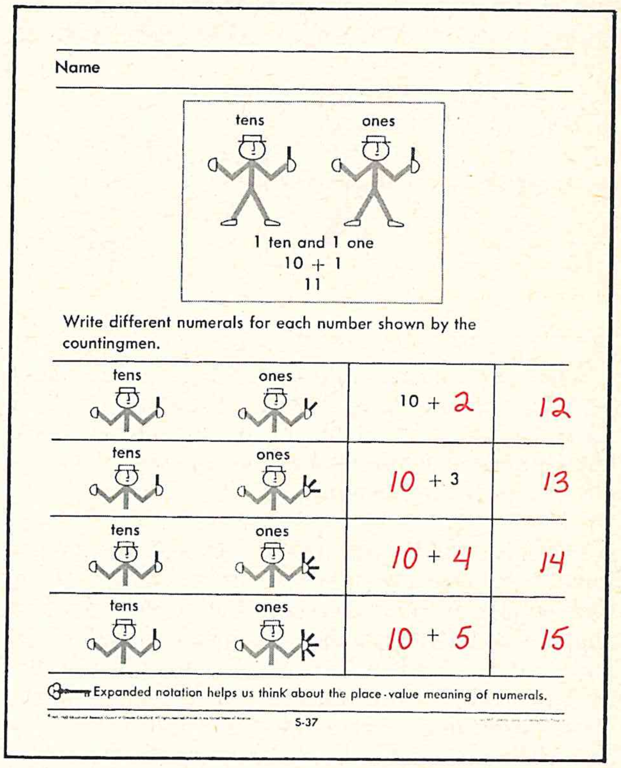
Place one more finger on the Ones-man and call on a child to tell how many tens and how many ones he now sees. Have the child write the standard place-value numeral 12 for the number twelve. Ask if some other child can write this in *expanded form.* Have 10 + 2 written on the chalkboard beside the 12. Introduce 10 + 3 as a different way of writing 13, and move ahead from 10 + 4 through 10 + 9. Relate the expanded notation to the place-value numeral each time.

▶ Have the children use their individual Ten-frames to show the place-value meaning of numbers eleven through nineteen. Direct them to place the ten-bar in the left side of the frame and then put a one-bar beside it. Ask a child to write a numeral on the chalkboard for the number of squares he has in his frame. If he writes 11, have another child write the expanded-form numeral (10 + 1) to show how these squares are grouped within the set. Repeat with other numbers through ten plus nine, or nineteen.

▶ Construct a set of cards bearing expanded-notation numerals for the numbers eleven through nineteen, and a set bearing the standard place-value numerals 11 through 19. Distribute these to the children; call on a child to stand, display his numeral, and say, "I'm looking for a partner." The child who has a different numeral for the number represented by the first child's numeral must stand and read his card. Place the two cards with an equal sign between them in the pocket chart. Have a child read the equation that is formed. Continue the game by calling for other pairs of numerals representing other numbers.

▶ Use work page S-37 to provide an opportunity for the children to write different numerals for a number. Talk about the picture of the Countingmen at the top of the page. Have the class read the different numerals for the number eleven which the Countingmen represent. Direct their attention to the lower portion of the page. Tell them that they are to think of the number represented by the Countingmen in the first box in each row and then to write different numerals for this number. In the second box they are to complete the numeral in expanded form, and in the last box they are to write the place-value numeral for the number.

Work page S-38 provides additional practice in writing different numerals for a number. Have the children complete each equation in the first column by writing the standard place-value numeral in the placeholder; have them complete each equation in the second column to show the expanded-notation numeral for each number.



Name

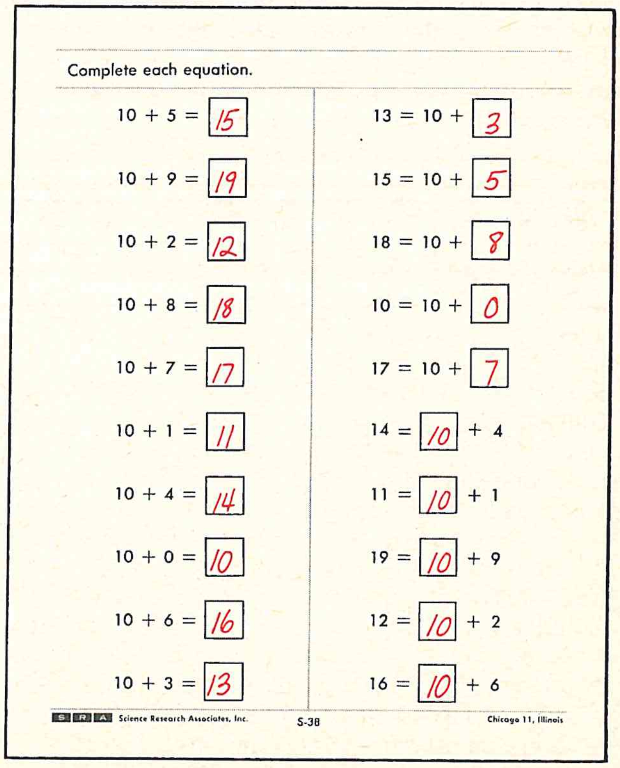
1 ten and 1 one
10 + 1
11

Write different numerals for each number shown by the countingmen.

tens	ones	10 + 2	12
tens	ones	10 + 3	13
tens	ones	10 + 4	14
tens	ones	10 + 5	15

Expanded notation helps us think about the place-value meaning of numerals.

S-37



Complete each equation.

$10 + 5 = \boxed{15}$
$10 + 9 = \boxed{19}$
$10 + 2 = \boxed{12}$
$10 + 8 = \boxed{18}$
$10 + 7 = \boxed{17}$
$10 + 1 = \boxed{11}$
$10 + 4 = \boxed{14}$
$10 + 0 = \boxed{10}$
$10 + 6 = \boxed{16}$
$10 + 3 = \boxed{13}$

$13 = 10 + \boxed{3}$
$15 = 10 + \boxed{5}$
$18 = 10 + \boxed{8}$
$10 = 10 + \boxed{0}$
$17 = 10 + \boxed{7}$
$14 = \boxed{10} + 4$
$11 = \boxed{10} + 1$
$19 = \boxed{10} + 9$
$12 = \boxed{10} + 2$
$16 = \boxed{10} + 6$

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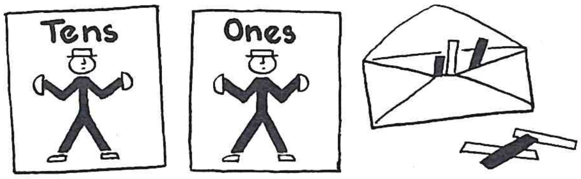
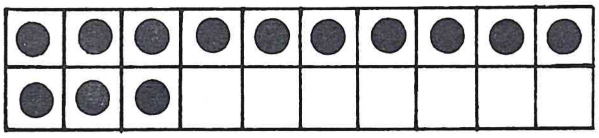
Supplemental Experiences

▶ Help each child make a pair of Countingman cards. Duplicate the picture of the Countingman on 8½″ x 11″ pieces of paper. Have the children color and paste them on heavy cardboard, or you may prefer to provide stencils of the Countingman so the children can draw directly on cardboard or tagboard. Have them label one card the Tens-man and the other card the Ones-man. Then help the children cut out brightly colored paper, or provide small flat sticks, to be used as fingers. Give each child an envelope in which to store the fingers.



Place a number of objects on the flannel board, and have the children record the count on their Countingman cards. Hold up a numeral and have the children show the number it represents by placing fingers on the Countingmen.

▶ On the chalkboard, draw a rectangle and divide it into twenty squares or boxes, ten in each row. Then draw circles or other objects in the boxes. Have the children count the number of objects in the boxes and record the count on their Countingman cards, or write the correct numeral on the chalkboard. When displaying a set of objects whose number is greater than ten, have the children explain the meaning of the numeral they write for the number. Relate the numeral to the number and to the arrangement of the objects in the boxes: one row of ten and one row of three ones is represented by the numeral 10 + 3 or 13.



▶ To play a relay game, select two teams of ten children. Write two columns of the place-value numerals 10 through 19 (not necessarily in order) on the chalkboard. Have the teams line up facing the board, and provide the first member of each team with a piece of chalk. At a signal the first child from each team must walk quickly to the board and write the expanded form for the first numeral in the column. He then hurries back to his team, passes the chalk to the next child, and goes to the end of the line. Each team member follows this procedure in turn. The team which is the first to write each numeral in expanded form correctly wins the game.

Addition

SUMS ELEVEN THROUGH NINETEEN

Work Pages S-39 and S-40

Objectives

To review sums eleven through nineteen.

To review the first step in addition with two-digit addends.

To help children apply their understanding of the associative property.

Fundamentals

When presented with a problem such as $11 + 2 = \square$, the child should be encouraged to use known concepts in order to solve it. From his knowledge of place value, he knows that another way to name eleven is $10 + 1$. He can think of the problem then as $(10 + 1) + 2 = \square$.

The child will next use his understanding of the associative property of addition and think of the problem as $10 + (1 + 2) = \square$, as he separates the ones and tens. He will add the one and two to get three; then he will use his understanding of place value to think of ten plus three as thirteen. These steps can be summarized in this way:

$11 + 2 = (10 + 1) + 2$ expanded notation
$= 10 + (1 + 2)$ associative property
$= 10 + 3$ addition combination
$= 13$ place value

Only one step will be presented at a time, and the child will have ample opportunity to become aware of the relation between steps.

The teacher will want to help the child solve every problem by first taking it apart, by working with the ones and then the tens, and finally putting the pieces back together again.

On the next few work pages, parentheses are used to help the child to think of different numerals for a number and the associative property of addition.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Understanding of the concept of place value.

Understanding of the concept of different numerals for the same number.

Understanding of the associative property.

Mastery of addition combinations through nine.

Ability to work with placeholder equations.

Developmental Experiences

MATERIALS	VOCABULARY
2 Countingmen	parentheses
flannel board	
felt numerals 0 through 9	
numeral cards	
plastic numerals and plus signs	

▶ To review the meaning of place value, use the Countingmen to represent the number eleven. Ask children to tell what they see: "One set of ten and one set of one." Then write the numeral 11 on the chalkboard, and ask what digits (two 1's) they see. Have a child tell what this symbol really means: "One ten and one one." Next to the place-value notation for the number eleven, write the expanded notation 10 + 1. Tell children that when eleven is written this way it reminds them of what the 1 in each position really means. You can emphasize the place-value meaning of the numbers twelve through nineteen in this way.

Hold up cards denoting the place-value numerals 11 through 19. At their desks, the children can use sets of plastic numerals and plus signs to build the expanded notation for the numbers.

Place the incomplete equation 11 + 1 = on the flannel board. Have children look at the 11 in the equation and decide on another way the number eleven could be represented. When the children respond, "Ten plus one," put the symbol 10 + 1 on the flannel board, enclose it with parentheses, and complete the equation. Explain to the class that this is just another way to show eleven plus one; stress that the *parentheses* will help them to visualize ten plus one as the number eleven. Continue the activity

$$11 + 1 = (10 + 1) + 1$$

with the children, replacing the place-value numeral in equations with the expanded notation representing the same number.

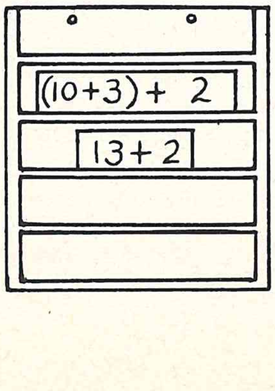
▶ Write a series of numerals on the chalkboard. Have children take turns writing these numerals in different ways to complete the equations:

$$12 + 2 = (10 + 2) + 2$$
$$15 + 3 = (10 + 5) + 3$$

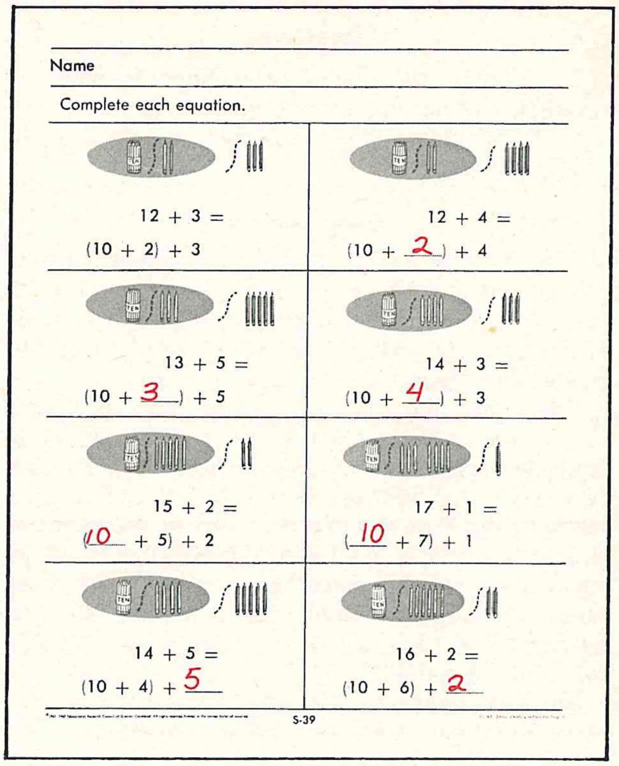
▶ Use work page S-39 to reinforce the children's concept of different numerals for a number. The colored parentheses around the expanded notation for the two-digit numerals will direct their attention to the two written forms for the same number. Work the entire page with the group. Explain that they are to fill in the numerals which are needed to complete each equation. Remind the class that the set pictures will help them.

Use work page S-40 to give the children an opportunity for testing their ability to rewrite numerals in equations without the aid of a set picture. Work the first problem with the group; then have them work independently to complete the page.

Supplemental Experiences



▶ Distribute cards on which there are such numerals as 11 + 4, 13 + 2, 12 + 5. One at a time place, in the pocket chart, cards on which there are different numerals representing the same numbers as the numerals cards held by the children: (10 + 1) + 4, (10 + 3) + 2, (10 + 2) + 5. Have each child match his card to the card in the chart, placing it in the pocket chart, too. Have the numeral read: "(10 + 3) + 2 names the same number as the numeral 13 + 2, or (10 + 3) + 2 is a different numeral for thirteen plus two." Have children use their hands to frame the expanded notation for the number thirteen in the one numeral and the place-value form for this number in the other numeral.


Name

Complete each equation.

12 + 3 = (10 + 2) + 3

12 + 4 = (10 + 2) + 4

13 + 5 = (10 + 3) + 5

14 + 3 = (10 + 4) + 3

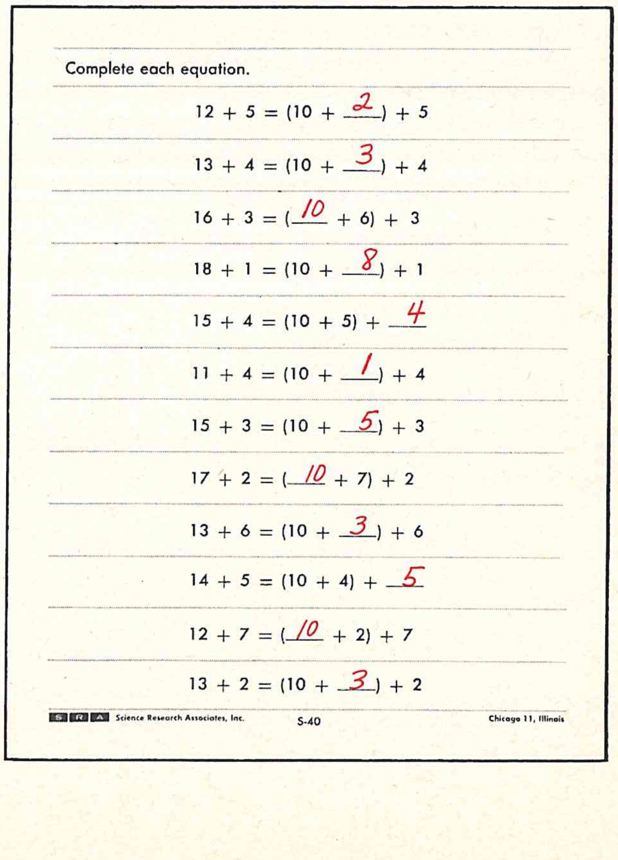
15 + 2 = (10 + 5) + 2

17 + 1 = (10 + 7) + 1

14 + 5 = (10 + 4) + 5

16 + 2 = (10 + 6) + 2

S-39


Complete each equation.

12 + 5 = (10 + 2) + 5

13 + 4 = (10 + 3) + 4

16 + 3 = (10 + 6) + 3

18 + 1 = (10 + 8) + 1

15 + 4 = (10 + 5) + 4

11 + 4 = (10 + 1) + 4

15 + 3 = (10 + 5) + 3

17 + 2 = (10 + 7) + 2

13 + 6 = (10 + 3) + 6

14 + 5 = (10 + 4) + 5

12 + 7 = (10 + 2) + 7

13 + 2 = (10 + 3) + 2

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Addition

SUMS ELEVEN THROUGH NINETEEN

Work Pages S-41 and S-42

Objectives

To continue the review of addition involving a two-digit addend and a one-digit addend.

To help children apply their understanding of the associative property.

Fundamentals

The child has learned to write a different numeral for the first number in equations such as $12 + 2 = \square$. The numeral the child was guided to write in a different way was the one which emphasized the place-value concept: $10 + 2$.

When the child can easily rewrite such problems as $12 + 2 = \square$ as $(10 + 2) + 2 = \square$, he is ready to apply the associative property, so that $(10 + 2) + 2$ becomes $10 + (2 + 2)$. He is then ready to think of the two plus two in the problem as four. He will be guided to think of the numerals enclosed in colored parentheses on the work page as representing one number, and he will learn to rewrite $10 + (2 + 2)$ as $10 + 4$. Pictures of sets will guide his thinking.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Understanding of the concept of place value.

Understanding of the concept of different numerals for the same number.

Understanding of the associative property.

Mastery of addition combinations through nine.

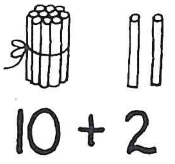
Ability to work with placeholder equations.

Developmental Experiences

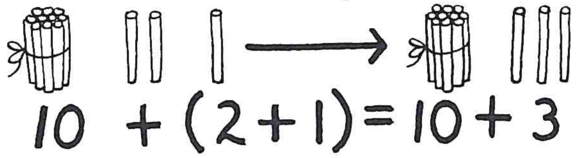
MATERIALS

plastic numerals
sticks
plus signs
parentheses

▶ Provide each child with a bundle of ten sticks and nine separate sticks. Have the children lay out twelve sticks—a set of ten and two more. Have them use plastic numerals to show the expanded notation for the number of the set.



Tell the children to put a set of one beside the set of ten and two, and display $10 + 2 + 1$ below the sticks. Have the children join the set of one to the set of ten and two. Guide them to observe that this gives a set of ten and a set of three. Have the children put parentheses around the $2 + 1$ to help them visualize that now the numbers of the set of two and the set of one belong together. The sets were joined, the numbers added.



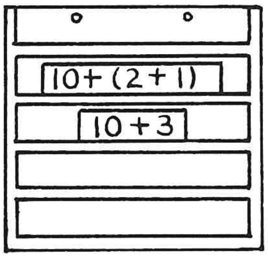
Ask children to find a different numeral for $2 + 1$ and to rewrite $10 + (2 + 1)$ as $10 + 3$. Follow this procedure with sets of thirteen and four, eleven and five, and fourteen and two.

▶ Place a series of numerals on the chalkboard. Have children take turns writing these numerals in a different way.

$$10 + (2 + 3) = 10 + 5$$

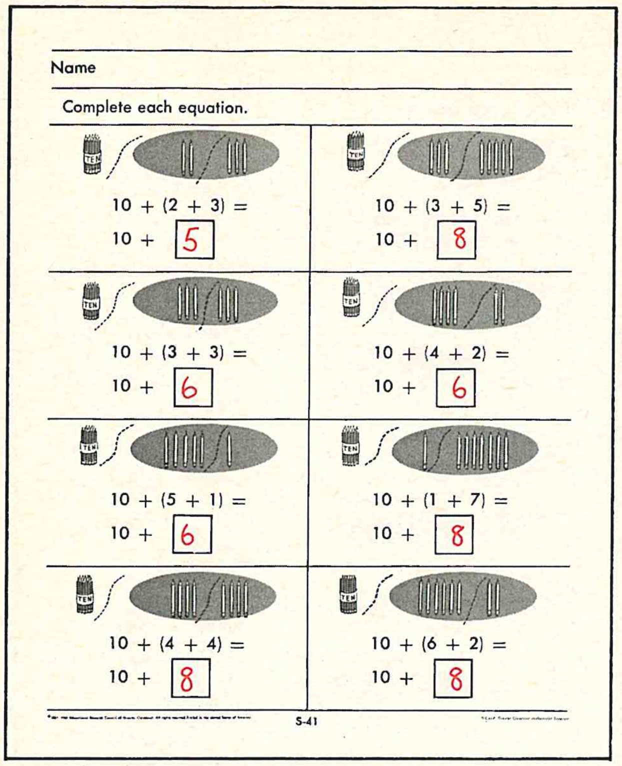
$$10 + (1 + 5) = 10 + 6$$

▶ Distribute cards showing the numerals $10 + 2$ through $10 + 9$ to the children. Place in the pocket chart, one at a time, a set of cards showing the same numbers in a different form: $10 + (1 + 1)$ through $10 + (6 + 3)$. Have the children match their cards with each card that is placed in the pocket chart. Have the child with the matching card place it in the pocket chart for comparison. Have the numerals read: "$10 + 6$ names the same number as $10 + (4 + 2)$; $10 + 6$ is a different numeral for $10 + (4 + 2)$." Have the child cup his hands to frame the numerals, on each card, that represent the same number.



▶ Use work page S-41 to reinforce the concept of using different numerals for the same number. The colored parentheses will help the children think about the number represented by the enclosed numeral. Help the children understand that they must write in the placeholder a different numeral for this number. Remind them that the pictures of the sets will help them visualize what happens in the problem. Work with the children through the entire page.

Use work page S-42 to give the children the opportunity to test themselves on their ability to rewrite numerals in equations without a set picture to help them. Work the first problem with the group. Have them complete the rest of the page on their own.



Name

Complete each equation.

10 + (2 + 3) =
10 + $\boxed{5}$

10 + (3 + 5) =
10 + $\boxed{8}$

10 + (3 + 3) =
10 + $\boxed{6}$

10 + (4 + 2) =
10 + $\boxed{6}$

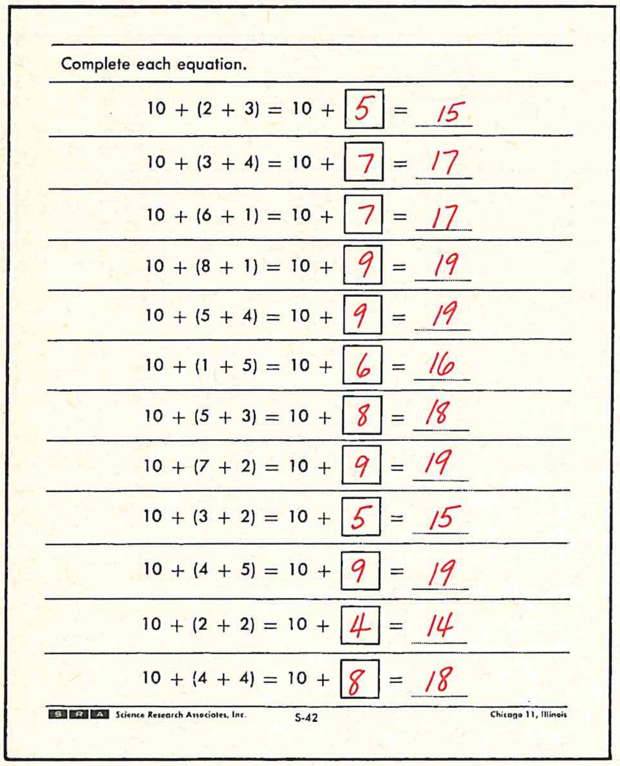
10 + (5 + 1) =
10 + $\boxed{6}$

10 + (1 + 7) =
10 + $\boxed{8}$

10 + (4 + 4) =
10 + $\boxed{8}$

10 + (6 + 2) =
10 + $\boxed{8}$

S-41



Complete each equation.

10 + (2 + 3) = 10 + $\boxed{5}$ = 15
10 + (3 + 4) = 10 + $\boxed{7}$ = 17
10 + (6 + 1) = 10 + $\boxed{7}$ = 17
10 + (8 + 1) = 10 + $\boxed{9}$ = 19
10 + (5 + 4) = 10 + $\boxed{9}$ = 19
10 + (1 + 5) = 10 + $\boxed{6}$ = 16
10 + (5 + 3) = 10 + $\boxed{8}$ = 18
10 + (7 + 2) = 10 + $\boxed{9}$ = 19
10 + (3 + 2) = 10 + $\boxed{5}$ = 15
10 + (4 + 5) = 10 + $\boxed{9}$ = 19
10 + (2 + 2) = 10 + $\boxed{4}$ = 14
10 + (4 + 4) = 10 + $\boxed{8}$ = 18

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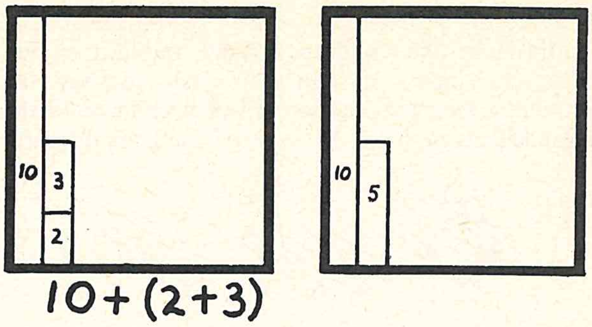
Supplemental Experiences

▶ Have the children use their Ten-frames to represent different forms for the same number. Direct them to place a ten-bar in the lower left corner of the frame. Have them place a two-bar next to the ten-bar. Ask them to join a set of three squares to the set of ten and two. Have a three-bar placed at the end of the two-bar. Have the children find a single bar to replace the two-bar and the three-bar.



Ask them to tell the story of the replacement and write the corresponding equation: 10 + (2 + 3) = 10 + 5. Continue in this way with other combinations.

Addition

SUMS ELEVEN THROUGH NINETEEN

Work Pages S-43 and S-44

Objectives

To continue the development of addition involving a two-digit addend and a one-digit addend.

To help children apply their understanding of the associative property.

Fundamentals

The child is now ready for the third step in the solution of such problems as 13 + 4 = ☐. The teacher will want to help him tie together the ideas presented on the two preceding work pages. She will need to guide him through the thinking steps involved in working the problem 13 + 4 = ☐. The child must first think of 13 + 4 as (10 + 3) + 4; then as 10 + (3 + 4); and finally as 10 + 7. The child will see that the original problem can be reduced to two simpler problems: 3 + 4 = ☐ and 10 + 7 = ☐.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Understanding of the concept of place value.

Understanding of the concept of different numerals for the same number.

Understanding of the associative property.

Mastery of addition combinations through nine.

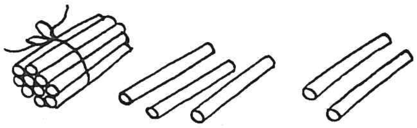
Ability to work with placeholder equations.

Developmental Experiences

MATERIALS

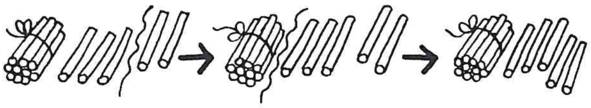
sticks
flannel board
felt numerals
symbols for operations and relations
felt placeholder
parentheses

▶ Place the problem 13 + 2 = ☐ on the flannel board. Have the children arrange bundles of ten sticks and single sticks on their desks to show the set picture for this equation. Have a child build the expanded notation for 13 + 2 on the flannel board.



Have another child show how these numbers can be associated in a different way: 10 + (3 + 2).

Now have the children join the set of two sticks to the set of three sticks to give a set of five sticks.



Have a child rewrite 10 + (3 + 2) so that it corresponds with the arrangement of sticks he now has on his desk: 10 + 5. Have him place this numeral under the others on the flannel board.

Guide the children to summarize what happened when two was added to thirteen. First the ones were separated from the set of ten, then all the ones were joined, and finally all of the ones were put back with the set of ten. Guide the children to an awareness of the use of the associative or grouping property involved in this problem.

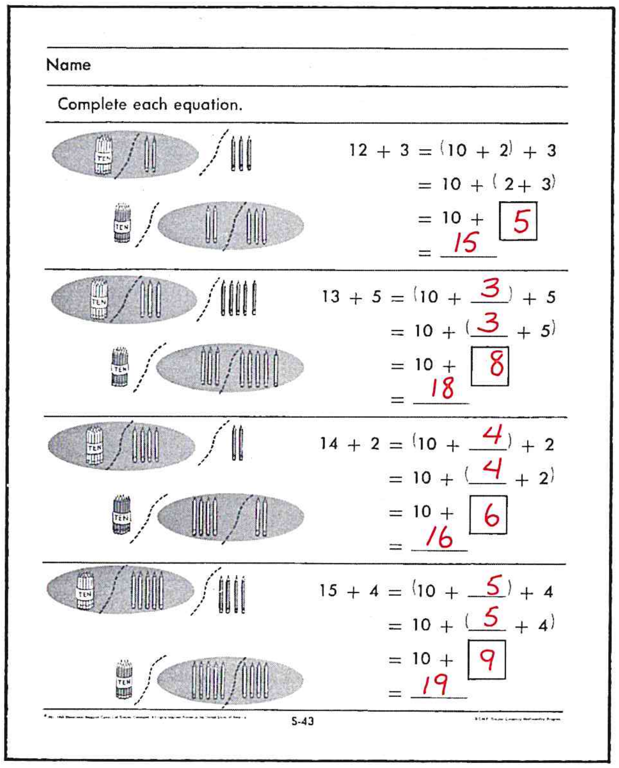
▶ Use work page S-43 to help children visualize what happens when a number represented by a one-digit numeral is added to a number represented by a two-digit numeral. Work with the children for the entire page, helping them think through the steps involved in problems of this type. Use the set pictures to guide the activity. Help the children as they write the required numeral in the placeholder and as they write the place-value numeral on the line at the end of the equation.

Use work page S-44 to give children the opportunity to test themselves on their ability to rewrite numerals and equations without a set picture to help them. Work the first problem with the group. Have them complete the rest of the page independently.

Supplemental Experiences

▶ Review easy combinations related to sums not greater than ten. The ability of the children to handle these combinations with ease is imperative as work with numbers advances.

Place several sets of numerals 0 through 9, together with plus and minus signs, on a table next to the flannel board. Place a line down the center of the board. At a given signal, have two children come forward and create a numeral that represents a given number. The teacher may say, "Make a numeral for the number five." Allow the children complete freedom in creating their numerals. One child



Name

Complete each equation.

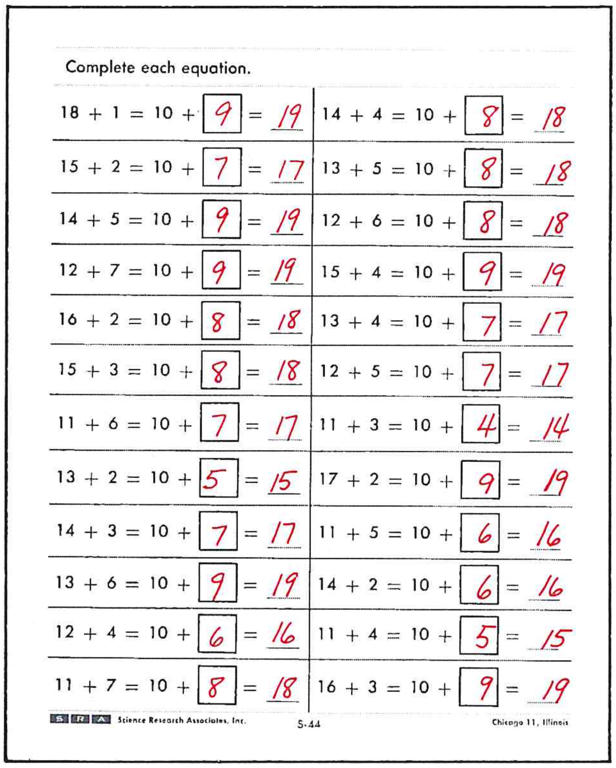
12 + 3 = (10 + 2) + 3
= 10 + (2 + 3)
= 10 + $\boxed{5}$
= 15

13 + 5 = (10 + 3) + 5
= 10 + (3 + 5)
= 10 + $\boxed{8}$
= 18

14 + 2 = (10 + 4) + 2
= 10 + (4 + 2)
= 10 + $\boxed{6}$
= 16

15 + 4 = (10 + 5) + 4
= 10 + (5 + 4)
= 10 + $\boxed{9}$
= 19

S-43



Complete each equation.

18 + 1 = 10 + $\boxed{9}$ = 19
15 + 2 = 10 + $\boxed{7}$ = 17
14 + 5 = 10 + $\boxed{9}$ = 19
12 + 7 = 10 + $\boxed{9}$ = 19
16 + 2 = 10 + $\boxed{8}$ = 18
15 + 3 = 10 + $\boxed{8}$ = 18
11 + 6 = 10 + $\boxed{7}$ = 17
13 + 2 = 10 + $\boxed{5}$ = 15
14 + 3 = 10 + $\boxed{7}$ = 17
13 + 6 = 10 + $\boxed{9}$ = 19
12 + 4 = 10 + $\boxed{6}$ = 16
11 + 7 = 10 + $\boxed{8}$ = 18

14 + 4 = 10 + $\boxed{8}$ = 18
13 + 5 = 10 + $\boxed{8}$ = 18
12 + 6 = 10 + $\boxed{8}$ = 18
15 + 4 = 10 + $\boxed{9}$ = 19
13 + 4 = 10 + $\boxed{7}$ = 17
12 + 5 = 10 + $\boxed{7}$ = 17
11 + 3 = 10 + $\boxed{4}$ = 14
17 + 2 = 10 + $\boxed{9}$ = 19
11 + 5 = 10 + $\boxed{6}$ = 16
14 + 2 = 10 + $\boxed{6}$ = 16
11 + 4 = 10 + $\boxed{5}$ = 15
16 + 3 = 10 + $\boxed{9}$ = 19

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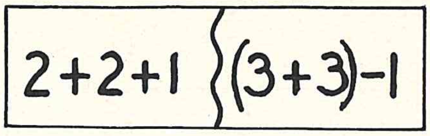
may put $2 + 2 + 1$ on the board; another may choose $(3 + 3) - 1$ as his numeral.



▶ Present simple chain problems orally. Say, "Two plus three *(pause)* plus two." Call upon someone to give the answer. Give another problem: "Eight minus two *(pause)* minus one." Call upon someone to give the answer.

Continue in this way with other oral problems. In the beginning phases of this activity with mental computation, some of the children may need to work with counters to get the answers. Encourage them to work as much of the time as possible, however, without any material aids. The children will enjoy working faster and faster as they become more familiar with the combinations.

Addition—Comparison

SUMS THROUGH NINETEEN

Work Pages S-45 Through S-48

Objectives

To help the children work with the addition algorism for sums not greater than nineteen.

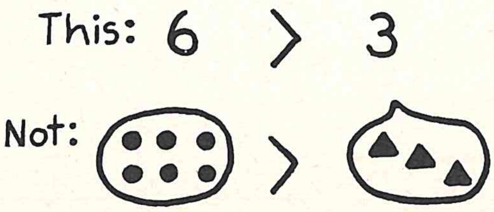
To compare numbers in the environment of familiar combinations.

To provide practice in finding missing addends in vertical notation.

Fundamentals

In the first grade the child had many experiences with the idea of one number being greater or less than another. He developed the ability to indicate these relations between numbers by using the symbol $>$ for "is greater than" and the symbol $<$ for "is less than."

The second-grade teacher will want to review the concept of comparison and the use of the symbols $>$ and $<$. She will want to stress that the relations "less than" and "greater than" are relations between two *numbers*. The symbols for these relations should be used only with numerals and not with sets.



When the child works with sets, he should say that one set has more objects or fewer objects than another set. He should say that the number of one set is greater than or less than the number of another set.

It is important that the teacher help the child understand that the relations symbolized by $>$ and $<$ do not express how much greater or less one number is than another. They merely state a fact of order in terms of a simple quantitative comparison. The inequality $6 > 3$ means that six is greater than three; there is no indication of how much greater.

The child will also need to understand that for any given pair of whole numbers, only one of three statements is true. The first number is less than the second, the numbers are equal, or the first number is greater than the second. The statement $9 > 6$ is true, whereas the statements $9 < 6$ and $9 = 6$ are false.

To enrich and vary his contact with sums of eleven through nineteen, the child will be asked to compare these sums. He should learn to be alert for problems which can be solved without computation. He should be encouraged to use his knowledge of the commutative property and his knowledge of different numerals for the same number in solving such problems as 12 + 3 ○ 3 + 12 and 10 + 5 ○ 10 + 2 + 3.

The child has had many opportunities to work through problems in addition of a number represented by a one-digit numeral and a number represented by a two-digit numeral with sums not greater than nineteen. A background for understanding the algorism has been established. The child will now work with the standard computational form of the problem.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Order—Relations—Operations; Addition of Whole Numbers.

Readiness for Understanding

An understanding of the concept of place value.

An understanding of the concept of different numerals for the same number.

An understanding of the associative property.

Mastery of addition combinations through nine.

Familiarity with vertical notation.

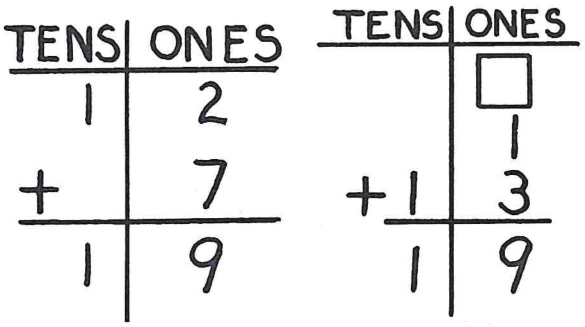
Ability to compare numbers.

Developmental Experiences

MATERIALS
- flannel board
- felt numerals
  - 0 through 9
- symbols for operations and relations
- felt circle
- felt cutouts

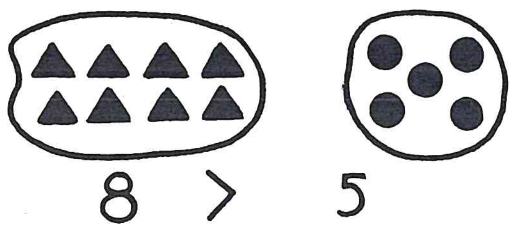
VOCABULARY
- greater than
- less than

▶ Write the problem 12 + 7 in vertical form on the chalkboard. Explain to the children that just as they worked addition and subtraction problems involving one-digit numerals in this form, they will also use the form for easier and quicker work in problems with a two-digit addend. Say that the important idea to keep in mind is the correct placement of the numerals representing the ones and tens. To help the children in the preliminary phases of the work, place the word "tens" and "ones" above lined columns. Remind the children that the ones will be added first in this form of the problem, just as they were in the equation form. After the sum of the ones is found, it is added to the tens for the final answer. In finding the answer to the problem 12 + 7 a child would say, "Two ones plus seven ones equals nine ones; one ten plus nine ones equals nineteen."



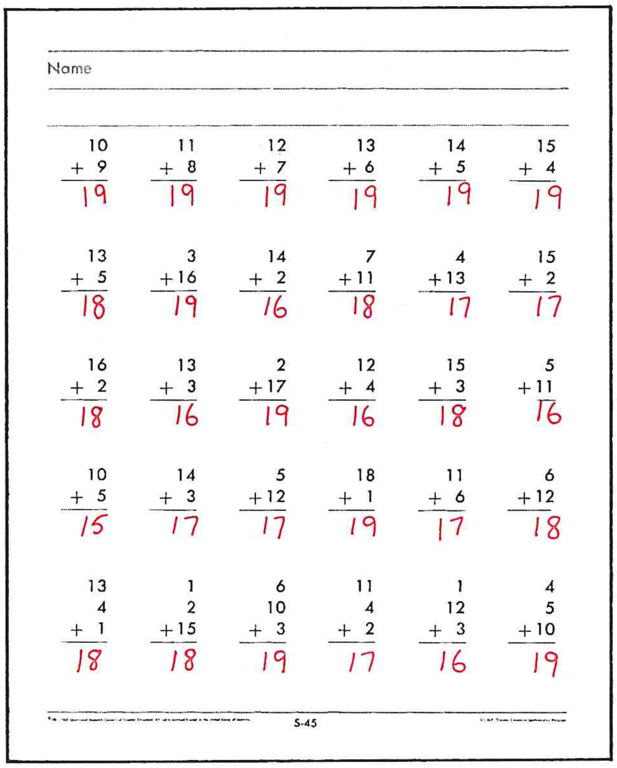
Continue working similar problems in this way. In some of them, include three or four addends in a column, such as ten, three, one; or eleven, zero, three, five. In others, have a numeral missing. Help the children think what number is needed in this type of problem to make it complete. They may think, "There are nine ones in the answer, and only four ones are now given in the problem. Five plus four is needed to make nine. Five must be the number missing in this problem."

▶ Review the meaning of the symbols > and <. Display two sets of objects on the flannel board. Have the children discuss the number of each set. Place the numeral for that number below the set.



Ask which set has more objects in it. Discuss which number is greater—eight or five. Have the children recall that there is a special way to show that eight is *greater than* five. Place the "greater than" symbol, >, between the numerals 8 and 5. Read, "Eight is greater than five." Compare other numbers less than ten. Explain that the opening of the symbol is next to the numeral for the greater number.

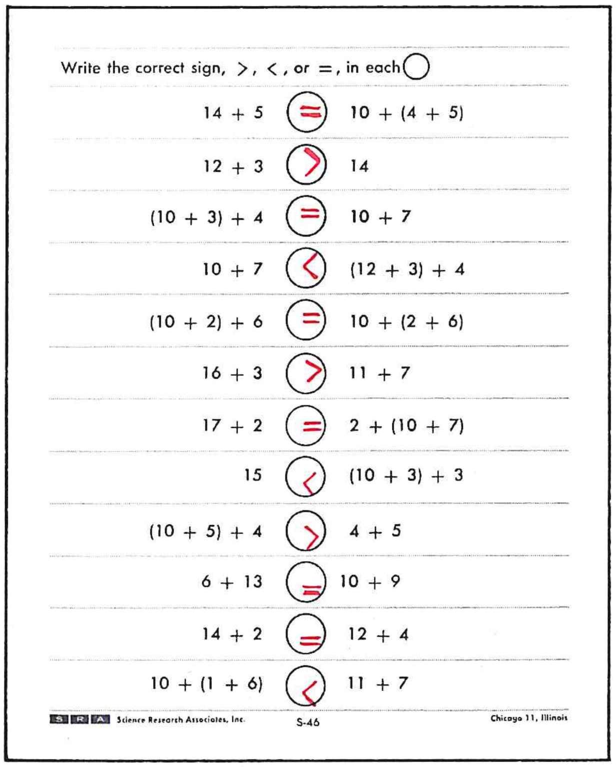
In a similar way, review the meaning of the *"less than"* symbol, <. Review the meaning of the equal sign. Have children read such comparisons as 9 > 6, 3 < 5, and 4 + 1 = 5.



Name

10 + 9 19	11 + 8 19	12 + 7 19	13 + 6 19	14 + 5 19	15 + 4 19
13 + 5 18	3 +16 19	14 + 2 16	7 +11 18	4 +13 17	15 + 2 17
16 + 2 18	13 + 3 16	2 +17 19	12 + 4 16	15 + 3 18	5 +11 16
10 + 5 15	14 + 3 17	5 +12 17	18 + 1 19	11 + 6 17	6 +12 18
13 4 + 1 18	1 2 +15 18	6 10 + 3 19	11 4 + 2 17	1 12 + 3 16	4 5 +10 19

S-45



Write the correct sign, >, <, or =, in each ○

14 + 5	=	10 + (4 + 5)
12 + 3	>	14
(10 + 3) + 4	=	10 + 7
10 + 7	<	(12 + 3) + 4
(10 + 2) + 6	=	10 + (2 + 6)
16 + 3	>	11 + 7
17 + 2	=	2 + (10 + 7)
15	<	(10 + 3) + 3
(10 + 5) + 4	>	4 + 5
6 + 13	=	10 + 9
14 + 2	=	12 + 4
10 + (1 + 6)	<	11 + 7

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▶ Place a series of numerals on the flannel board. Have the children compare the numbers represented by these numerals. Have them place the correct relation symbol in the circle and then explain why they chose that symbol.

12+3 ○ 13+2
10+2 ○ 10+(2+1)
15+3 ○ 10+(3+5)

In solving the first example, the children may say that since the order of two and three is all that is different in the problem, it is not necessary to add to get the answer.

In the second problem they might reason that since there is one more one represented in the numeral on the right side, then that numeral must stand for the greater number.

In the third problem, they may decide that since there are as many ones and tens respresented in the numeral on the right as are represented in the numeral on the left, then the numbers must be equal.

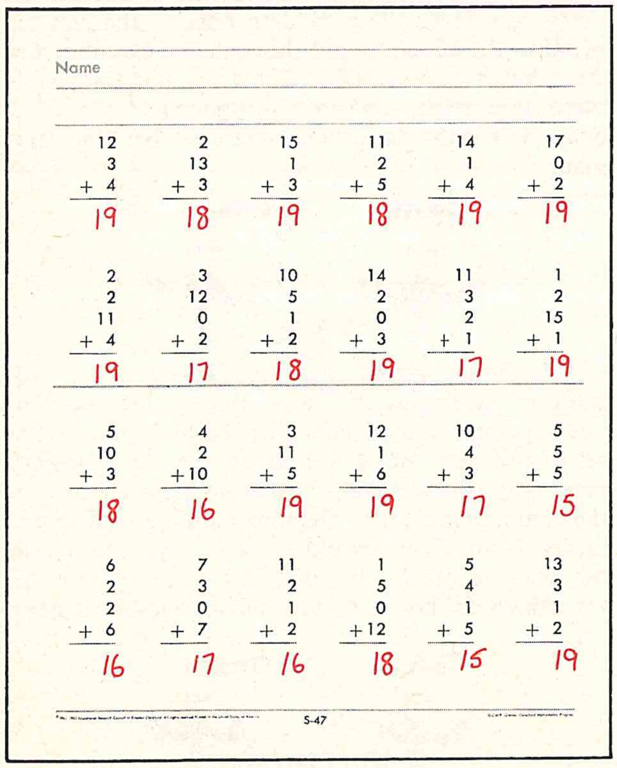
Continue in this way to encourage the children to explain the thinking behind their decisions. Begin to build, in accordance with the maturity level of the children, the foundation for an "if-then" approach to problem situations.

▶ Use work page S-45 to give practice in working one- and two-digit addition problems in vertical form. Remind the children to keep the numerals representing ones and tens lined up as carefully as possible in their respective columns, and tell them to find the sums.

Work page S-46 offers the children an opportunity to use their reasoning ability. Work the first problem with the group, and then have them do the rest of the page independently. Encourage them to find answers without computing whenever possible and then to double-check themselves by adding. When work on this page is completed, discuss each problem with the children; have them explain how they thought out their answers.

▶ Use work page S-47 to provide practice in working one- and two-digit addition problems involving three and four addends in vertical form. Remind the children about lining up the numerals representing ones and tens, and tell them to find the sums.

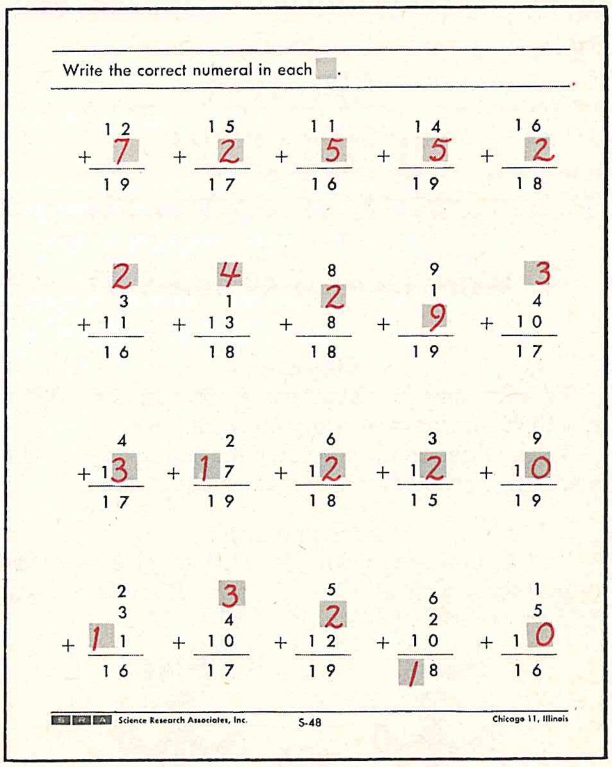
Use work page S-48 to provide practice in finding the numeral for a missing number in a column addition problem involving a two-digit addend. Work the first problem with the children, and have them complete the page independently. When they have done so, discuss each problem with them. Have them explain how they reasoned as they worked these problems.



Name

12 3 + 4 19	2 13 + 3 18	15 1 + 3 19	11 2 + 5 18	14 1 + 4 19	17 0 + 2 19
2 2 11 + 4 19	3 12 0 + 2 17	10 5 1 + 2 18	14 2 0 + 3 19	11 3 2 + 1 17	1 2 15 + 1 19
5 10 + 3 18	4 2 +10 16	3 11 + 5 19	12 1 + 6 19	10 4 + 3 17	5 5 + 5 15
6 2 2 + 6 16	7 3 0 + 7 17	11 2 1 + 2 16	1 5 0 +12 18	5 4 1 + 5 15	13 3 1 + 2 19

S-47



Write the correct numeral in each □.

12 + 7 19	15 + 2 17	11 + 5 16	14 + 5 19	16 + 2 18
2 3 + 11 16	4 1 + 13 18	8 2 + 8 18	9 1 + 9 19	3 4 + 10 17
4 + 13 17	2 + 17 19	6 + 12 18	3 + 12 15	9 + 10 19
2 3 + 11 16	3 4 + 10 17	5 2 + 12 19	6 2 + 10 18	1 5 + 10 16

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Supplemental Experiences

▶ Place a series of vertical notation problems on the chalkboard. Have the children copy these problems on their writing paper and then work each one. Remind them that they must carefully line up the ones and tens.

$$\begin{array}{rrrr} 4 & 14 & 7 & 17\\ +5 & +5 & +2 & +2\\ \hline \\ 3 & 13 & 0 & 10\\ +5 & +5 & +9 & +9\\ \hline \end{array}$$

▶ Show the record of additions by presenting problems in which the words "ones" and "tens" are used.

$$\begin{array}{r} 12\\ +\ 6\\ \hline 18\end{array}\qquad \begin{array}{lr} & 1\text{ ten }\ 2\text{ ones}\\ + & 6\text{ ones}\\ \hline & 1\text{ ten }\ 8\text{ ones or } 18\end{array}$$

▶ Display a card on which five numerals are written in random order. Call on a child to unscramble these numerals and rewrite them on the chalkboard to represent numbers in order from the least to the greatest. After he does this, place relation symbols between the numerals: $3 < 5 < 7 < 9 < 10$. Have the series of relations read: "Three is less than five, five is less than seven, seven is less than nine, nine is less than ten." Continue in this way with other cards.

Addition

SUMS ELEVEN THROUGH NINETEEN

Work Pages S-49 and S-50

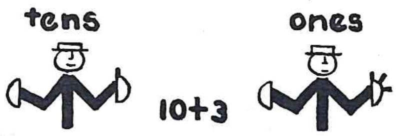
Objectives

To continue the development of addition with a two-digit addend and a one-digit addend.

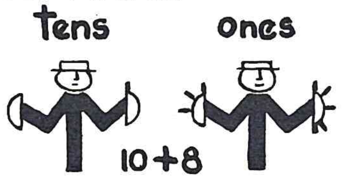
To help children apply their understanding of the associative property.

Fundamentals

The Countingmen will be used to give meaning to problems such as 13 + 5 = ☐. This device will help the child first think of 13 as 10 + 3.



Five ones will be put with the three ones on the Ones-man to make eight ones. The child can then visualize the answer to the problem—one ten and eight ones or 10 + 8 or 18.



When he works with problems such as 13 + 5 = ☐, the child should be guided to visualize the number of ones in the problem, to add the ones, and finally to add the ones to the tens. He should be given many opportunities to develop the ability to think of 13 + 5 as 10 + 8.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

An understanding of the concept of place value.

An understanding of the concept of different numerals for the same number.

An understanding of the associative property.

Mastery of addition combinations through nine.

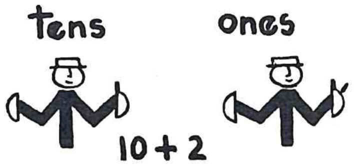
Ability to work with placeholder equations.

Developmental Experiences

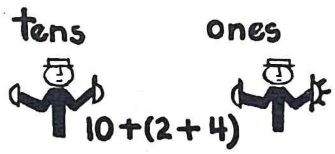
MATERIALS

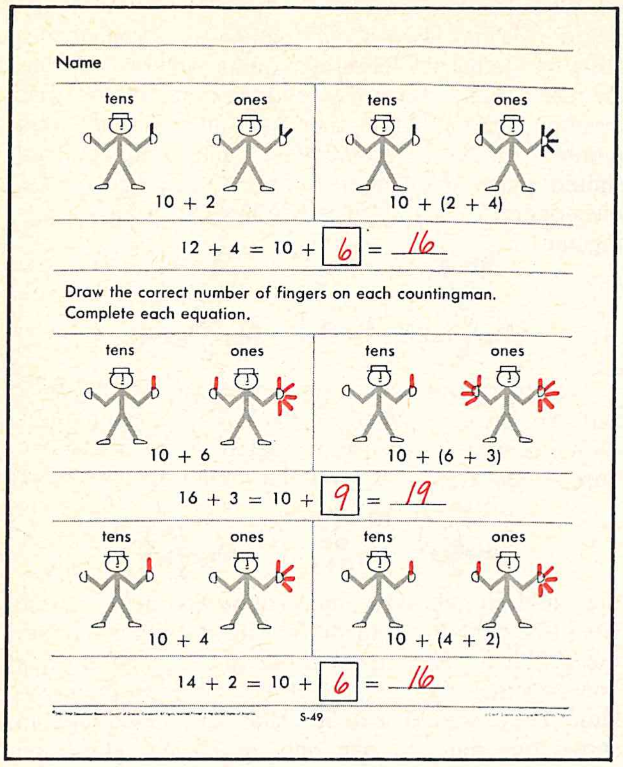
2 Countingmen
tagboard cards

▶ Place the problem 12 + 4 = ☐ on the chalkboard. Display two Countingmen. Call on a child to operate the Tens-man and another to operate the Ones-man. Have these children observe the first numeral in the equation and show on the Countingmen the number it represents. The tens-operator should place one finger on his Countingman; the ones-operator should place two fingers on his Countingman.



Tell the operators that they must look at the problem once more and decide what to do next. The ones-operator should join a set of four ones to the set of two ones which are already on the Countingman. After the class tells how many ones are now on the Countingman, have them explain the final step in the problem. They should think in terms of putting the ones together with the tens and say that the Countingman shows one ten and six ones, or sixteen.





Name

10 + 2

10 + (2 + 4)

12 + 4 = 10 + $\boxed{6}$ = 16

Draw the correct number of fingers on each countingman.
Complete each equation.

10 + 6

10 + (6 + 3)

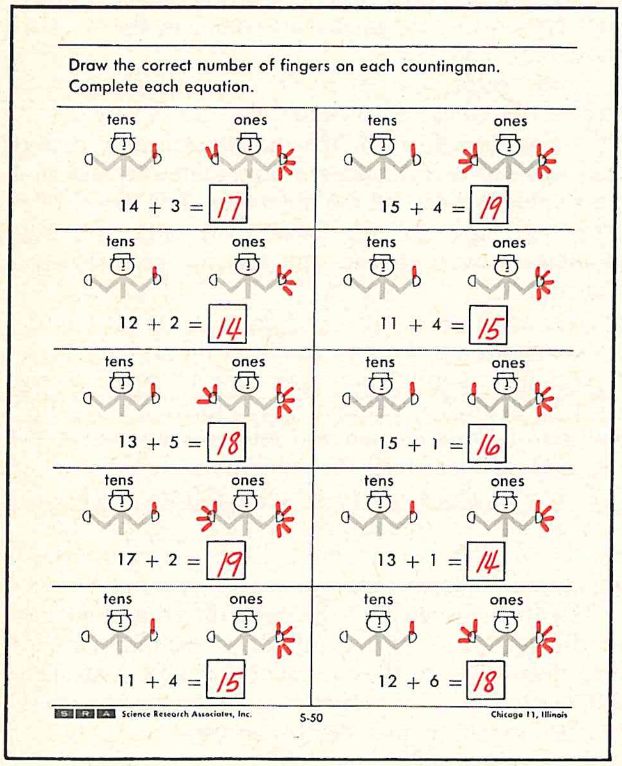
16 + 3 = 10 + $\boxed{9}$ = 19

10 + 4

10 + (4 + 2)

14 + 2 = 10 + $\boxed{6}$ = 16

S-49



Draw the correct number of fingers on each countingman.
Complete each equation.

14 + 3 = $\boxed{17}$

15 + 4 = $\boxed{19}$

12 + 2 = $\boxed{14}$

11 + 4 = $\boxed{15}$

13 + 5 = $\boxed{18}$

15 + 1 = $\boxed{16}$

17 + 2 = $\boxed{19}$

13 + 1 = $\boxed{14}$

11 + 4 = $\boxed{15}$

12 + 6 = $\boxed{18}$

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Encourage the children to summarize the thinking steps in this problem. Guide them as they express their ideas: 12 + 4 becomes 10 + 2 + 4; 10 + 2 + 4 becomes 10 + 6; 10 + 6 is 16. The ultimate aim is to have children think in terms of the decimal system of numeration in problems of this kind. They will learn to think of 12 + 4 as 10 + 6.

Have children work other problems of this kind on the Countingmen.

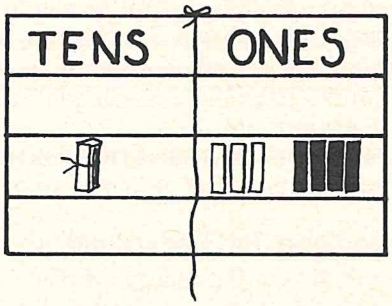
▶ Show cards, one at a time, on which appear placeholder equations such as 12 + 6 = ☐ or 15 + 4 = ☐. Have the children quickly tell how many ones they see and then how many tens they see. Have them give the answer. For those children who have difficulty quickly seeing the ones in these problems, cover up the one 10 in the two-digit numeral.

▶ Use work page S-49 to reinforce the understanding of the first phase in the development of two-digit addition. Work the first problem on the page with the children. Explain that they must examine the placeholder equation carefully and that the Countingman picture at the left represents the first number in the problem. Say that the numeral under the Countingman picture will help them think of the place-value interpretation of the number. As they look at the equation they will discover how many are to be added to this first number. Ask whether it is a set of ones or a set of tens that will be joined to the sets already on the Countingmen. Have the children draw the correct number of fingers on the Ones-man in the picture at the right side of the page to show this joining of sets. Tell them to complete the equation. Have them complete the rest of the problems on S-49 independently.

Work page S-50 provides review of addition combinations with sums not greater than nineteen. Direct the children to draw the correct number of fingers on each Countingman and then to complete each equation.

Supplemental Experiences

▶ Use the pocket chart as a place-value device. Extend a piece of yarn down the center as a separating line. Label the right side of the chart "ones" and the left side "tens." Place one bundle of ten red strips of paper in the tens place and three separate red strips of paper in the ones place. To the three strips join four blue strips. Call on a child to write the related equation, 13 + 4 = 17, on the chalkboard. Have the equation read.



Continue in this way to build set stories related to addition with two-digit and one-digit addends with no carrying. Have the children write a related number sentence for each activity.

Subtraction

COMBINATIONS THROUGH NINETEEN

Work Pages S-51 and S-52

Objectives

To review subtraction combinations related to sums not greater than nineteen.

To review the first step in the development of subtraction with two-digit numerals.

Fundamentals

The child will review two-digit subtraction without borrowing. An example will illustrate the mathematical difficulty involved:

$$\begin{array}{r} 17 \\ -4 \\ \hline \end{array}$$

$$17 - 4 = (10 + 7) - 4$$

While it is true that addition is associative, this property does not allow us to conclude that $(10 + 7) - 4 = 10 + (7 - 4)$ nor have we developed any other property to justify the truth of this equation. Notice the difficulty which arises in a problem of the following type:

$$(10 - 4) - 3 = 10 - (4 + 3).$$

Another way must be found to justify subtracting the four from seven. One possibility is to start at seventeen and count backward by ones four times. This is not an efficient way to subtract. Another possibility is to appeal to the inverse concept and look for the number which we add to four to get seventeen. This is neither an easy nor an efficient way to subtract.

A third possibility is to apply methods that have been used in addition. Since it was possible to add ones first and then add their sum to tens, the child can explore this idea for subtraction.

An advantage of this method in subtraction is that it can be tried and then checked by addition. When the child explores this possibility, he will find that it does work. The Countingmen will be used to help visualize the problem.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concept of place value.

An understanding of the concept of different numerals for the same number.

Mastery of number combinations related to sums not greater than nine.

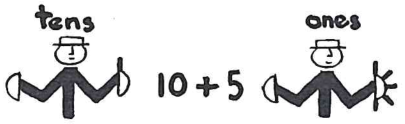
Ability to work with placeholder equations and the Countingman.

Developmental Experiences

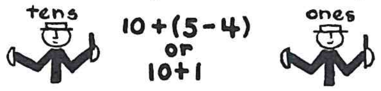
MATERIALS

2 Countingmen
tagboard cards

▶ Write the problem 15 − 4 = □ on the chalkboard. Display two Countingmen and call on a child to operate the Tens-man and another to operate the Ones-man. Have the children think of the first number represented in the equation and show this number on the Countingmen. The tens-operator should place one finger on his Countingman; the ones-operator should place five fingers on his Countingman.



Tell the operators that they must look at the problem and decide what to do next. The ones-operator should remove a set of four ones from the set of five ones which are already on the Countingman. After



the children tell how many ones are now on the Countingman, have them explain the final step in the problem. They must think in terms of putting ones together with tens to get the answer to the problem. They should explain that the Countingman shows one ten and one one, or eleven. Have the difference recorded in the equation on the chalkboard.

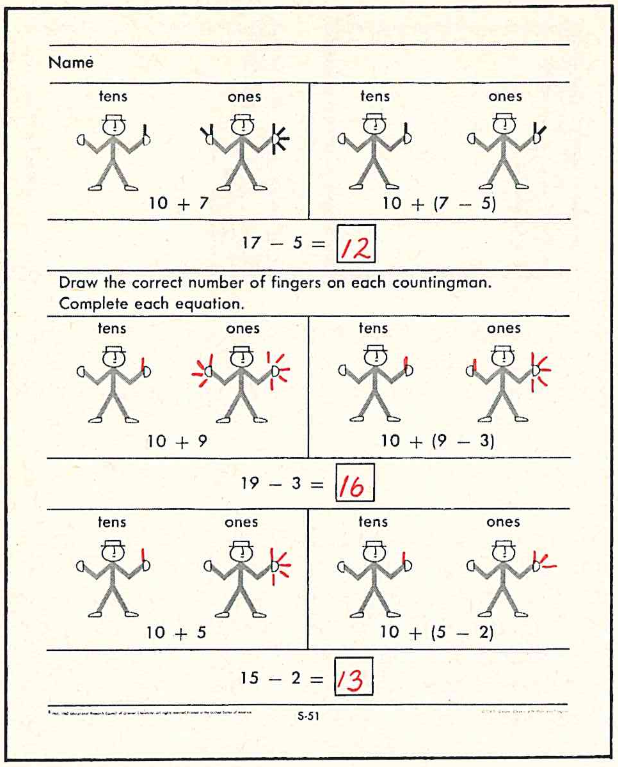
Have the children work other problems of this kind on the Countingmen and complete the related placeholder equations.

▶ Show cards, one at a time, on which appear placeholder equations such as 16 − 2 = □ or 15 − 3 = □. Have the children quickly tell how many ones and how many tens are represented by the numerals. Have them tell the difference. For those children who have difficulty seeing the ones in these problems, cover up the one ten in the two-digit numeral.

▶ Use work page S-51 to reinforce the children's understanding of the first phase in the development of two-digit subtraction. Work the first problem on the page with the children. Have them look at the first set of Countingmen and tell what number they show. Have them read the subtraction equation and tell what number must be subtracted from seventeen. Explain that the second set of Countingmen shows the result of this subtraction. Have the placeholder equation completed.

Tell the children that the parentheses around the subtraction (7 − 5) are used in this problem to help them think of the subtraction of ones first. The difference that results after the subtraction of ones, is added to tens to give the final answer.

Have the children complete the remainder of work page S-51 and all of the problems on S-52 independently. Explain that they are to complete each placeholder equation. Remind them that they must draw the correct number of fingers on each set of Countingmen. Tell them that the parentheses around the numerals in these problems will help them think of the numbers that the numerals represent.



Name

tens ones tens ones

10 + 7

10 + (7 − 5)

17 − 5 = $\boxed{12}$

Draw the correct number of fingers on each countingman.
Complete each equation.

tens ones tens ones

10 + 9

10 + (9 − 3)

19 − 3 = $\boxed{16}$

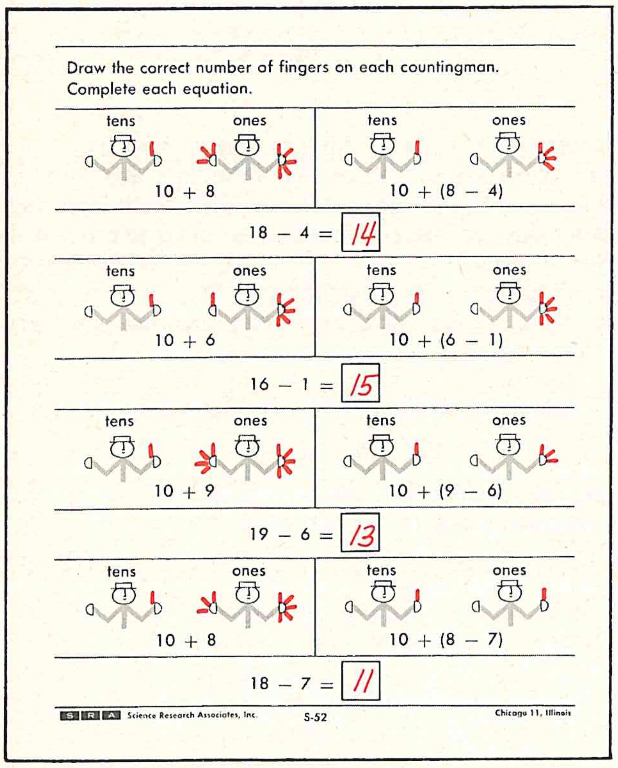
tens ones tens ones

10 + 5

10 + (5 − 2)

15 − 2 = $\boxed{13}$

S-51



Draw the correct number of fingers on each countingman.
Complete each equation.

tens ones tens ones

10 + 8

10 + (8 − 4)

18 − 4 = $\boxed{14}$

tens ones tens ones

10 + 6

10 + (6 − 1)

16 − 1 = $\boxed{15}$

tens ones tens ones

10 + 9

10 + (9 − 6)

19 − 6 = $\boxed{13}$

tens ones tens ones

10 + 8

10 + (8 − 7)

18 − 7 = $\boxed{11}$

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Supplemental Experiences

▶ Write a series of addition and subtraction combinations on the chalkboard. Have the children place the correct sign (>, <, or =) in the circle to compare the numbers these numerals represent.

16 − 4 ○ 17 − 5
12 + 2 ○ 14 − 2
15 − 4 ○ 19 − 5
16 + 3 ○ 16 − 3

If the children find it necessary to compute the answers to these problems, permit them to do so on the chalkboard. Have them explain their thinking as they arrive at their answer.

▶ Distribute cards bearing different numerals for the numbers eleven through nineteen. Place the standard place-value numeral 15 in the reading pocket chart. Have children holding cards with different numerals for fifteen raise their hands. Ask them to bring their numerals forward, one at a time, and place them in the chart under 15.

15
---
19 − 4
16 − 1
11 + 4
12 + 3

Vary the activity. Write the numerals 12 through 19 on the chalkboard. Give each child a turn to choose one of these numerals and write below it a different numeral for the number it represents.

Addition—Subtraction

COMBINATIONS THROUGH NINETEEN

Work Pages S-53 Through S-56

Objectives

To emphasize the inverse relation between addition and subtraction.

To help the children work with addition and subtraction algorisms.

To evaluate the children's ability to work with addition and subtraction combinations through nineteen.

Fundamentals

The inverse relation between addition and subtraction was evident in working with numbers related to sums not greater than ten. Does the inverse relation also hold true for number combinations related to sums greater than ten? The child should be given an opportunity to answer this question for himself.

Vertical notation, the standard form for subtracting a number represented by a one-digit numeral from a number represented by a two-digit numeral, is presented after the child is made aware of the steps involved in working with equations such as 15 − 2 = □. Although this faster method of subtraction is presented, the teacher will want to continue to stress the concepts behind the algorism. She can accomplish this by conducting discussion sessions after the children work with problems in which the vertical-notation form is used.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Awareness of the inverse relation between addition and subtraction.

An understanding of the concept of place value.

Mastery of number combinations to nine.

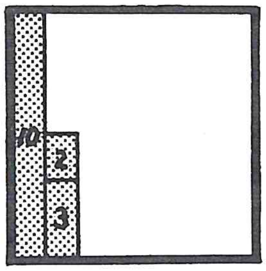
Ability to work with vertical notation and equation form in addition and subtraction problems.

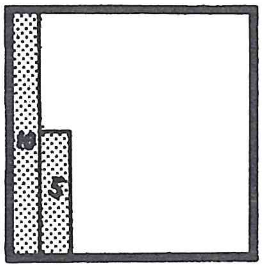
Developmental Experiences

MATERIALS

individual Ten-frames
flannel board
number-strips for flannel board
strips of colored paper
pocket chart

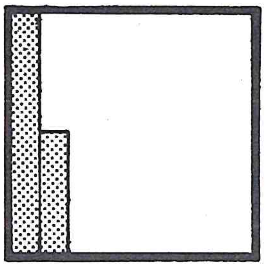
▶ The inverse relation between addition and subtraction will be re-emphasized in this activity. Write the equations 13 + 2 = ☐ and 15 − 2 = ☐ on the chalkboard, one under the other. Have the children represent thirteen in their Ten-frames. Ask them to find the two bars that best illustrate the place-value meaning of thirteen. Now tell them to join a set of two to the set of thirteen. They will discover that they must join two to three as a first step.

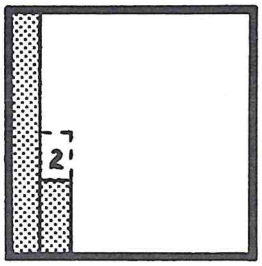




Ask the children to find the bar that has the same number of squares as the three-bar and the two-bar together; have them replace these bars with the five-bar. Have them tell what number the bars now in the Ten-frame represent. Have the addition placeholder equation completed. The teacher may wish to guide this activity by using number-strips on the flannel board.

Tell the children that you now want them to remove two blocks from the fifteen blocks. In order to do this they must again think of the set of five squares as a set of three and a set of two. Have them replace the five-bar with the three-bar and the two-bar. Ask them to take the set of two squares from the set of five squares. Have them tell how many squares remain and complete the subtraction equation. Have the two equations on the chalkboard read aloud. Continue in this way with other pairs of equations involving two-digit addition and subtraction. Guide the children to observe that the inverse relation holds for two-digit addition and subtraction as well as for combinations related to sums less than ten.





▶ In order to review work with vertical notation in subtraction problems related to sums not greater than nineteen, write a problem in vertical notation

$$\begin{array}{r} 17 \\ -\ 2 \\ \hline \end{array}$$

form on the chalkboard. Explain to the children that this form is quicker and easier to work with than an equation when the problem contains a two-digit numeral. Remind the children that the important idea to keep in mind is the correct placement of numerals representing ones and tens. Place the words "tens" and "ones" above the lined columns on the

TENS	ONES
1	7
−	2
1	5

chalkboard. Tell the children that in the vertical notation form the ones will be subtracted first, just as they were in the equation form. Tell the children that after this difference is found, it is added to the tens to get the final answer. In explaining the problem on the chalkboard, a child should say, "Seven ones minus two ones equals five ones; one ten plus five ones equals fifteen."

Use the same method to work other subtraction problems of this type. Include one or two addition problems in the activity.

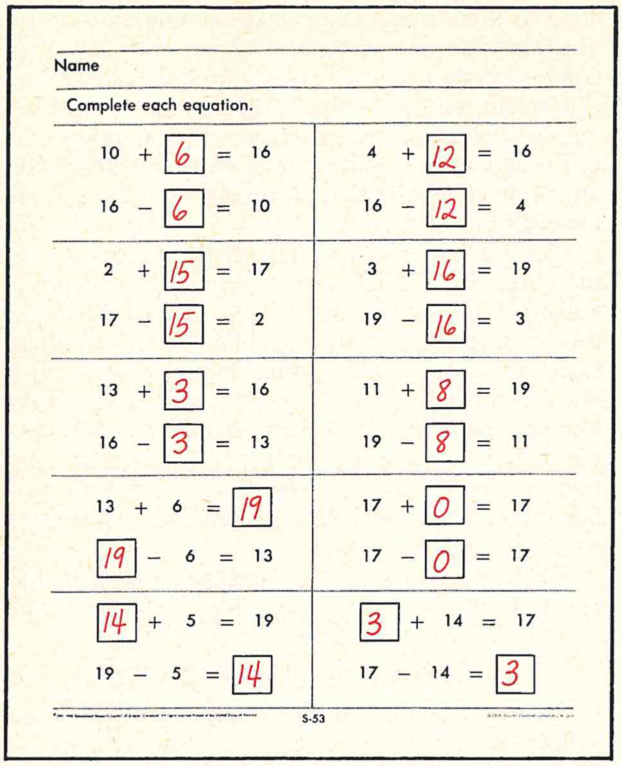
▶ Use the pocket chart as a place-value device. Put one bundle of ten red strips of paper in the tens place and eight separate blue strips of paper in the ones place. Remove four strips from the eight strips. Call on a child to write the related vertical notation form of the problem on the chalkboard:

$$\begin{array}{r} 18 \\ -\ 4 \\ \hline 14 \end{array}$$

Have the problem read.

Continue in this way to build similar set combinations and have the children write the corresponding algorisms. Include one or two addition problems in the activity.

▶ Work page S-53 re-emphasizes the relation between addition and subtraction. Work the first pair of equations with the children, and let them work the

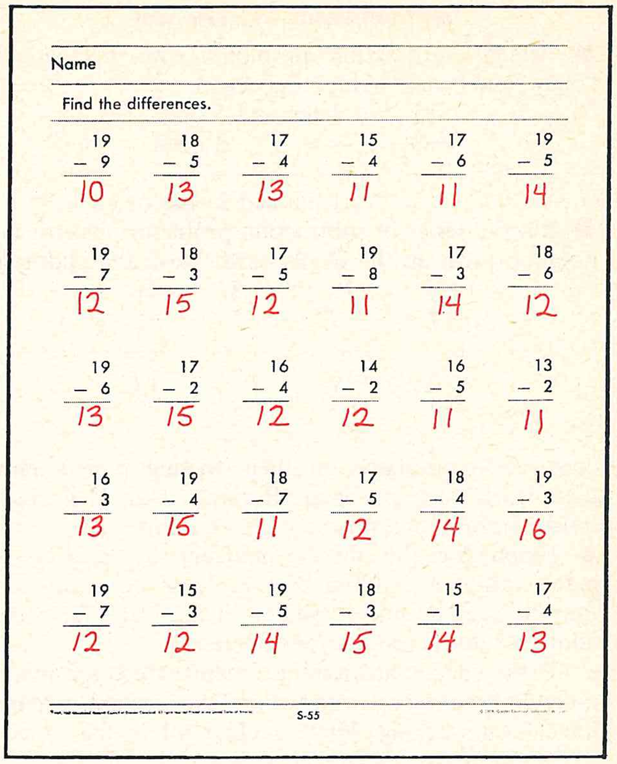


Name

Complete each equation.

$10 + \boxed{6} = 16$ $16 - \boxed{6} = 10$	$4 + \boxed{12} = 16$ $16 - \boxed{12} = 4$
$2 + \boxed{15} = 17$ $17 - \boxed{15} = 2$	$3 + \boxed{16} = 19$ $19 - \boxed{16} = 3$
$13 + \boxed{3} = 16$ $16 - \boxed{3} = 13$	$11 + \boxed{8} = 19$ $19 - \boxed{8} = 11$
$13 + 6 = \boxed{19}$ $\boxed{19} - 6 = 13$	$17 + \boxed{0} = 17$ $17 - \boxed{0} = 17$
$\boxed{14} + 5 = 19$ $19 - 5 = \boxed{14}$	$\boxed{3} + 14 = 17$ $17 - 14 = \boxed{3}$

S-53

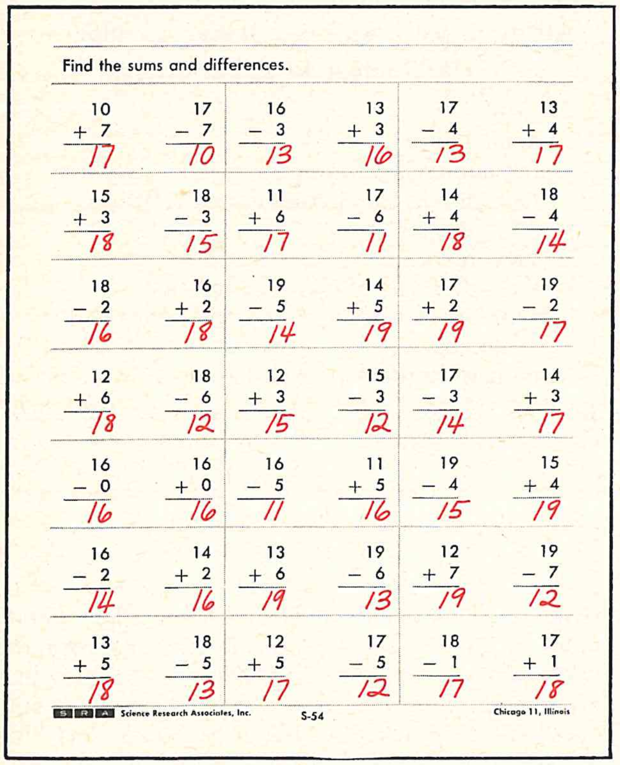


Name

Find the differences.

19 − 9 10	18 − 5 13	17 − 4 13	15 − 4 11	17 − 6 11	19 − 5 14
19 − 7 12	18 − 3 15	17 − 5 12	19 − 8 11	17 − 3 14	18 − 6 12
19 − 6 13	17 − 2 15	16 − 4 12	14 − 2 12	16 − 5 11	13 − 2 11
16 − 3 13	19 − 4 15	18 − 7 11	17 − 5 12	18 − 4 14	19 − 3 16
19 − 7 12	15 − 3 12	19 − 5 14	18 − 3 15	15 − 1 14	17 − 4 13

S-55



Find the sums and differences.

10 + 7 17	17 − 7 10	16 − 3 13	13 + 3 16	17 − 4 13	13 + 4 17
15 + 3 18	18 − 3 15	11 + 6 17	17 − 6 11	14 + 4 18	18 − 4 14
18 − 2 16	16 + 2 18	19 − 5 14	14 + 5 19	17 + 2 19	19 − 2 17
12 + 6 18	18 − 6 12	12 + 3 15	15 − 3 12	17 − 3 14	14 + 3 17
16 − 0 16	16 + 0 16	16 − 5 11	11 + 5 16	19 − 4 15	15 + 4 19
16 − 2 14	14 + 2 16	13 + 6 19	19 − 6 13	12 + 7 19	19 − 7 12
13 + 5 18	18 − 5 13	12 + 5 17	17 − 5 12	18 − 1 17	17 + 1 18

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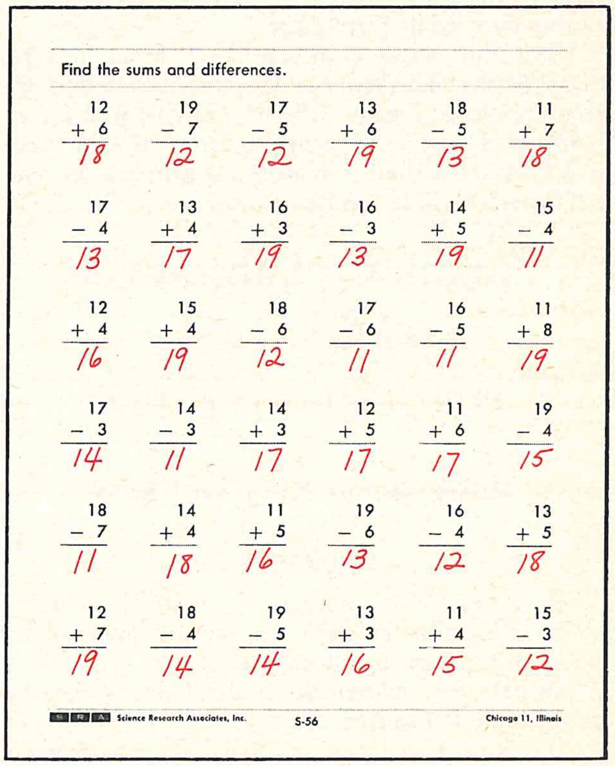

Find the sums and differences.

12 + 6 18	19 − 7 12	17 − 5 12	13 + 6 19	18 − 5 13	11 + 7 18
17 − 4 13	13 + 4 17	16 + 3 19	16 − 3 13	14 + 5 19	15 − 4 11
12 + 4 16	15 + 4 19	18 − 6 12	17 − 6 11	16 − 5 11	11 + 8 19
17 − 3 14	14 − 3 11	14 + 3 17	12 + 5 17	11 + 6 17	19 − 4 15
18 − 7 11	14 + 4 18	11 + 5 16	19 − 6 13	16 − 4 12	13 + 5 18
12 + 7 19	18 − 4 14	19 − 5 14	13 + 3 16	11 + 4 15	15 − 3 12

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rest of the page independently.

Work page S-54 provides practice in addition and subtraction combinations related to sums not greater than nineteen, by using the vertical notation form. Direct the children to find the sums or differences. Their awareness of the inverse relation between addition and subtraction should enable them to complete this page quickly.

Work pages S-55 and S-56 provide review of addition and subtraction combinations with sums not greater than nineteen. Have the children find the sums or differences.

Supplemental Experiences

▶ Present subtraction problems with the words "ones" and "tens" written into them.

$$\begin{array}{r} 15 \\ -3 \\ \hline \end{array} \qquad \begin{array}{rr} 1 \text{ ten and } 5 \text{ ones} \\ - \qquad 3 \text{ ones} \\ \hline 1 \text{ ten and } 2 \text{ ones or } 12 \end{array}$$

▶ Place a series of subtraction problems in vertical-notation form on the chalkboard. Have the children copy these problems on their writing papers and then work each problem. Remind them that they must carefully line up the ones and tens.

$$\begin{array}{r} 9 \\ -7 \\ \hline \end{array} \quad \begin{array}{r} 19 \\ -7 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -5 \\ \hline \end{array} \quad \begin{array}{r} 18 \\ -5 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 16 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 17 \\ -3 \\ \hline \end{array}$$

▶ Emphasize the inverse concept by playing a game with the children that involves no computation, but just an understanding of the idea that what addition "does" subtraction "undoes."

On the chalkboard write problems which are more difficult than those with which the second-graders have been working. Have a child fill in the placeholder and explain his thinking: *"If* forty-three plus twenty-two equals sixty-five, *then* sixty-five minus twenty-two equals forty-three."

$$43 + 22 = 65$$
$$65 - 22 = \square$$

Vary the activity; present oral brain teasers. Say, "Bobby had fifty marbles. One day he lost fifteen marbles in a game. The next day he won fifteen marbles. How many marbles does Bobby have now?" Ask the child to reason this problem out and tell the class how he got his answer.

Addition—Subtraction

COMBINATIONS

THROUGH NINETEEN

Work Pages S-57 and S-58

Objectives

To emphasize the concept of equality in equations.

To strengthen the children's understanding of the concepts of addition and subtraction.

To help the children think about the operational sign required in a given equation.

To provide practice in completing placeholder equations.

Fundamentals

The teacher will want to continue to encourage the child to use the facts he already knows about the operations of addition and subtraction to find the missing operational symbol in equations such as $14 \triangle 12 = 2$. She should continue to challenge the child to think about the particular operation on two numbers that gives a particular result. When the child is taught to think only in terms of one aspect of a problem, he is severely limited in his approach to number concepts. The skillful teacher offers the child many and varied opportunities to be creative in his attack on numbers; she encourages him to re-examine each known idea.

The teacher continually checks to see whether the child has a good understanding of concepts involving numbers, operations, and relations. She tests this understanding by posing different kinds of questions pertaining to these concepts: "What number must be added to twelve to give nineteen?" "What number plus thirteen gives eighteen?" She asks the child to explain how he got his answers to questions such as these. She guides the child to think flexibly and to find relations between the ideas to which he has been exposed.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of While Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concepts of addition and subtraction.

Ability to determine the operational sign required in given equations.

Ability to work with addition and subtraction combinations through nineteen.

Ability to work with placeholder equations.

Developmental Experiences

MATERIALS

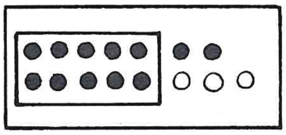
- flannel board
- felt numerals 0 through 9
- felt symbols for operations and relations
- felt placeholder
- set cards
- individual sets of plastic numerals
- symbols and triangles for operations and relations
- numeral cards 0 through 19

▶ Write placeholder equations such as the following on the chalkboard:

$$3 + \square = 7 \qquad \square - 7 = 2$$
$$13 + \square = 17 \qquad \square - 7 = 12$$

$$\square - 5 = 1 \qquad 8 - \square = 4$$
$$\square - 5 = 11 \qquad 18 - \square = 14$$

Have the children complete each equation. Guide them to an awareness of the relation between the two equations in each pair. Help them realize that knowing how to work the equation involving only ones will help when working equations involving tens and ones. Remind them to do the problems with two-digit numerals by adding or subtracting the ones first, and then putting this result with the tens to get the answer.



▶ Have children use plastic numerals and other symbols on their desks to build the equation 12 △ 3

= 15. Hold up a picture card showing a set illustration of the number combination. Ask the children to tell what they think was done with these sets in order to make a set of fifteen objects. Have them place the correct operational sign in the triangle in their equation and read the completed statement.

Have the children place the equation 18 △ 2 = 16 on their desks. Hold up a card showing a set illustration related to the subtraction combination. Ask the children to tell what action they think took place in order to have sixteen objects. Have them place the correct operational sign in the triangle and read the completed equation. Continue in this way to work with sets and equations in which the operational sign is required. Use addition and subtraction combinations related to sums of eleven through nineteen.

Vary the activity. Give directions for the equation you want them to build: 11 △ 2 = 13, or 14 △ 3 = 11. After the children have replaced the triangle with the correct operational sign, have the equation read. Have the children tell the name of the operational sign they chose, the name of the operation the symbol represents, and why they chose this symbol.

▶ Play the game, "Number Please." Scatter a set of numeral cards 0 through 19 on a table next to the flannel board. Put the placeholder equation 16 − ☐ = 13 on the flannel board. Tell the children that they are going to play a "pretend game." They will have to pretend that they can "hear the equation talk." The equation will ask them to find the number needed to make it a complete sentence. From the numeral cards on the table, they must choose the one needed to replace the placeholder.

The teacher pretends to be "the voice of the placeholder." When she says, "Number please, John," the child called upon must complete the equation. Have counters available for children who need this concrete aid to find the solution to the problem.

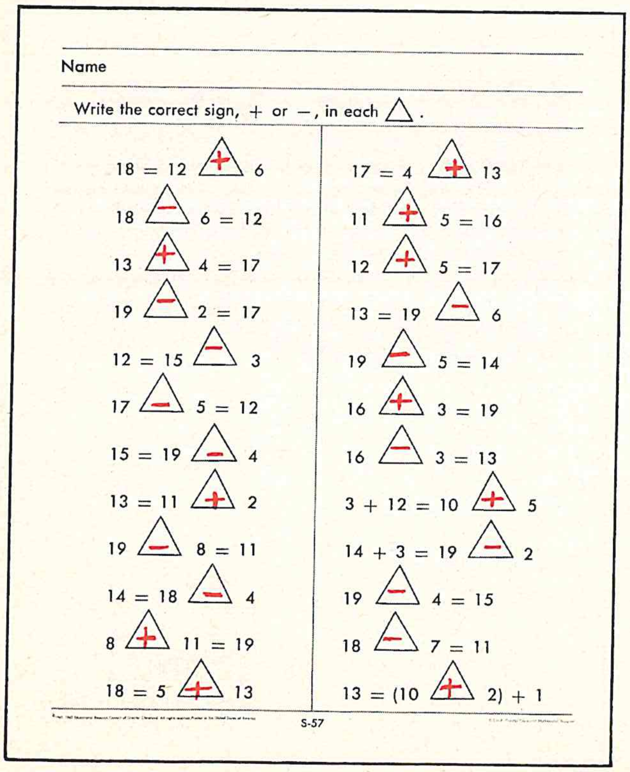
Continue in this way with addition and subtraction combinations related to sums not greater than nineteen. Keep the placeholder in the second position: 12 + ☐ = 16 or 14 − ☐ = 11.

▶ Put the placeholder equation ☐ + 1 = 16 on the flannel board. Ask the children to explain how they will solve this problem. They will probably say that they must add one to some number (fifteen) to get sixteen. Tell them that you want them to try another way to get the answer. If no one suggests subtraction, ask someone to try subtracting in this problem and see what happens. Ask him to tell what number he will subtract. Place the subtraction equation 16 − 1 = ☐ below the addition placeholder equation. When the answer fifteen results from the subtraction, have the numeral 15 taken from the subtraction equation and placed in the addition equation. Have the addition equation read. Ask whether it is now a true statement with 15 in the place of the placeholder.

Continue to have children "try out" this way of getting answers in addition placeholder equations. Keep the placeholder in the first position: ☐ + 2 = 14 or ☐ + 3 = 16.

▶ Use work page S-57 to give children additional opportunities to select the appropriate operational signs to complete equations correctly. Work the first equation with the group, and then have them complete the rest of the page independently.

Work page S-58 offers the children an opportunity to test themselves on their understanding of one-



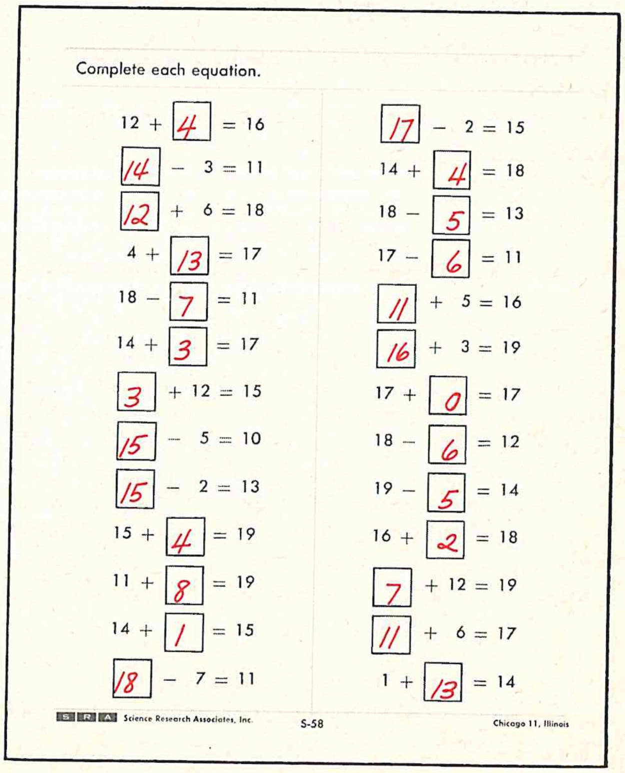
Name

Write the correct sign, + or −, in each △.

$18 = 12 \overset{+}{\triangle} 6$
$18 \overset{-}{\triangle} 6 = 12$
$13 \overset{+}{\triangle} 4 = 17$
$19 \overset{-}{\triangle} 2 = 17$
$12 = 15 \overset{-}{\triangle} 3$
$17 \overset{-}{\triangle} 5 = 12$
$15 = 19 \overset{-}{\triangle} 4$
$13 = 11 \overset{+}{\triangle} 2$
$19 \overset{-}{\triangle} 8 = 11$
$14 = 18 \overset{-}{\triangle} 4$
$8 \overset{+}{\triangle} 11 = 19$
$18 = 5 \overset{+}{\triangle} 13$

$17 = 4 \overset{+}{\triangle} 13$
$11 \overset{+}{\triangle} 5 = 16$
$12 \overset{+}{\triangle} 5 = 17$
$13 = 19 \overset{-}{\triangle} 6$
$19 \overset{-}{\triangle} 5 = 14$
$16 \overset{+}{\triangle} 3 = 19$
$16 \overset{-}{\triangle} 3 = 13$
$3 + 12 = 10 \overset{+}{\triangle} 5$
$14 + 3 = 19 \overset{-}{\triangle} 2$
$19 \overset{-}{\triangle} 4 = 15$
$18 \overset{-}{\triangle} 7 = 11$
$13 = (10 \overset{+}{\triangle} 2) + 1$

S-57



Complete each equation.

$12 + \boxed{4} = 16$
$\boxed{14} - 3 = 11$
$\boxed{12} + 6 = 18$
$4 + \boxed{13} = 17$
$18 - \boxed{7} = 11$
$14 + \boxed{3} = 17$
$\boxed{3} + 12 = 15$
$\boxed{15} - 5 = 10$
$\boxed{15} - 2 = 13$
$15 + \boxed{4} = 19$
$11 + \boxed{8} = 19$
$14 + \boxed{1} = 15$
$\boxed{18} - 7 = 11$

$\boxed{17} - 2 = 15$
$14 + \boxed{4} = 18$
$18 - \boxed{5} = 13$
$17 - \boxed{6} = 11$
$\boxed{11} + 5 = 16$
$\boxed{16} + 3 = 19$
$17 + \boxed{0} = 17$
$18 - \boxed{6} = 12$
$19 - \boxed{5} = 14$
$16 + \boxed{2} = 18$
$\boxed{7} + 12 = 19$
$\boxed{11} + 6 = 17$
$1 + \boxed{13} = 14$

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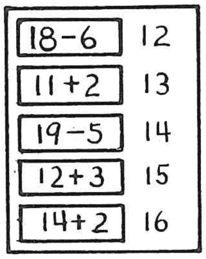
and two-digit addition and subtraction. Have them fill in the placeholder to complete each equation. When work on this page is completed, have the class discuss the thinking steps they used to arrive at their answers.

Supplemental Experiences

▶ Scatter five cards, on the flannel board, showing different numerals for numbers related to sums eleven through nineteen. Have a child unscramble the cards and place them on the flannel board in such a way that the numbers these numerals represent are ordered from least to greatest. Have a second child place the place-value numeral for these numbers next to the numeral cards as a check on the accuracy of the ordering. Continue in this way with other numerals for numbers related to sums of eleven through nineteen.



Story Problems

Work Pages S-59 and S-60

Objectives

To develop the ability to analyze and solve story problems.

To give practice in the use of addition and subtraction combinations to solve problems related to everyday life.

Fundamentals

One of the important areas in arithmetic is the application of known skills in problem-solving situations. A child may know how to add, but unless he knows when to add and why he must add, his knowledge is of little value to him. In early work with story problems, the teacher should give much help and guidance in reading and interpreting the problems. Some children may need assistance with the vocabulary encountered in the stories. Others must be guided in thinking through the problem situation presented in each story. By asking carefully structured questions, the teacher can lead the child to discover the answer to the problem.

Readiness for Understanding

Knowledge of addition and subtraction combinations to twenty.

Ability to read and understand simple story problems.

Developmental Experiences

MATERIALS

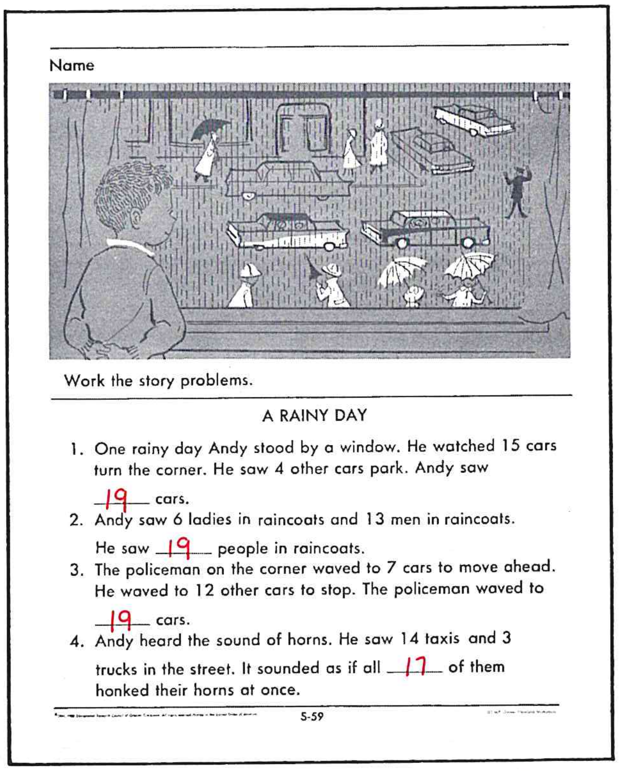
cards bearing
addition and
subtraction
combinations

▶ Have different children tell things they have done on a rainy day. Pass out work page S-59 and discuss the picture at the top of the page. Read the first story to find what Andy did on a rainy day. Ask one child to tell what he is asked to find in the story and a second child to tell what facts are given. Ask a third child to tell what operation to use in solving the problem and why he chose this operation. Now ask a fourth child to give his answer orally and tell how he got it, or have him work it on the chalkboard for all to see. Discuss with the class whether the answer is reasonable.

Read aloud the rest of the story problems on work page S-59 if the vocabulary is not familiar. Then have the children work independently.

Use the same procedure to introduce problems involving subtraction on work page S-60.

Go over the pages together by having each child check his own work. Find which stories caused trouble and give extra help to those children who had difficulty thinking through the problems.



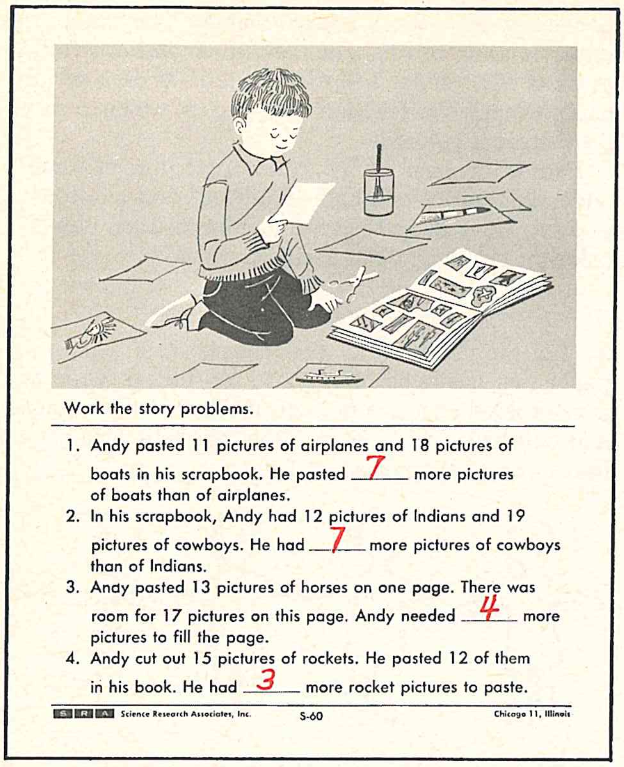
Name

Work the story problems.

A RAINY DAY

1. One rainy day Andy stood by a window. He watched 15 cars turn the corner. He saw 4 other cars park. Andy saw 19 cars.
2. Andy saw 6 ladies in raincoats and 13 men in raincoats. He saw 19 people in raincoats.
3. The policeman on the corner waved to 7 cars to move ahead. He waved to 12 other cars to stop. The policeman waved to 19 cars.
4. Andy heard the sound of horns. He saw 14 taxis and 3 trucks in the street. It sounded as if all 17 of them honked their horns at once.

S-59



Work the story problems.

1. Andy pasted 11 pictures of airplanes and 18 pictures of boats in his scrapbook. He pasted 7 more pictures of boats than of airplanes.
2. In his scrapbook, Andy had 12 pictures of Indians and 19 pictures of cowboys. He had 7 more pictures of cowboys than of Indians.
3. Andy pasted 13 pictures of horses on one page. There was room for 17 pictures on this page. Andy needed 4 more pictures to fill the page.
4. Andy cut out 15 pictures of rockets. He pasted 12 of them in his book. He had 3 more rocket pictures to paste.

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Supplemental Experiences

▶ Play the game of "Train" with cards bearing such equations as $7 + \square = 15$, $\square - 6 = 6$, and $8 + 9 = \square$. One child is the conductor and stands by the desk of another child. The teacher holds up a card and both children try to give the answer to the equation. If the conductor says the correct answer first, he moves on to the next child. He continues to be conductor as long as he gives the answer first. When another child responds with the correct answer first, he is the new conductor. The winner continues around the room until he misses.

Children who give an incorrect answer may be "switched" to a side track to study the missed combination. They can rejoin the "train" by giving the correct answer to a card missed by another student.

Fractional Numbers

Work Pages S-61 Through S-66

Objectives

To review the meaning of the fractional numbers one-half, one-third, and one-fourth.

To review the meaning of the fractions ½, ¼, ⅓.

Fundamentals

The child's work with fractional numbers in the first grade has given him some understanding of one-half as one part of an object which has been divided into two parts of the same size, and as one-half of a given set. He also has had experience with one-fourth and one-third. This understanding will be reinforced by further experiences in the second grade.

Just as a distinction is made between whole numbers and numerals, a distinction should also be made between fractional numbers and their written symbols, which are called fractions. For work with children the vertical form $\frac{1}{2}$ is preferred to the form ½.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Fractional Numbers and Fractions.

Readiness for Understanding

Ability to understand and recognize halves, thirds, and fourths as parts of a whole.

Ability to recognize equivalent sets.

Developmental Experiences

MATERIALS

paper shapes
felt shapes
flannel board

VOCABULARY

separate
one-half
halves
one-third
thirds
one-fourth
fourths
fraction

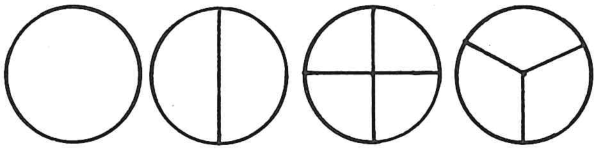
▶ Review the meaning of *"one-half."* Draw various shapes on the chalkboard and have different children come up and draw a line to *separate* each shape into two parts of the same size. Remind the children that each of the two parts is one-half of the whole shape and that there are two *halves* in one whole. Let the children fold and unfold papers to show two halves. Cut one of these papers in half to demonstrate that the halves are the same size. Ask if someone knows how to write a numeral that shows one-half. Let a child write ½ on the chalkboard. Say that this numeral is called a *fraction*. Each child may then write ½ on each half of his paper, and he will see that two halves equal one whole.

Work with halves of sets of objects in the room. Have the children count the number of desks in a row and determine how many there are in one-half of the row. Repeat this procedure with pencils on the teacher's desk and with other objects in the room.

▶ Review *one-fourth* and *one-third* by drawing shapes on the chalkboard and separating them into four or three parts of the same size. Have the children point out one-fourth or one-third of each shape. Relate the fractions ¼ and ⅓ to these fractional numbers. Have the children tell how many *fourths* or *thirds* are in one whole.

Arrange sets of three or four cutouts on the flannel board. Have the children point out one-fourth or one-third of each set. Repeat with sets of six, eight, nine, and twelve. Emphasize that the sets must be separated in such a way that there are the same number in each subset.

▶ Discuss the relative sizes of fractional parts of different shapes. Place a felt circle, two half-circles, four quarter-circles, and three one-third circles on the flannel board. Have the children compare the various parts of a circle. Lead them to conclude that one-half is greater than one-third, that one-fourth is less than one-third, and that one-half is greater than one-fourth.



▶ Use work pages S-61 and S-62 to reinforce the concept of one-half. On work page S-61 discuss the pictures at the top of the page. Direct the children to color one-half of each of the remaining pictures. On work page S-62 direct them to color one-half of the pictures in each box; remind them that the two parts of each set must contain the same number of objects.

▶ Pass out work page S-63. Discuss the pictures at the top of the page and then ask the children to tell how many fourths there are in one whole. Have them color one-fourth of each remaining picture.

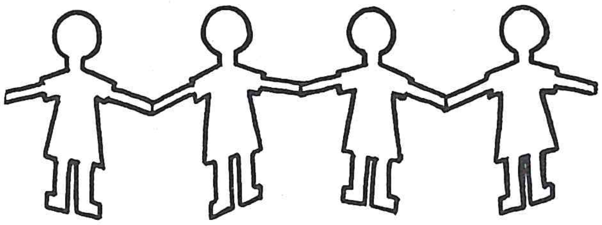
After a review of thirds and a discussion of the pictures, work page S-64 should be done independently. Have the children color one-third of each picture.

▶ Use work page S-65 to evaluate the children's understanding of one-half, one-third, and one-fourth as parts of a whole. Tell them to look at each picture and to mark the fraction that shows which part of the picture is colored.

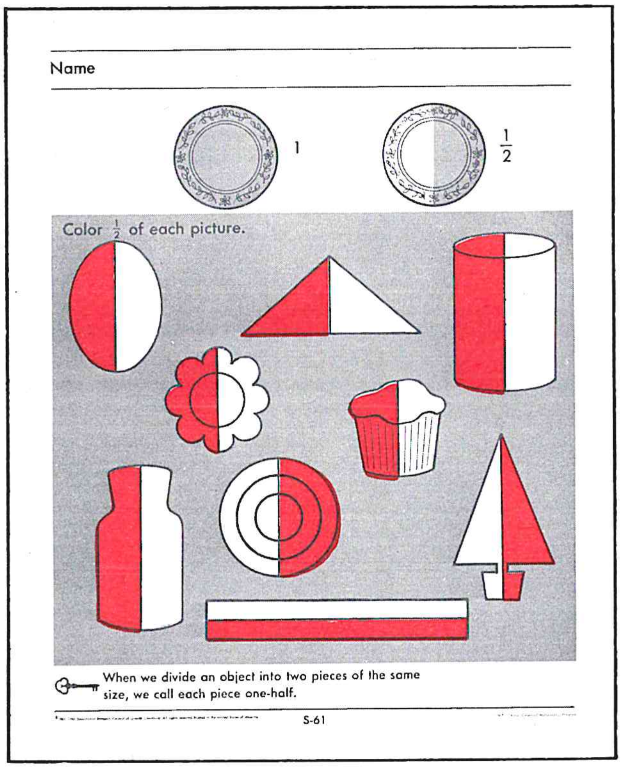
Use work page S-66 to evaluate the children's understanding of one-half, one-third, and one-fourth of a set. Have them read the fraction in each box and color the correct part of each set.

Supplemental Experiences

▶ Let the children cut paper dolls from a piece of paper that has been folded into fourths. If the hands at the folded edge are not cut, the dolls will be joined and can be opened out to show one-half of a set of four or one-fourth of a set of four.



▶ Provide packs of cards showing fractions, and give a pack to each child. The children play in pairs. At a given signal each child turns the top card up, showing a fraction. The one who has the fraction for the greater fractional number gets both cards. The winner is the one who has the most cards at the end of the game.



Name

1

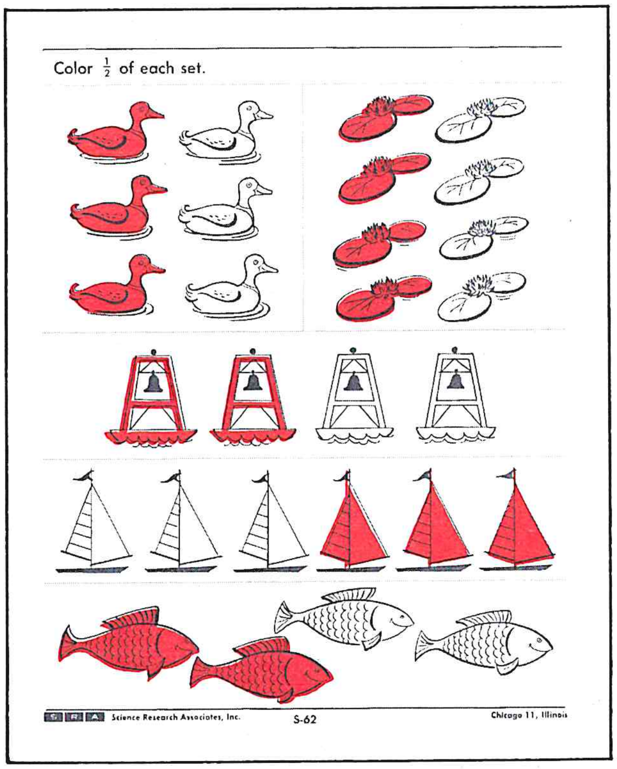
$\frac{1}{2}$

Color $\frac{1}{2}$ of each picture.

When we divide an object into two pieces of the same size, we call each piece one-half.

S-61

★ More than one correct answer is possible for each problem.



Color $\frac{1}{2}$ of each set.

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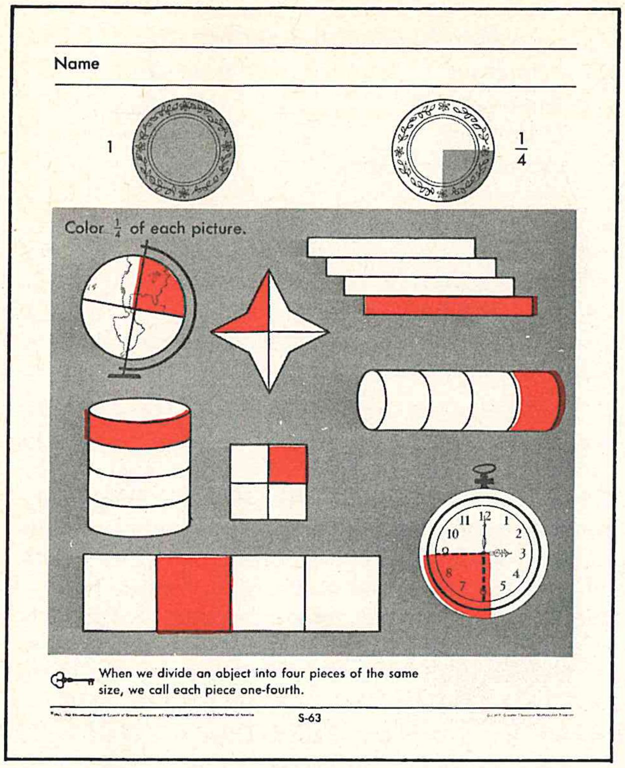
★ More than one correct answer is possible for each problem.



Name

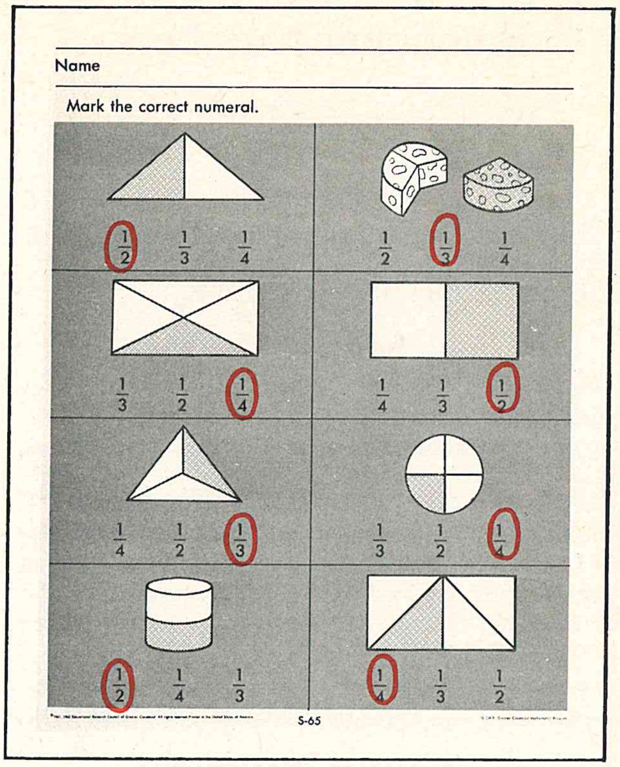
1 $\frac{1}{4}$

Color $\frac{1}{4}$ of each picture.

When we divide an object into four pieces of the same size, we call each piece one-fourth.

S-63

★ More than one correct answer is possible for each problem.



Name

Mark the correct numeral.

[✓ $\frac{1}{2}$] $\frac{1}{3}$ $\frac{1}{4}$

$\frac{1}{2}$ [✓ $\frac{1}{3}$] $\frac{1}{4}$

$\frac{1}{3}$ $\frac{1}{2}$ [✓ $\frac{1}{4}$]

$\frac{1}{4}$ $\frac{1}{3}$ [✓ $\frac{1}{2}$]

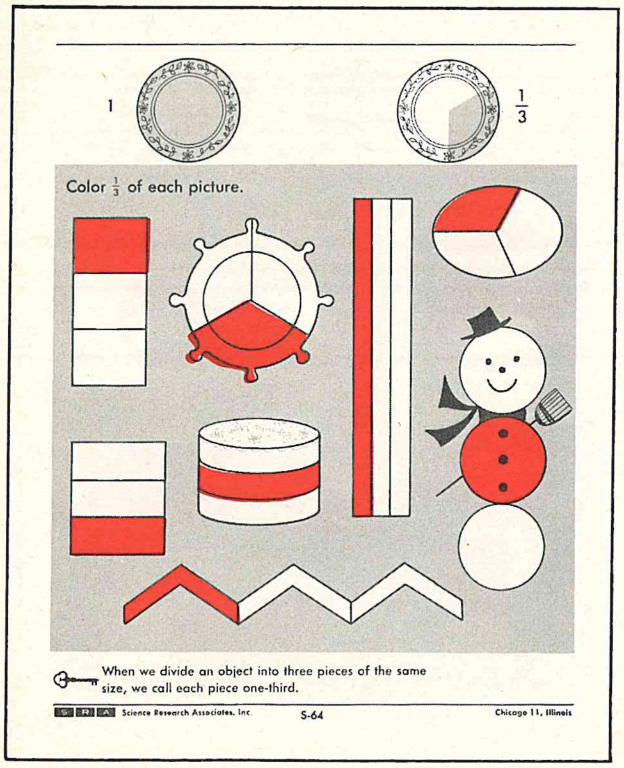
$\frac{1}{4}$ $\frac{1}{2}$ [✓ $\frac{1}{3}$]

$\frac{1}{3}$ $\frac{1}{2}$ [✓ $\frac{1}{4}$]

[✓ $\frac{1}{2}$] $\frac{1}{4}$ $\frac{1}{3}$

[✓ $\frac{1}{4}$] $\frac{1}{3}$ $\frac{1}{2}$

S-65



1 $\frac{1}{3}$

Color $\frac{1}{3}$ of each picture.

When we divide an object into three pieces of the same size, we call each piece one-third.

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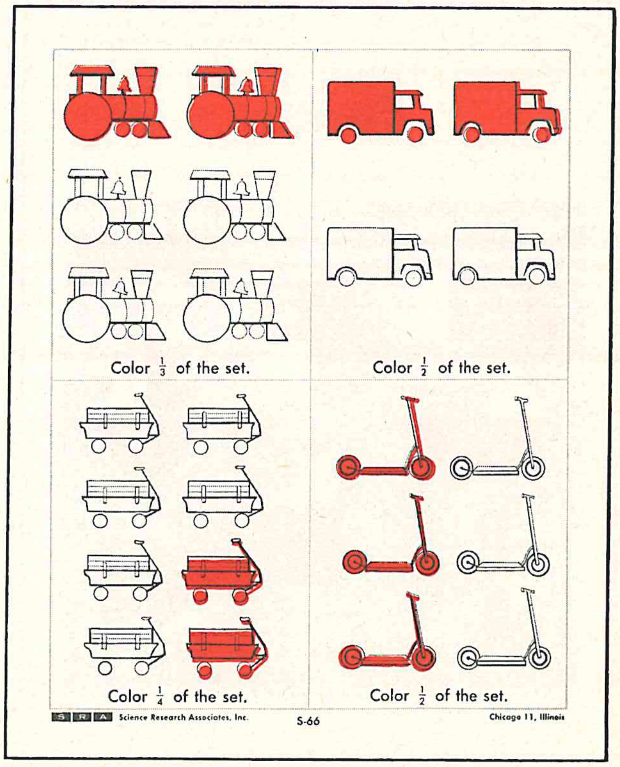
★More than one correct answer is possible for each problem.



Color $\frac{1}{3}$ of the set.

Color $\frac{1}{2}$ of the set.

Color $\frac{1}{4}$ of the set.

Color $\frac{1}{2}$ of the set.

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★ More than one correct answer is possible for each problem.

Fractional Numbers

Work Pages S-67 and S-68

Objectives

To extend the understanding of fractional numbers to include three-fourths and two-thirds.

To introduce the fractions ¾ and ⅔.

Fundamentals

Since the child has worked with one-half, one-third, and one-fourth, he should have an understanding of these fractional parts of a whole object. He has also separated sets into two, three and four equivalent subsets and has learned that each subset is one-half, one-third, or one-fourth of a given set. He will now have experiences to help him understand three-fourths and two-thirds as fractional parts of a whole. The fractions ¾ and ⅔ will be introduced to represent these fractional numbers.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Fractional Numbers and Fractions.

Readiness for Understanding

Ability to understand and recognize halves, thirds, and fourths as parts of a whole.

Ability to recognize equivalent sets.

Ability to recognize and write fractions ½, ⅓, and ¼.

Developmental Experiences

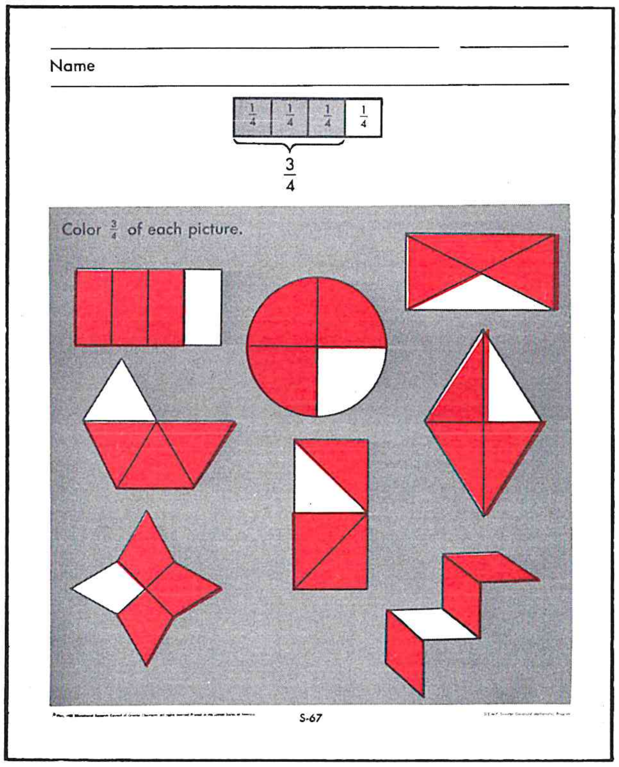
MATERIALS	VOCABULARY
flannel board	three-fourths
felt cutouts	two-thirds
cutout paper shapes	

▶ Take a large sheet of paper, fold it in half, and have a child name each part "one-half." Have him write the fraction ½ on the chalkboard. Now fold the paper in fourths. Call on four children to come forward and name each part "one-fourth." Have each one write the fraction ¼ on the chalkboard.

▶ To develop an understanding of *three-fourths,* draw a square on the chalkboard and mark it off into fourths. Remind the children of the work page where they colored one-fourth of each picture. Shade one-fourth of the square and have a child write the numeral ¼ on the shaded part. Shade one more fourth and ask how many fourths are now shaded. Have a child write ¼ on the second portion. Ask "How many fourths are shaded?" and "Who could write the fraction for two-fourths?" After $^2/_4$ has been written, shade one more fourth and have the children tell how many parts are shaded. Have one child write ¾. Ask how many fourths are not shaded and how many fourths are in the whole square.

Repeat this with a circle, having children come to the chalkboard and write each fraction as before.

▶ Develop an understanding of *two-thirds* by drawing different shapes on the chalkboard, dividing them

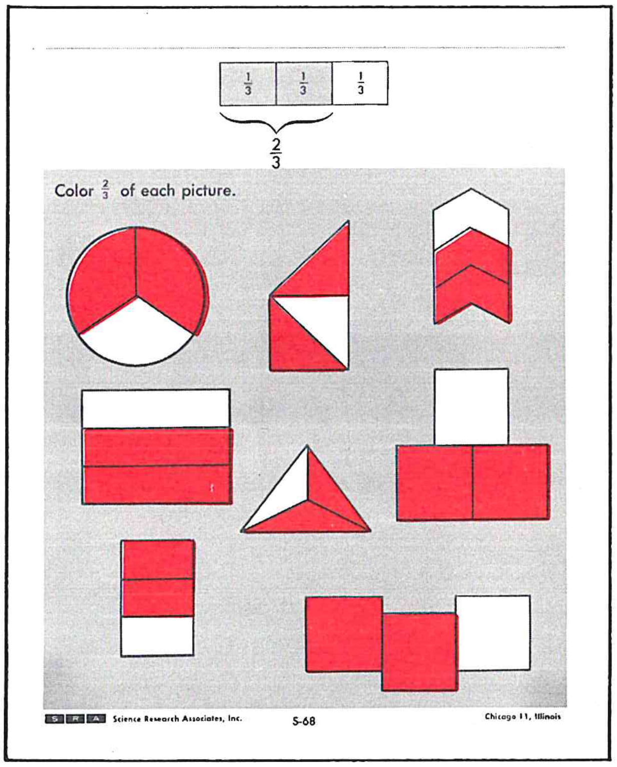


Name

$\underbrace{\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}}_{\frac{3}{4}}\ \frac{1}{4}$

Color $\frac{3}{4}$ of each picture.

S-67



$\underbrace{\frac{1}{3}\ \frac{1}{3}}_{\frac{2}{3}}\ \frac{1}{3}$

Color $\frac{2}{3}$ of each picture.

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★ More than one correct answer is possible for each problem.

★ More than one correct answer is possible for each problem.

into thirds and proceeding as above. Introduce the fraction ⅔ as representing two of three parts all of which are the same size.

▶ Pass out work page S-67 and have the children examine the picture at the top of the page. Ask them to look at the shaded part and tell what part of the whole it is. Have the children color three-fourths of each remaining picture. Work with the children on the first shape, and then let them finish alone.

Work page S-68 deals with thirds. Discuss the picture at the top of the page, asking how many parts it is divided into and how many thirds are shaded. Direct the children to color two-thirds of each of the other pictures. Work with them on the first two pictures then have them work independently.

Supplemental Experiences

▶ Let the children draw a row of patterns made by repeating some picture or geometric figure four times. Color three-fourths of the patterns. Make the next row a pattern repeated three times and color two-thirds of the patterns. These exercises can be repeated down the page in alternate rows.

▶ Cut rectangles, squares, and triangles into pieces like a puzzle and put them into envelopes. Each piece should be one-half, one-third, one-fourth, two-thirds, or three-fourths of the original figure. Have enough pieces in each envelope to form a whole shape.

When the child has completed a puzzle he writes the fractional parts used, as ¼ and ¾ or ⅓ and ⅓ and ⅓.

Place Value

Work Pages S-69 Through S-74

Objectives

To strengthen the children's understanding of place-value concepts.

To review the meaning and order of the numbers zero through ninety-nine.

To reinforce the children's understanding of the place-value numerals 10 through 99.

Fundamentals

As the place-value concept becomes clearer, the child learns that the cardinal number represented by each digit in a two-digit numeral is dependent both upon the digit itself and upon the place it occupies. In the numeral 66, for example, the 6 on the right (in the ones place) indicates that six ones are represented; the 6 on the left (in the tens place) indicates that six sets of ten are represented. The teacher will want to continue to point out this important feature of our decimal system of numeration.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Systems of Numeration.

Readiness for Understanding

Understanding of the numbers zero through nineteen.

Ability to recognize the numerals 0 through 19.

Some understanding of place-value concepts.

Developmental Experiences

MATERIALS

2 Countingmen
sticks
bead frame
numeral cards 0 through 99

▶ Use two Countingmen to review the place-value meaning of the numerals 10 through 19. Call on a child to place fingers on the Countingmen to show the number sixteen. Ask him to explain the way he put the fingers on the men ("The one finger on the Tens-man represents one ten, and the six fingers on the Ones-man represents six ones"). Call on another child to write the place-value numeral for this number on the chalkboard and to explain each of the digits in the numeral. Give several other children turns at showing numbers from zero through nineteen on the Countingmen and writing the corresponding place-value numerals.

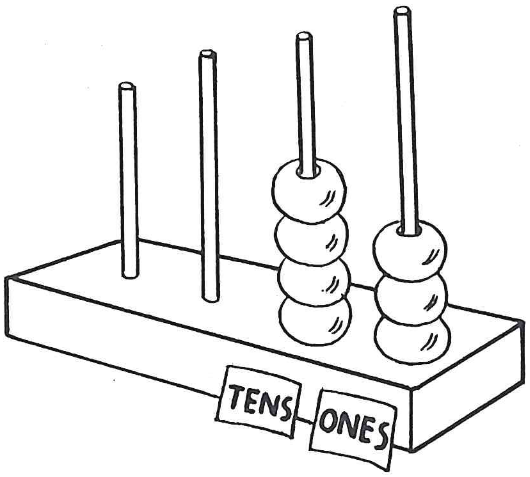
▶ Place fingers on the Countingmen to represent the number nineteen. Ask the children to tell how the Countingmen can show one more than nineteen. Place one more finger on the Ones-man. Some child may observe that he sees one ten and ten ones. Help the children recall that the ten ones can be recorded as one ten by placing another finger on the Tens-man and removing the ten fingers from the Ones-man. Then explain that the Ones-man is now ready to go on with his count. Guide the children to say that the Countingmen now represent two tens and zero ones, or twenty. Ask a child to write the numeral 20 on the chalkboard and to explain each of the digits in the numeral.

Have a child give the name of the number which is one more than twenty and show this number on the Countingmen. Have a second child write the corresponding numeral on the chalkboard and explain the place-value meaning of each of the digits in this numeral. Continue in similar fashion to develop an understanding of the meaning of the numbers twenty-two through ninety-nine. Have as many of these numbers as time allows represented on the Countingmen and the corresponding numerals displayed.

▶ Lay sixteen sticks on a desk. Tell the children that they are to pretend they don't know how to count any farther than ten. Lead them to see that they can still determine the number of sticks in the pile. First, have them count to ten and place a rubber band around this set of ten sticks. Then as they count beyond ten, they should count "one ten and one, one ten and two . . . one ten and six." They know how to record this count as 16. They have found the number of sticks without counting directly beyond ten. Repeat the activity with several other numbers from eleven through ninety-nine. Each time the count of ten is reached, have the children bundle the sticks together and precede their count with the number of tens they have on their desks. For example, two tens and three, two tens and four, and so forth.

The children might enjoy a verse about this method of counting:

Every time I count to ten,
I stop and start the count again.



▶ Use the bead frame to strengthen the children's understanding of the place-value numerals for the numbers eleven through ninety-nine. Label one of the rods in the bead frame "tens" and the other rod "ones." Have the children use colored beads to represent the number forty-three on the frame. (They should place four beads on the tens-rod and three beads on the ones-rod.) Ask a child to write the place-value numeral for this number on the chalkboard. Repeat the activity with several other numbers from eleven through ninety-nine.

▶ Play a matching game. Write a column of numerals on the chalkboard; to the right of this column, write a column of phrases that describe the place-value meaning of the numerals.

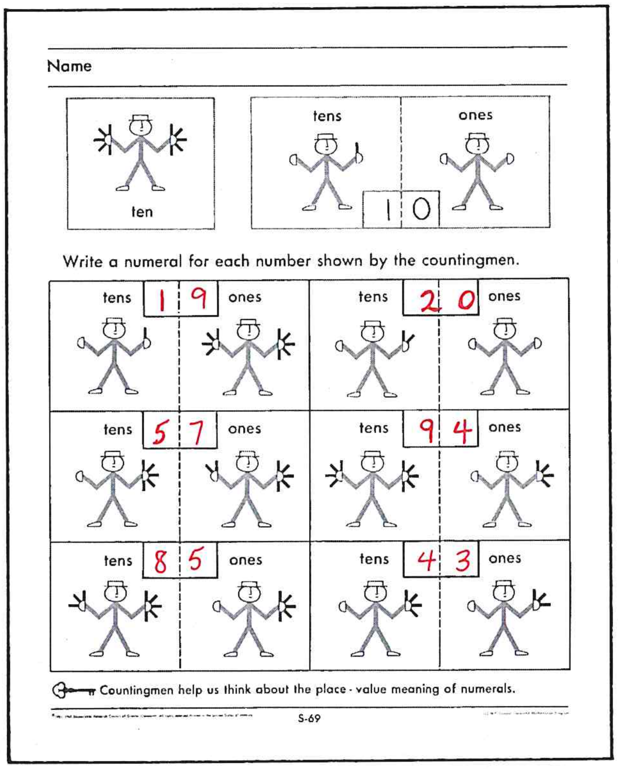
23	one ten and four ones
32	four tens and one one
41	three tens and two ones
14	two tens and three ones

Call on a child to read the first numeral in the first column. Then have him search the second column for the meaning of this numeral. Have him draw a line from the 23 to the phrase which reads, "two tens and three ones." Continue in similar fashion to have the other numerals matched with their meanings.

▶ Pass out five numeral cards, bearing the numerals 5, 16, 34, 38, and 47, to five of the children. Have two children at a time compare their cards to see which numeral represents the greater number. After this is done, have the five children line up in order from the least number to the greatest number.

Have the children discover shortcuts to quick recognition of the greater number by first looking to see which has the greater number of tens. For example, they can quickly see that 47 represents a greater number than 38 merely by comparing four tens and three tens. On the other hand, if the tens digit is the same in two numerals, such as 34 and 38, they must then examine the digits in the ones place to determine which numeral represents the greater number.

▶ Use work page S-69 to strengthen the concept of ten and to give the children practice in thinking about the place-value meaning of the digits in two-digit numerals. Discuss the pictures at the top of the page, pointing out that ten can be thought of as ten ones or as one ten and zero ones. In each of the boxes at the bottom of the page, have the children write the numeral for each number shown by the Countingmen.


Name

ten

tens | ones 1 0

Write a numeral for each number shown by the countingmen.

tens 1 9 ones

tens 2 0 ones

tens 5 7 ones

tens 9 4 ones

tens 8 5 ones

tens 4 3 ones

Countingmen help us think about the place-value meaning of numerals.

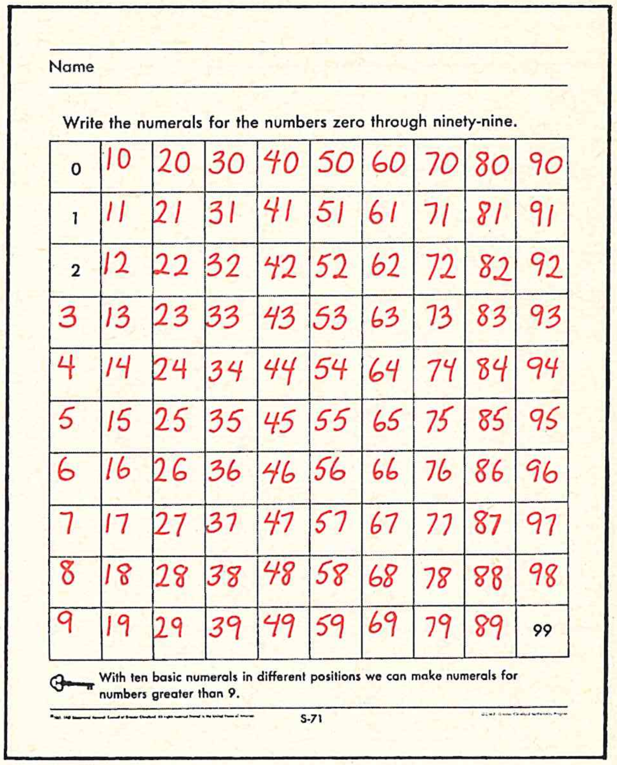
S-69

On work page S-70 the children will be given an opportunity to write two-digit numerals. Have them read each phrase and write the corresponding place-value numeral.

▶ The table on work page S-71 will be particularly valuable in that it points out the patterns evident in the numerals that represent the numbers zero through ninety-nine. Tell the children that they are to think of the order of these numbers and write numerals for them in the columns provided. Work down the first column and the first several numerals in column two with the children. Encourage them to complete the page independently. After they have completed the chart, discuss the pattern of the digits. Lead the children to discover that in each column the tens digit is the same, but that the ones digit increases from zero through nine. In each row, moving across the page from left to right, the digit in the ones position remains the same while the digit in the tens position increases by one. Some child may observe that this happens because ten is being added each time. Another child may note a very interesting pattern on the main diagonal where each numeral after zero has the same digit in the ones place as in the tens place, 11, 22, 33 . . . 99. Stress once again that only ten basic numerals, or digits, are used to make the numerals for the numbers greater than nine.

Use work page S-72 to evaluate the children's understanding of the place value of numerals. Tell them to fill each blank to tell the meaning of each digit in the numeral.

▶ Use work page S-73 for additional practice in recognition of two-digit numerals. Have the children read the numerals and draw the correct number of fingers on each Countingman. When this has been



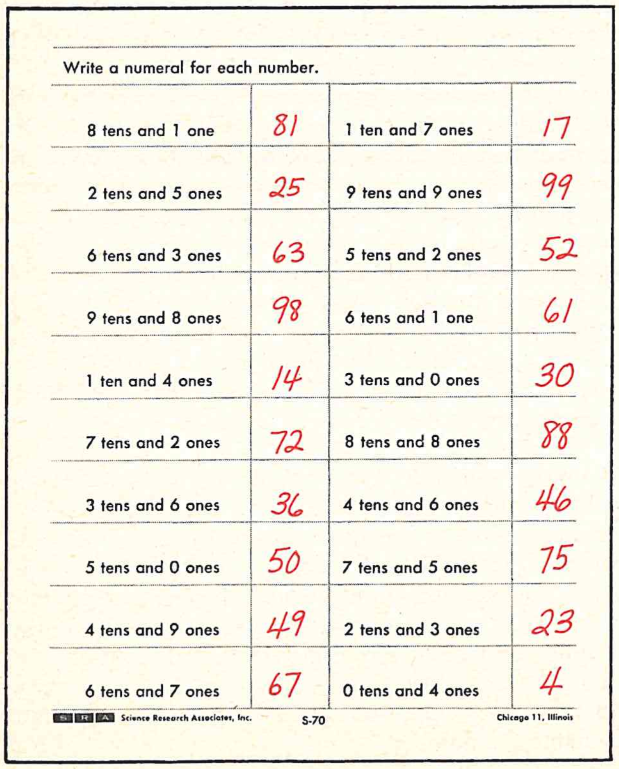
Name

Write the numerals for the numbers zero through ninety-nine.

0	10	20	30	40	50	60	70	80	90
1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99

With ten basic numerals in different positions we can make numerals for numbers greater than 9.

S-71



Write a numeral for each number.

8 tens and 1 one	81	1 ten and 7 ones	17
2 tens and 5 ones	25	9 tens and 9 ones	99
6 tens and 3 ones	63	5 tens and 2 ones	52
9 tens and 8 ones	98	6 tens and 1 one	61
1 ten and 4 ones	14	3 tens and 0 ones	30
7 tens and 2 ones	72	8 tens and 8 ones	88
3 tens and 6 ones	36	4 tens and 6 ones	46
5 tens and 0 ones	50	7 tens and 5 ones	75
4 tens and 9 ones	49	2 tens and 3 ones	23
6 tens and 7 ones	67	0 tens and 4 ones	4

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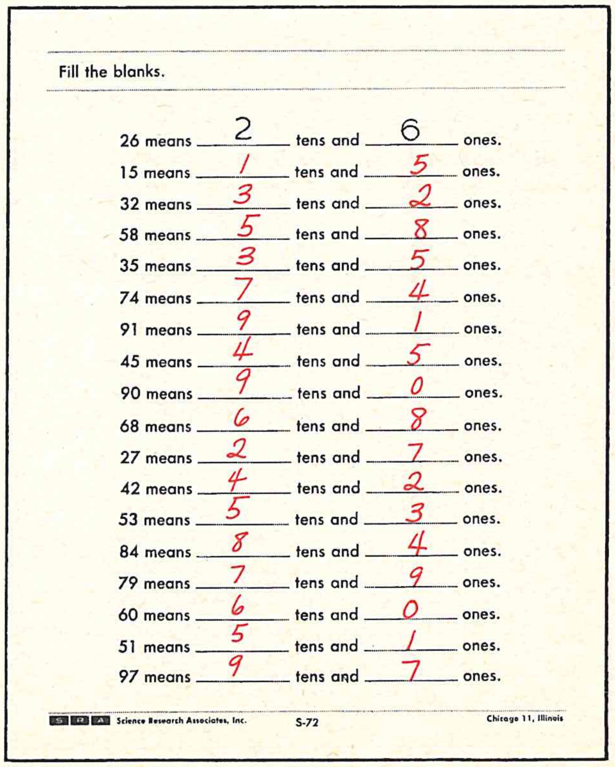

Fill the blanks.

26 means 2 tens and 6 ones.
15 means 1 tens and 5 ones.
32 means 3 tens and 2 ones.
58 means 5 tens and 8 ones.
35 means 3 tens and 5 ones.
74 means 7 tens and 4 ones.
91 means 9 tens and 1 ones.
45 means 4 tens and 5 ones.
90 means 9 tens and 0 ones.
68 means 6 tens and 8 ones.
27 means 2 tens and 7 ones.
42 means 4 tens and 2 ones.
53 means 5 tens and 3 ones.
84 means 8 tens and 4 ones.
79 means 7 tens and 9 ones.
60 means 6 tens and 0 ones.
51 means 5 tens and 1 ones.
97 means 9 tens and 7 ones.

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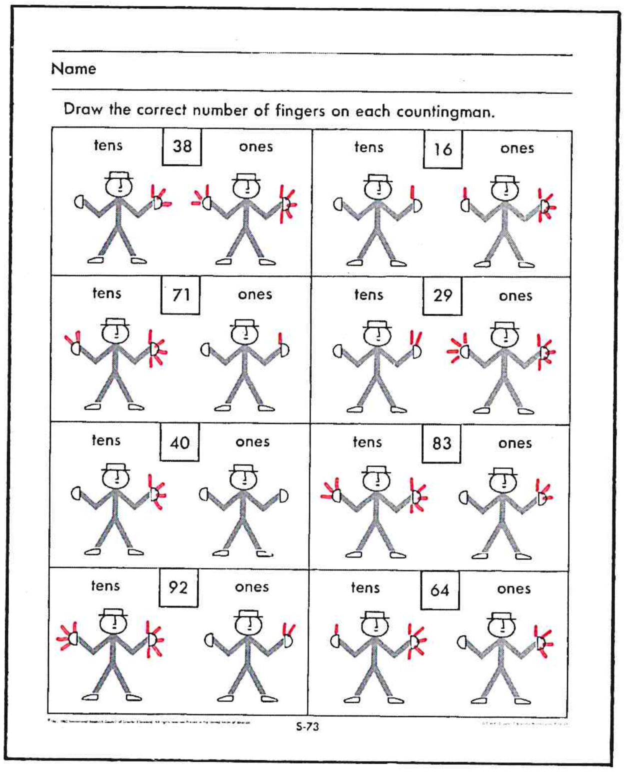
completed, have them explain the place-value meaning of each numeral.

On work page S-74 tell the children to write the standard place-value numeral for the number of each set pictured.



Name

Draw the correct number of fingers on each countingman.

tens	38	ones	
		tens	16	ones

tens 38 ones

tens 16 ones

tens 71 ones

tens 29 ones

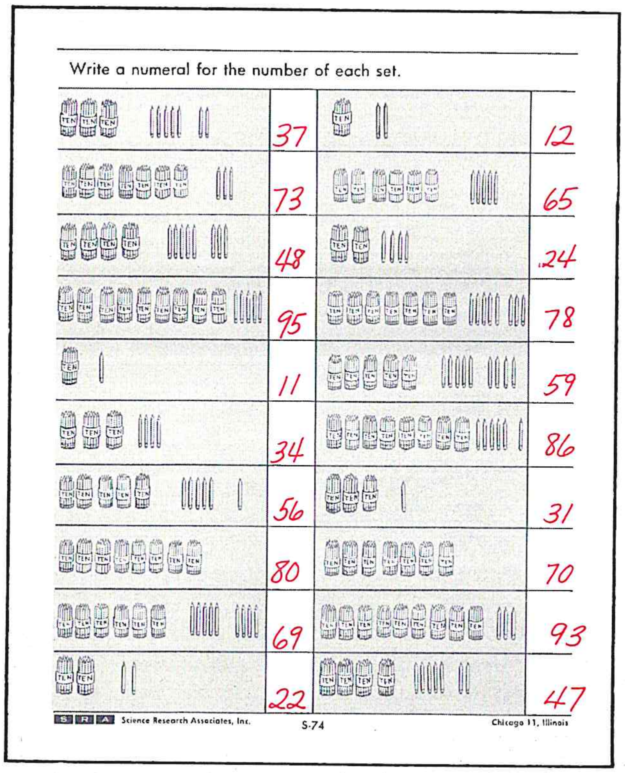
tens 40 ones

tens 83 ones

tens 92 ones

tens 64 ones

S-73



Write a numeral for the number of each set.

37	12
73	65
48	24
95	78
11	59
34	86
56	31
80	70
69	93
22	47

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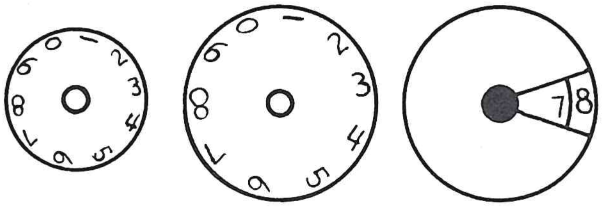
Supplemental Experiences

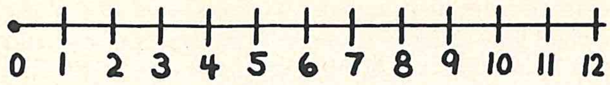
▶ Construct a place-value wheel for work with small groups and with individual children. Cut two twelve-inch circles and one eight-inch circle from heavy tagboard. Write the numerals 0 through 9 round the eight-inch circle and one of the twelve-inch circles, as illustrated. Cut a wedge from the other twelve-inch circle. Place the eight-inch circle on top of the numbered twelve-inch circle, and the remaining circle on top of them both. Fasten the three circles together through the center with a paper fastener. Dial a numeral on the wheel and ask a child to tell what number this numeral represents. Ask him to tell the place-value meaning of the numeral. Dial a different numeral and continue the activity. On some occasions ask a child to find a numeral for a given number. When he has found this numeral, ask him to explain it in terms of place value.



▶ Arrange ten rows of ten cup hooks each on a plywood board two feet square. Write the numerals 0 through 99 on key tags. Keep the tags labeled 0 through 3, and pass the others out to the children. Now place the tag labeled 0 on the first hook, the 1 on the second hook in the first column, the 2 on the third hook, and the 3 below the 2. Ask the children to name the numbers that these numerals stand for. Now ask who has the numeral for the number which is one more than three. Have the child with the 4 come to the front of the room and place his numeral tag on the hook below the 3. Continue in the same way until all the tags are placed in order by columns on the board. When all the tags have been correctly placed, discuss the pattern of the numerals and the digits that appear in these numerals.

Ask the children to close their eyes. Quickly remove one of the key tags from the board. As the children open their eyes ask them to tell which tag is missing. Ask them to explain how they determined their answer.

▶ Write numerals 0 through 9 on one end of each of ten tongue depressors. Place these tongue depressors numeral end down in a can which has been labeled "ones." Write numerals 1 through 9 on nine more tongue depressors. Place them numeral end down in a can which has been labeled "tens." Call on a child to come to the front of the room and choose one stick from each can. Have him tell how many tens and how many ones these digits represent and to name the number.



▶ The use of a number line often helps children become more aware of the orderly arrangement of the whole numbers. Such a line can be drawn on the chalkboard, or painted on the floor or on the playground. The children can make their own number line using adding-machine tape or strips of heavy brown paper. Help them mark off lengths of the same measure and label each mark beginning with 0 and extending through 99. Display this number line in a prominent place. Call on a child to begin at 16 and count aloud by ones to 24, pointing out the successive marks on the number line. Help the children to see that the greater the number, the greater its distance from zero.

Addition

SUMS OF ELEVEN AND TWELVE

Work Pages S-75 Through S-78

Objectives

To introduce addition of two numbers, each less than ten, whose sum is eleven or twelve.

To help children apply the associative property.

Fundamentals

The child applied the associative property when he learned to add two numbers such as eleven and two. When he now learns to add two numbers, each of which is less than ten and whose sum is greater than ten, he will again apply this property.

If the child were to count from nine to eleven, and think in terms of place-value concepts, he would count, nine, ten, ten plus one. In counting from eight to thirteen he would think of eight, nine, ten, ten plus one, ten plus two, ten plus three. In each instance the child arrives at ten in the counting process, then starts to count again from one, prefacing the count each time with the ten.

This is an important concept for the child to understand; when he counts in the decimal system of numeration, he pauses at ten (or a multiple of ten) and then continues the count. If he wishes to arrive at the answer to $8 + 4 = \square$ quickly, he arrives mentally at the ten first and then goes on from there: eight plus two, ten, ten plus two, twelve. A closer examination of the steps in his thinking reveals that the associative property of addition is involved.

In solving the problem, the child will learn to think of four as two plus two: $8 + 4 = 8 + (2 + 2)$. Since he needs two to associate with eight to get to ten, he must use this particular numeral for the number four. Arriving at ten, he adds the other two and has the answer: $10 + 2$ or 12.

We can summarize what really happened in the process of adding eight and four.

$$\begin{aligned} 8 + 4 &= 8 + (2 + 2) && \text{different numeral for the number four} \\ &= (8 + 2) + 2 && \text{associative property} \\ &= 10 + 2 && \text{addition } (8 + 2) \\ &= 12 && \text{place-value notation} \end{aligned}$$

The child will learn to think about these properties in easy steps. Beginning his work with sets, he learns to regroup the set of eight and the set of four as a set of ten and a set of two. He will eventually learn to think of the numeral $8 + 4$ and the numeral $10 + 2$ as representing the number twelve. He will be carefully guided to think about the ideas involved in this type of problem. He will not be asked to write all of the steps. Under direct teacher guidance, he will be asked to fill in parts of the steps to indicate his understanding of the concept. The child will have taken a giant step forward in his understanding of number concepts when he becomes fully aware of what happens in problems of this type.

Children in the second grade may have been exposed to these concepts at the first-grade level but the importance of complete understanding of these harder combinations makes it imperative that a carefully structured development be presented at this grade level.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Understanding of the associative property.

Understanding of the concept of different numerals for a number.

Understanding of the concept of place value.

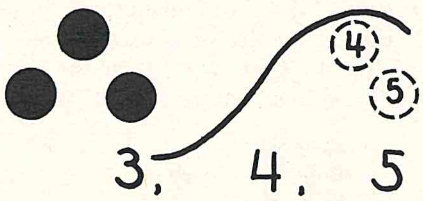
Mastery of addition combinations to ten.

Ability to work with placeholder equations.

Developmental Experiences

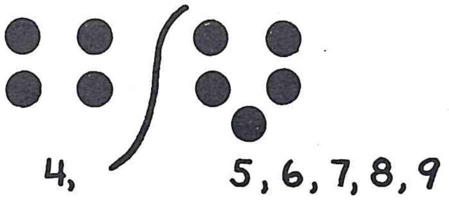
MATERIALS

tagboard cards
flannel board
felt counters
pocket chart



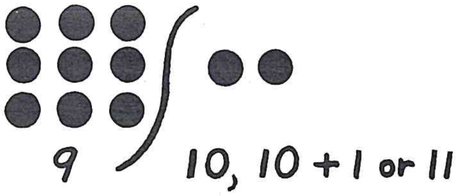
▶ Ask the children to think back to their work with easy combinations such as three plus two as you place a set of three cutouts and a set of two cutouts on the flannel board and join the sets. Remind the children that they could find out how many are in the joined sets by starting with three, the number of one of the sets, and counting four, five to get the answer. Help them recall that they know a quicker way of doing problems like this; they know how to add. They now say, "Three plus two equals five." It is important for the children to realize that they would know what to do even if they couldn't remember the answer to 3 + 2 = ☐. They would know how to count by ones, starting with three and moving to five. Stress the importance of knowing what to do, just in case the answers to problems are forgotten.

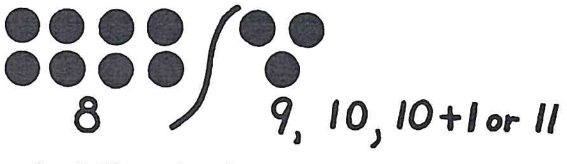
Ask the children to explain what they would do if they wanted to add four and five. They could start with four and then count five, six, seven, eight, nine.



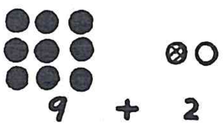
Have someone tell of a quicker way to do this problem: four plus five equals nine.

Tell the class that it is possible to do the same thing when they work with such numbers as nine and two. If they want to find out how much nine and two is, they can start with nine and count ten, ten plus one, or eleven. Ask the children if they noticed what happens when the count passes ten. Guide them to an awareness of the fact that they start to count all over again, remembering the ten. Say to them, "Let's find out how much eight and three are by counting." Have the class count, eight, nine, ten, ten plus one, eleven.

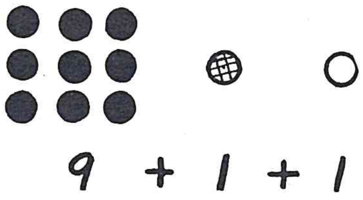


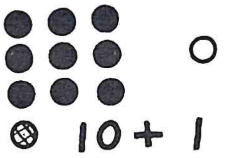


Pose a problem: What really happens when we add nine and two? Place the related sets on the flannel board. Have counters of a second color in the set of two. Put this part, 9 + 2, of the story on a tagboard card and place the card in the pocket chart.

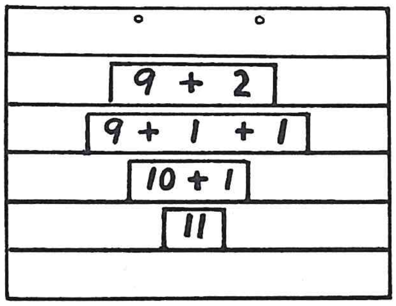


Ask, "When counting to numbers greater than nine, what number did we stop at before we could continue counting?" Tell the children that they will also stop at ten when working with the two sets on the flannel board. Say, "Let's regroup the sets of nine and two so that one of the sets makes ten." Ask what must be done first in order to make a set of ten. As a child gets ready to take one from the set of two and put it with the set of nine, stop the action. Put the story, 9 + 1 + 1, on a card and put it in the pocket chart. Now have the child put the set of one with the set of nine as he started out to do. Put the story, 10 + 1, in the pocket chart, and show the short way of writing it as the numeral 11.





Summarize the steps involved in adding nine and two for the class. Refer to the cards in the pocket chart as you review the ideas: 9 + 2 becomes 9 + (1 + 1), 1 + 1 is another numeral for the number two; (9 + 1) + 1 can be thought of as 10 + 1 (10 is another numeral for nine plus one); and a short form for 10 + 1 is the numeral 11.

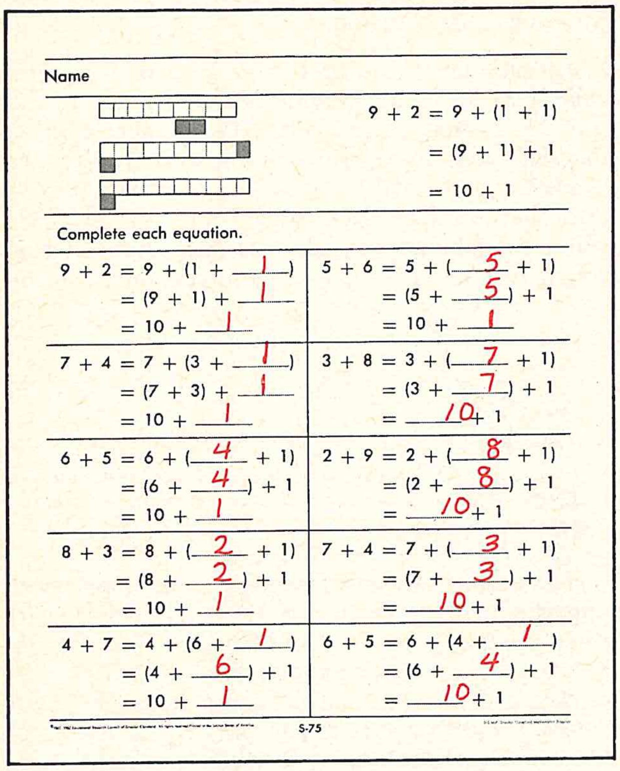


Follow this same procedure to develop 8 + 3, 7 + 4, 6 + 5, 5 + 6, 4 + 7, 3 + 8, and 9 + 2, as well as all the addition combinations of twelve.

▶ Use work pages S-75 and S-76 to provide the children with practice in finding sums of eleven and twelve. Work through each page with them. Show the pupils how the parentheses will help them discover the different numerals for numbers involved in the problem.

▶ On work page S-77 the necessary steps used to help the children justify the addition of two numbers whose sum is eleven or twelve are condensed. This encourages the children to think through the addition of seven and four in terms of ten, the base of our system of numeration. Work the first two problems on the page with the children. Then encourage them to complete the remaining equations by themselves.

Use work page S-78 to review addition combinations of eleven and twelve. Tell the children that whenever they are required to write a numeral in a placeholder, they are to use the standard place-value numerals rather than the expanded-notation form.


Name

$9 + 2 = 9 + (1 + 1)$
$= (9 + 1) + 1$
$= 10 + 1$

Complete each equation.

$9 + 2 = 9 + (1 + 1)$
$= (9 + 1) + 1$
$= 10 + 1$

$7 + 4 = 7 + (3 + 1)$
$= (7 + 3) + 1$
$= 10 + 1$

$6 + 5 = 6 + (4 + 1)$
$= (6 + 4) + 1$
$= 10 + 1$

$8 + 3 = 8 + (2 + 1)$
$= (8 + 2) + 1$
$= 10 + 1$

$4 + 7 = 4 + (6 + 1)$
$= (4 + 6) + 1$
$= 10 + 1$

$5 + 6 = 5 + (5 + 1)$
$= (5 + 5) + 1$
$= 10 + 1$

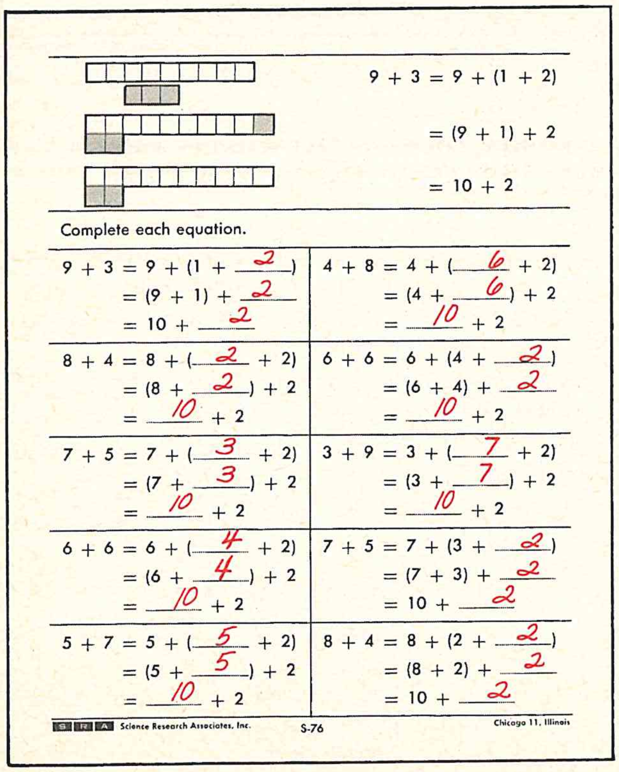
$3 + 8 = 3 + (7 + 1)$
$= (3 + 7) + 1$
$= 10 + 1$

$2 + 9 = 2 + (8 + 1)$
$= (2 + 8) + 1$
$= 10 + 1$

$7 + 4 = 7 + (3 + 1)$
$= (7 + 3) + 1$
$= 10 + 1$

$6 + 5 = 6 + (4 + 1)$
$= (6 + 4) + 1$
$= 10 + 1$

S-75


$9 + 3 = 9 + (1 + 2)$
$= (9 + 1) + 2$
$= 10 + 2$

Complete each equation.

$9 + 3 = 9 + (1 + 2)$
$= (9 + 1) + 2$
$= 10 + 2$

$8 + 4 = 8 + (2 + 2)$
$= (8 + 2) + 2$
$= 10 + 2$

$7 + 5 = 7 + (3 + 2)$
$= (7 + 3) + 2$
$= 10 + 2$

$6 + 6 = 6 + (4 + 2)$
$= (6 + 4) + 2$
$= 10 + 2$

$5 + 7 = 5 + (5 + 2)$
$= (5 + 5) + 2$
$= 10 + 2$

$4 + 8 = 4 + (6 + 2)$
$= (4 + 6) + 2$
$= 10 + 2$

$6 + 6 = 6 + (4 + 2)$
$= (6 + 4) + 2$
$= 10 + 2$

$3 + 9 = 3 + (7 + 2)$
$= (3 + 7) + 2$
$= 10 + 2$

$7 + 5 = 7 + (3 + 2)$
$= (7 + 3) + 2$
$= 10 + 2$

$8 + 4 = 8 + (2 + 2)$
$= (8 + 2) + 2$
$= 10 + 2$

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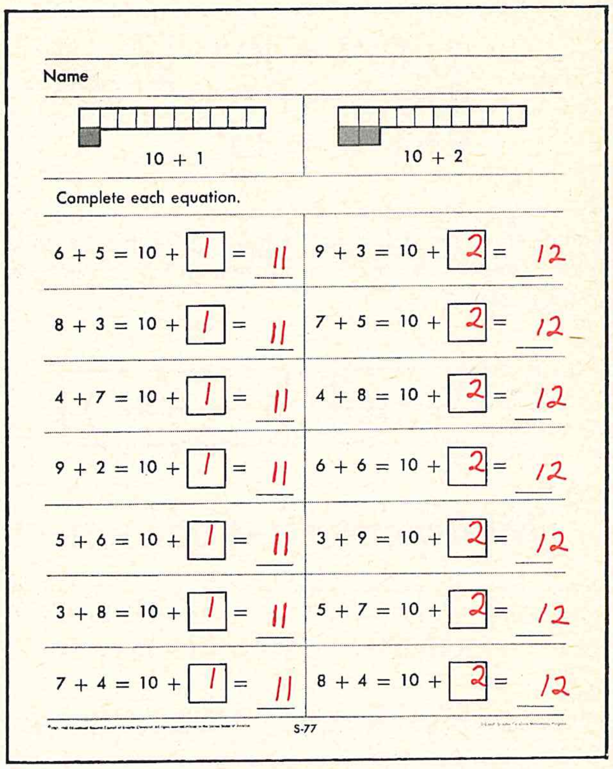

Name

10 + 1

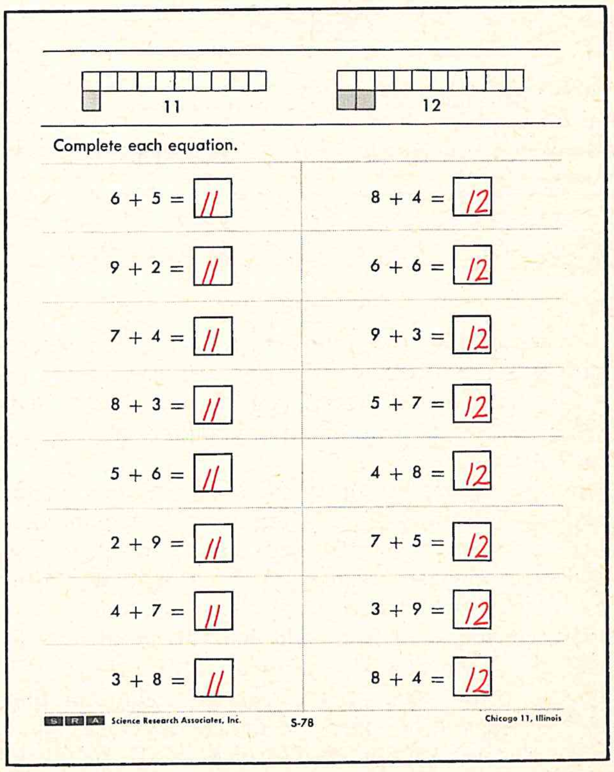
10 + 2

Complete each equation.

$6 + 5 = 10 + \boxed{1} = 11$
$8 + 3 = 10 + \boxed{1} = 11$
$4 + 7 = 10 + \boxed{1} = 11$
$9 + 2 = 10 + \boxed{1} = 11$
$5 + 6 = 10 + \boxed{1} = 11$
$3 + 8 = 10 + \boxed{1} = 11$
$7 + 4 = 10 + \boxed{1} = 11$

$9 + 3 = 10 + \boxed{2} = 12$
$7 + 5 = 10 + \boxed{2} = 12$
$4 + 8 = 10 + \boxed{2} = 12$
$6 + 6 = 10 + \boxed{2} = 12$
$3 + 9 = 10 + \boxed{2} = 12$
$5 + 7 = 10 + \boxed{2} = 12$
$8 + 4 = 10 + \boxed{2} = 12$

S-77


11

12

Complete each equation.

$6 + 5 = \boxed{11}$
$9 + 2 = \boxed{11}$
$7 + 4 = \boxed{11}$
$8 + 3 = \boxed{11}$
$5 + 6 = \boxed{11}$
$2 + 9 = \boxed{11}$
$4 + 7 = \boxed{11}$
$3 + 8 = \boxed{11}$

$8 + 4 = \boxed{12}$
$6 + 6 = \boxed{12}$
$9 + 3 = \boxed{12}$
$5 + 7 = \boxed{12}$
$4 + 8 = \boxed{12}$
$7 + 5 = \boxed{12}$
$3 + 9 = \boxed{12}$
$8 + 4 = \boxed{12}$

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Supplemental Experiences

▶ To provide additional practice in working with different numerals for numbers, design a series of tagboard cards using equations such as:
Expose the cards, one at a time, asking the children to read the equation and give the correct answer.

$$\begin{aligned} 9+1+1 &= 10+\square \\ 9+2 &= 10+\square \\ 9+2 &= \square \end{aligned}$$

▶ Design flash cards for classroom work with combinations of ten. They are helpful in building a solid foundation for the children in their quick recognition of these sums.

$\boxed{3+\square=10}$ $\boxed{\square+6=10}$

Subtraction—Addition

COMBINATIONS OF ELEVEN AND TWELVE

Work Pages S-79 Through S-82

Objectives

To introduce subtraction combinations of eleven and twelve.
To re-emphasize the inverse relation between addition and subtraction.
To provide practice in working with addition and subtraction combinations of eleven and twelve.

Fundamentals

When adding two numbers less than ten whose sum is greater than ten, the child learns to think of getting to ten and then going beyond. He is now ready to "un-do" what he did by adding; he is ready to subtract.

Since subtracting a number is the inverse of adding the same number, it is logical to think that the same ideas will be used, but in reverse. The child learns that he can count from eight to eleven in solving the problem $8 + 3 = \square$, but that a more efficient way is to add the two numbers. He will also learn that in the problem $11 - 8 = \square$, he can count back from eleven to eight, but that subtracting will be more efficient. If the child counts back for the solution of $11 - 8 = \square$, he might think eleven (or ten plus one), ten, nine, eight. If he were to count back in the problem $13 - 6 = \square$, he might count thirteen (or ten plus three), twelve (or ten plus two), eleven (or ten plus one), ten, nine, eight, seven. In both problems, the child starts with numbers represented by ten and a digit, arrives at ten, then continues with numbers represented by a single digit numeral.

To subtract in the problem $13 - 6 = \square$, the child must first learn to think of the number he must subtract in order to get to ten. The number is three, thirteen minus three brings him to ten. He has subtracted a part of the six that is to be subtracted; three must still be subtracted. From ten, the child will now subtract the three to get seven. Let us summarize the steps involved in solving this problem:

$$\begin{aligned} 13 - 6 &= (13 - 3) - 3 \\ &= 10 - 3 \\ &= 7 \end{aligned}$$

The child will learn to solve subtraction problems of this type in easy steps. He will be carefully guided to think about the ideas and to complete the related equations.

The teacher will continue to give the child every opportunity to apply his understanding of the inverse relation between addition and subtraction as he works with subtraction equations related to sums of eleven and twelve. She will continue to provide an environment conducive to the discovery of new ideas. She will continue to give the child every chance to apply his skills and understandings to new problems.

The equation form, or the mathematical sentence, is used to help the child focus his full attention on the ideas involved. After he has an understanding of the harder combinations, then vertical notation is introduced.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concept of different numerals for the same number.

Awareness of the inverse relation between addition and subtraction.

Mastery of subtraction combinations related to sums of ten.

Ability to work with placeholder equations and vertical notation.

Developmental Experiences

MATERIALS

flannel board
felt counters
pegboard cards
pocket chart

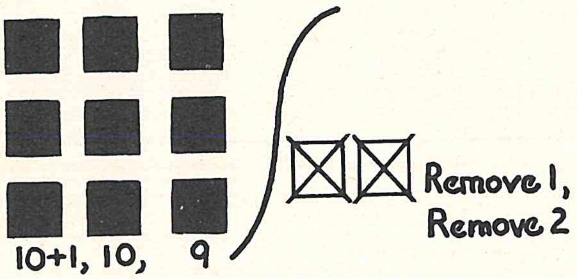
▶ Write on the chalkboard, in descending order, the numerals from 10 + 9 to 1. Ask the children to name the numbers from ten plus nine (nineteen) to one, in descending order.

10 + 9		9
10 + 8		8
10 + 7		7
10 + 6		6
10 + 5	$\boxed{10}$	5
10 + 4		4
10 + 3		3
10 + 2		2
10 + 1		1

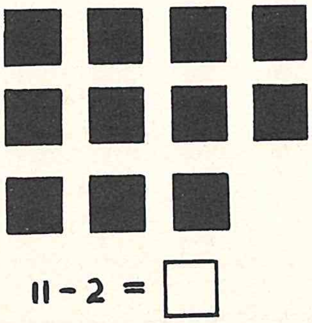
Ask the children to tell what number they stopped at in the process of counting back. Guide them to an understanding of the concept that from 10 on back, they are reading numerals with only one digit.

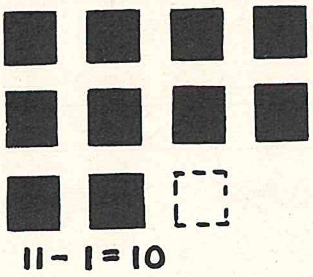
▶ Join a set of two to a set of nine on the flannel board. Have the children recall the slow way of finding the number of the joined set. They could start with the number of the first set and count: nine, ten, ten plus one, or eleven.

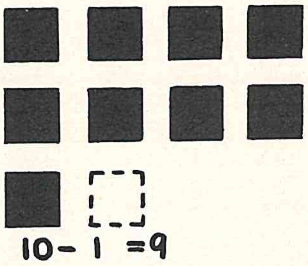
Remind the class that if they want to find out the number of objects that remain after two objects have been removed from a set of eleven, they could also count: ten plus one, ten, nine. Ask the children to tell you the name of the operation related to removing a part of a set: they call this operation subtraction.



Put the equation 11 − 2 = ☐ in the pocket chart. Then tell the children that they are to remove two from the set of eleven; that they should do it slowly, at first, in order to see what happens. Remind them to move to ten first, and to remove as many objects as necessary to form a set of ten. After removing the one object, place a card that shows this much of the action, 11 − 1 = 10, in the pocket chart. Ask the children to tell how many objects were to be removed according to the equation in the pocket chart; and if any more objects must be removed. After one more object has been removed, place the equation 10 − 1 = 9 related to this action, in the pocket chart.

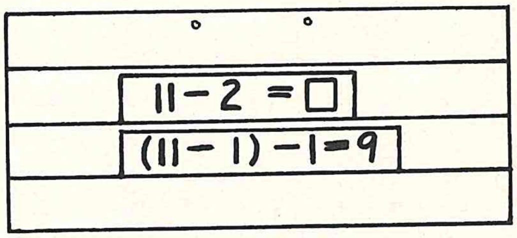






Remove the two cards 11 − 1 = 10 and 10 − 1 = 9 from the pocket chart. Replace them with a card that shows a shorter way of writing the story: (11 − 1) − 1 = 9.

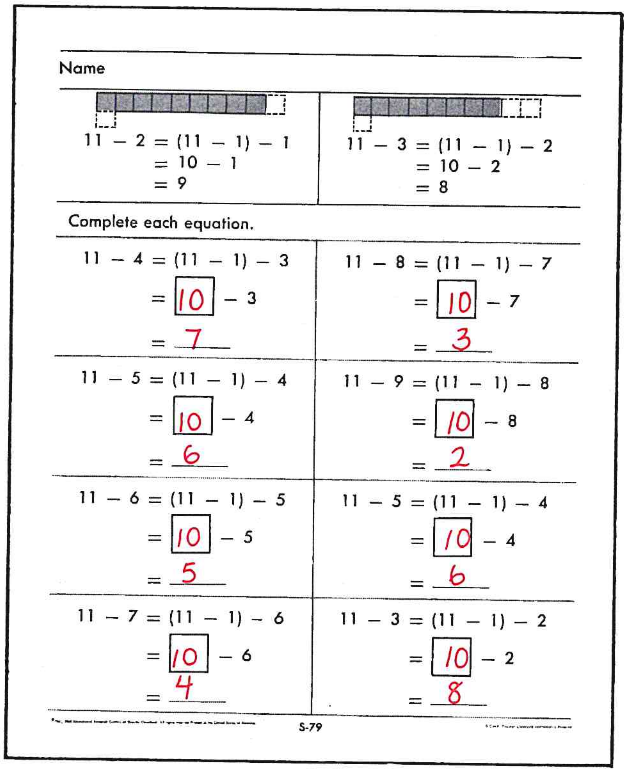
Summarize the steps involved in thinking through the problem 11 − 2 = ☐. Refer to the cards in the pocket chart and have the children help review the ideas: Eleven minus two can be thought of as (eleven minus one) minus one; (eleven minus one) minus one is thought of as ten minus one; ten minus one equals nine. The subtraction of two from eleven can be completed in two steps.



Follow the same procedure to develop other subtraction combinations 11 − 3 = ☐ through 11 − 9 = ☐ and 12 − 3 = ☐ through 12 − 9 = ☐.

▶ Use work pages S-79 and S-80 to provide the children with practice in working through the steps involved in subtraction problems related to sums of eleven and twelve. Work both pages with the children. The parentheses will help point out the different numerals for numbers involved in the problem as the children subtract "in pieces."

▶ Work page S-81 will re-emphasize the inverse relation between addition and subtraction. If children work easily with this concept in the simpler setting of combinations less than ten, they should be reassured by the fact that the inverse idea holds with combinations related to sums greater than ten. Work the



Name

$11 - 2 = (11 - 1) - 1$
$= 10 - 1$
$= 9$

$11 - 3 = (11 - 1) - 2$
$= 10 - 2$
$= 8$

Complete each equation.

$11 - 4 = (11 - 1) - 3$
$= \boxed{10} - 3$
$= 7$

$11 - 8 = (11 - 1) - 7$
$= \boxed{10} - 7$
$= 3$

$11 - 5 = (11 - 1) - 4$
$= \boxed{10} - 4$
$= 6$

$11 - 9 = (11 - 1) - 8$
$= \boxed{10} - 8$
$= 2$

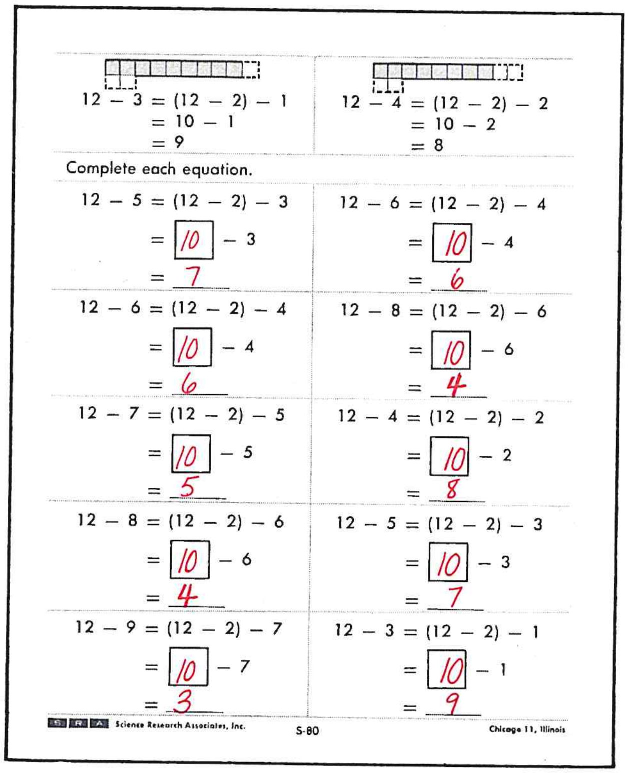
$11 - 6 = (11 - 1) - 5$
$= \boxed{10} - 5$
$= 5$

$11 - 5 = (11 - 1) - 4$
$= \boxed{10} - 4$
$= 6$

$11 - 7 = (11 - 1) - 6$
$= \boxed{10} - 6$
$= 4$

$11 - 3 = (11 - 1) - 2$
$= \boxed{10} - 2$
$= 8$

S-79



$12 - 3 = (12 - 2) - 1$
$= 10 - 1$
$= 9$

$12 - 4 = (12 - 2) - 2$
$= 10 - 2$
$= 8$

Complete each equation.

$12 - 5 = (12 - 2) - 3$
$= \boxed{10} - 3$
$= 7$

$12 - 6 = (12 - 2) - 4$
$= \boxed{10} - 4$
$= 6$

$12 - 6 = (12 - 2) - 4$
$= \boxed{10} - 4$
$= 6$

$12 - 8 = (12 - 2) - 6$
$= \boxed{10} - 6$
$= 4$

$12 - 7 = (12 - 2) - 5$
$= \boxed{10} - 5$
$= 5$

$12 - 4 = (12 - 2) - 2$
$= \boxed{10} - 2$
$= 8$

$12 - 8 = (12 - 2) - 6$
$= \boxed{10} - 6$
$= 4$

$12 - 5 = (12 - 2) - 3$
$= \boxed{10} - 3$
$= 7$

$12 - 9 = (12 - 2) - 7$
$= \boxed{10} - 7$
$= 3$

$12 - 3 = (12 - 2) - 1$
$= \boxed{10} - 1$
$= 9$

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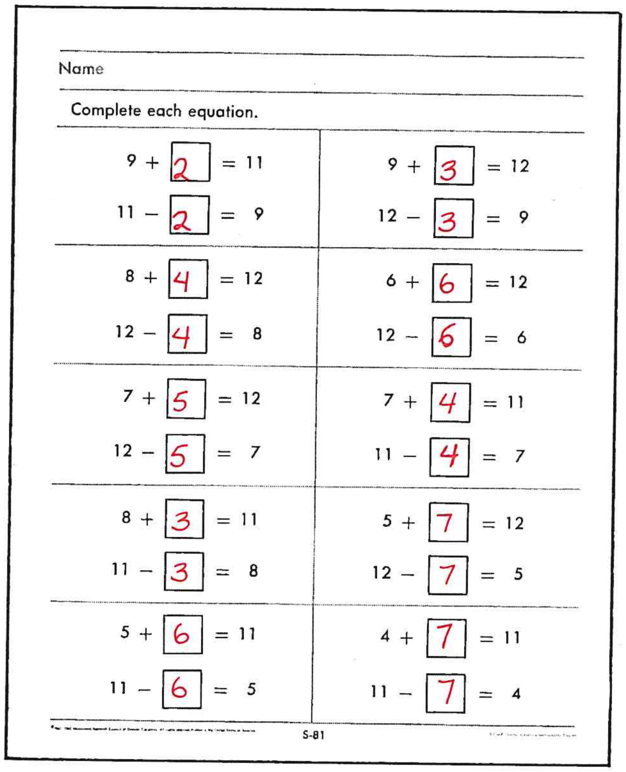


Name

Complete each equation.

$9 + \boxed{2} = 11$
$11 - \boxed{2} = 9$

$9 + \boxed{3} = 12$
$12 - \boxed{3} = 9$

$8 + \boxed{4} = 12$
$12 - \boxed{4} = 8$

$6 + \boxed{6} = 12$
$12 - \boxed{6} = 6$

$7 + \boxed{5} = 12$
$12 - \boxed{5} = 7$

$7 + \boxed{4} = 11$
$11 - \boxed{4} = 7$

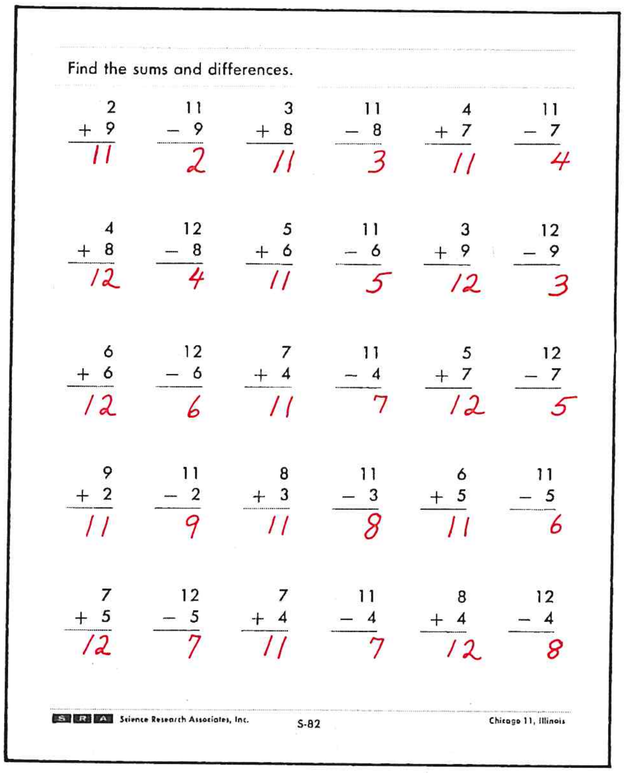
$8 + \boxed{3} = 11$
$11 - \boxed{3} = 8$

$5 + \boxed{7} = 12$
$12 - \boxed{7} = 5$

$5 + \boxed{6} = 11$
$11 - \boxed{6} = 5$

$4 + \boxed{7} = 11$
$11 - \boxed{7} = 4$

S-81



Find the sums and differences.

2 + 9 11	11 − 9 2	3 + 8 11	11 − 8 3	4 + 7 11	11 − 7 4
4 + 8 12	12 − 8 4	5 + 6 11	11 − 6 5	3 + 9 12	12 − 9 3
6 + 6 12	12 − 6 6	7 + 4 11	11 − 4 7	5 + 7 12	12 − 7 5
9 + 2 11	11 − 2 9	8 + 3 11	11 − 3 8	6 + 5 11	11 − 5 6
7 + 5 12	12 − 5 7	7 + 4 11	11 − 4 7	8 + 4 12	12 − 4 8

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first pair of equations with the children; discuss the inverse concept, and then have the class work the remainder of the page on their own.

Work page S-82 introduces the use of vertical notation with the "harder" combinations. Ask the children to find the sums or differences; remind them to keep the numerals in the ones and tens positions lined up as carefully as possible. In working these problems, an understanding of the inverse concept will be a real advantage to the children.

Supplemental Experiences

▶ To provide additional practice in working with subtraction "in pieces," design a series of tagboard cards using equations such as:

$\boxed{15 - 5 = \square}$ $\boxed{\square - 5 = 10}$

$\boxed{(12 - 2) - 1 = \square - 1}$ $\boxed{15 - 6 = (15 - \square) - 1}$

Expose the cards, one at a time, and have the children read and determine the answers which complete the equations.

▶ Use flash cards with subtraction combinations of ten. This activity will help build a solid foundation for the quick recognition of these number facts.

$\boxed{10 - \square = 6}$ $\boxed{\square - 3 = 7}$

$\boxed{10 - 4 = \square}$

Addition

SUMS ELEVEN THROUGH FOURTEEN

Work Pages S-83 Through S-86

Objectives

To extend addition of two numbers, each less than ten, to sums of thirteen and fourteen.

To help the children apply their knowledge of the associative property.

Fundamentals

To help the child understand addition combinations of thirteen and fourteen when the addends are less than ten, the teacher will want to start once again with sets of concrete objects. The child will regroup the sets and relate a mathematical sentence to each step. He will learn the steps involved in the operation of adding two numbers of nine or less, whose sum is not greater than fourteen.

Interesting patterns will develop as the work progresses. Perhaps the child will discover that for a given sum, as the first addend becomes less by one, the second addend becomes greater by one. The work pages are designed to make this pattern evident.

The aim is to condense the steps used to help the child justify his addition of two numbers such as seven and six. The child's ability to think of seven plus six as ten plus three, without having to go through the intermediate steps each time, should be increasing. The teacher, however, will often review the intermediate steps with the child, to make certain that he is not only able to say *what* the answer is, but can explain the *how* and *why* of reasoning through the problem.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

An understanding of the associative property.

An understanding of the concept of different numerals for a number.

An understanding of the concept of place value.

Mastery of the addition combinations of ten.

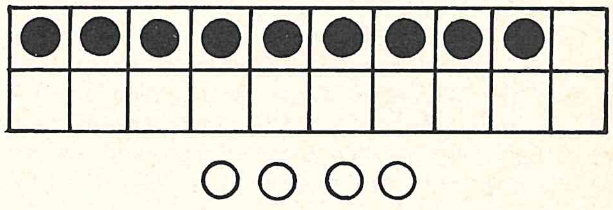
Ability to work with placeholder equations and vertical notation.

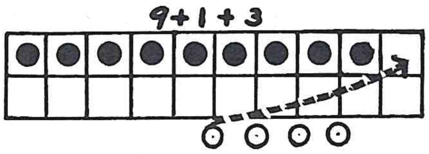
Developmental Experiences

MATERIALS

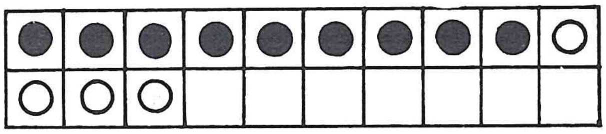
latticework
flannel board
felt disks
tagboard cards
pocket chart
2 Countingmen

▶ Tell the children that they are going to play a game called "Operation Big Ten." On the flannel board, place a latticework with two rows of ten openings; put a set of nine disks in the first row. Place a set of four disks below the frame. Then insert in the pocket chart a tagboard card bearing the equation $9 + 4 = \square$. The aim of the game is to make a set of ten as the first step in the operation of joining the set of nine and the set of four. Call on a child to carry out "Operation Big Ten" with the sets on the flannel board. As the child gets ready to take one counter from the set of four and put it with the set of nine, tell the class that you want to stop the action here in order to take a picture. Pretend that you are photographing the sets on the flannel board, then put the picture $9 + 1 + 3 = \square$ in the pocket chart.

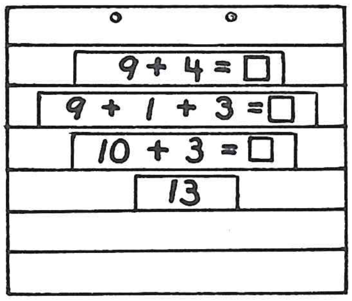




Now have the child complete the action of putting the set of one with the set of nine. Then have him join the other three disks to the set of ten by placing



them in the second row. Again pretend to take a picture of the sets now on the flannel board and put the picture 10 + 3 = □ in the pocket chart, along with the card for the short way of writing 10 + 3.



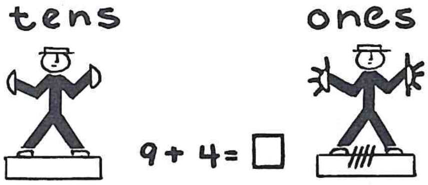
Summarize the thinking steps involved in solving the problem 9 + 4 = □. Refer to the cards in the pocket chart and have the children help review the ideas. Ask pertinent questions such as

- What did we do as a first step in "Operation Big Ten"?
  (Think of the number needed to add to nine to give ten.)
- Where did the one added to nine to give ten come from?
  (Four was thought of as one plus three.)
- After one was added to nine to give ten what happened to the three from one plus three?
  (It was added to ten to give thirteen.)

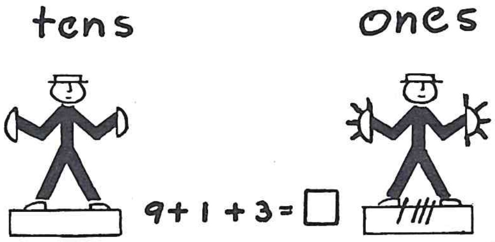
Use the same procedure to develop other addition combinations of thirteen and fourteen.

▶ When children work with the harder combinations of 9 + 4 = □ or 9 + 5 = □, the Countingmen will prove helpful.

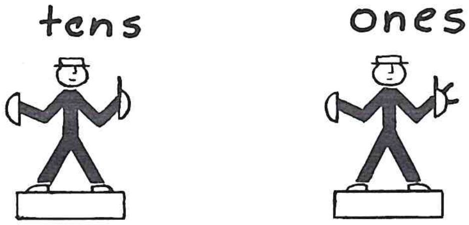
Display two Countingmen. Then call upon two children, one to operate the Tens-man and one to operate the Ones-man. Write the equation 9 + 4 = □ on the chalkboard. Have the ones-operator place nine fingers on his Countingman, and display four fingers on the base to show the set which will be joined to the set of nine.



Have the children think first of how many fingers are needed to make a set of ten. Have the ones-operator rearrange the fingers on the base of the Countingman to show that the set of four can be thought of as a set of one and a set of three. Write the equation 9 + 1 + 3 = □, which corresponds to this phase of the activity, under the placeholder equation on the chalkboard.



Have the ones-operator place one finger on his Countingman to make ten ones. At this point the tens-operator should record one ten on his Countingman; the ones-operator should rapidly remove the fingers, and then move the remaining three fingers from the base to the hands of the Countingman.



On the chalkboard write the equation 10 + 3 = □, which corresponds to this final phase of the activity. Ask the children to give another name for ten plus three. Referring back to the placeholder equation on the chalkboard, emphasize that the Countingmen helped the children find the answer, 9 + 4 = 13.

Summarize the thinking steps involved in solving the problem 9 + 4 = □. Refer to the chalkboard and have the children help review the ideas which are represented by the symbols:

9 + 4 = □
9 + 1 + 3 = □
10 + 3 = □

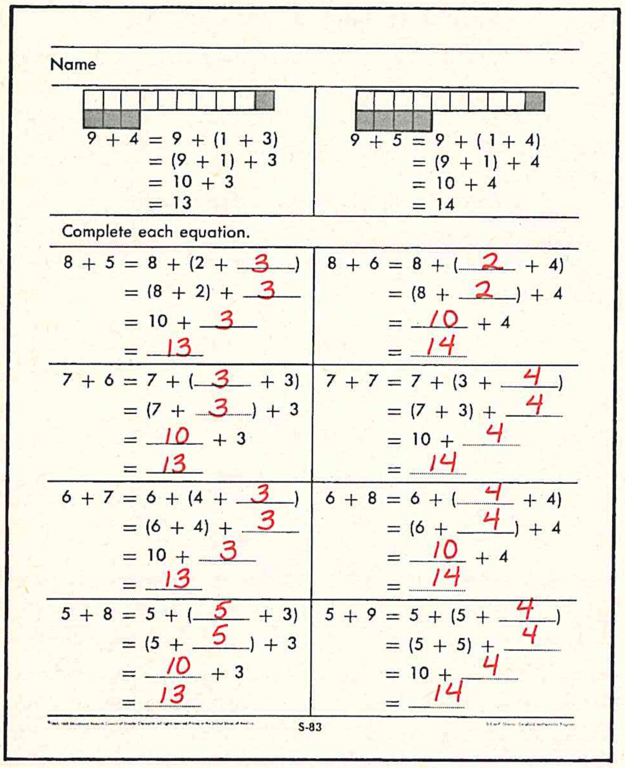
Follow this procedure to develop other addition combinations of thirteen and fourteen.

▶ Use work page S-83 to provide practice for the class in working with the steps involved in finding sums of thirteen and fourteen. Work the entire page with the children. The parentheses will help the children to recognize the different numerals for the numbers involved in the problem as regrouping takes place.

On work page S-84 encourage children to think for a second time about their addition; this time in terms of tens and ones. They can use the pictures at the top of the work page if they need a visual clue. Work the first one or two problems with the children, then encourage them to complete the other equations on the page by working independently.

▶ Work page S-85 will provide a review of the addition combinations related to sums of eleven through fourteen. Tell the children that when they write the numeral for the sum in each placeholder, they are to write the standard place-value numeral, not the expanded-notation form. The children can work the entire page on their own.

Work page S-86 provides practice with the vertical-notation form for work with the harder combinations. Direct the children to find the sums. Remind them to keep the digits in the ones and the tens positions lined up as carefully as possible.



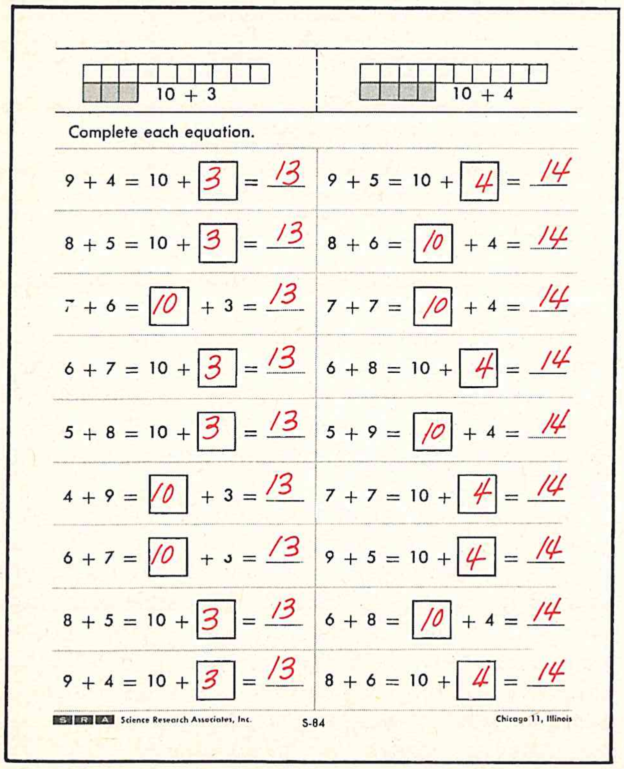
Name

9 + 4 = 9 + (1 + 3) = (9 + 1) + 3 = 10 + 3 = 13	9 + 5 = 9 + (1 + 4) = (9 + 1) + 4 = 10 + 4 = 14

Complete each equation.

8 + 5 = 8 + (2 + 3) = (8 + 2) + 3 = 10 + 3 = 13	8 + 6 = 8 + (2 + 4) = (8 + 2) + 4 = 10 + 4 = 14
7 + 6 = 7 + (3 + 3) = (7 + 3) + 3 = 10 + 3 = 13	7 + 7 = 7 + (3 + 4) = (7 + 3) + 4 = 10 + 4 = 14
6 + 7 = 6 + (4 + 3) = (6 + 4) + 3 = 10 + 3 = 13	6 + 8 = 6 + (4 + 4) = (6 + 4) + 4 = 10 + 4 = 14
5 + 8 = 5 + (5 + 3) = (5 + 5) + 3 = 10 + 3 = 13	5 + 9 = 5 + (5 + 4) = (5 + 5) + 4 = 10 + 4 = 14

S-83



10 + 3	10 + 4

Complete each equation.

9 + 4 = 10 + $\boxed{3}$ = 13	9 + 5 = 10 + $\boxed{4}$ = 14
8 + 5 = 10 + $\boxed{3}$ = 13	8 + 6 = $\boxed{10}$ + 4 = 14
7 + 6 = $\boxed{10}$ + 3 = 13	7 + 7 = $\boxed{10}$ + 4 = 14
6 + 7 = 10 + $\boxed{3}$ = 13	6 + 8 = 10 + $\boxed{4}$ = 14
5 + 8 = 10 + $\boxed{3}$ = 13	5 + 9 = $\boxed{10}$ + 4 = 14
4 + 9 = $\boxed{10}$ + 3 = 13	7 + 7 = 10 + $\boxed{4}$ = 14
6 + 7 = $\boxed{10}$ + 3 = 13	9 + 5 = 10 + $\boxed{4}$ = 14
8 + 5 = 10 + $\boxed{3}$ = 13	6 + 8 = $\boxed{10}$ + 4 = 14
9 + 4 = 10 + $\boxed{3}$ = 13	8 + 6 = 10 + $\boxed{4}$ = 14

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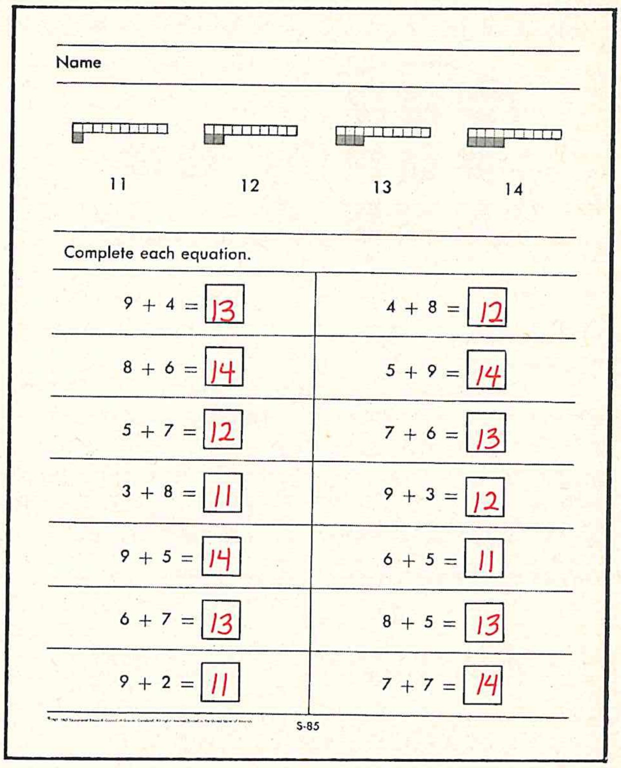


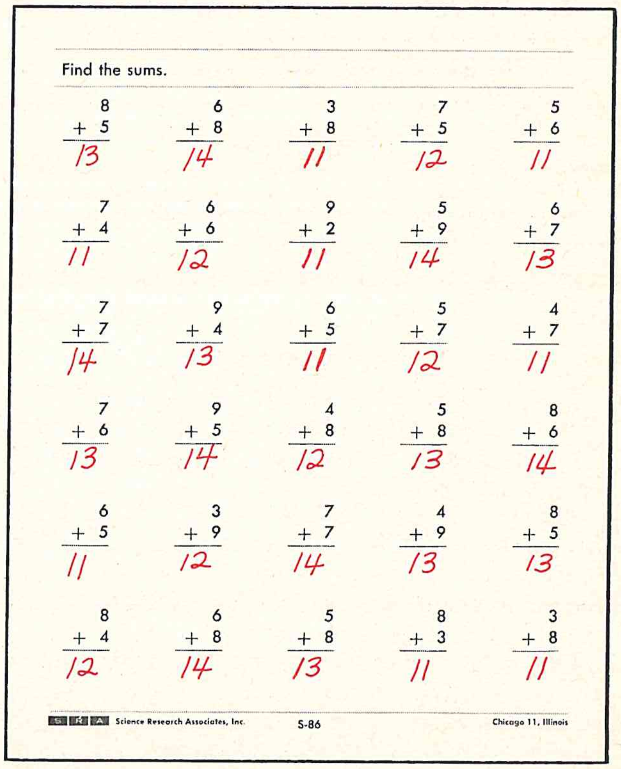
Name

11 12 13 14

Complete each equation.

9 + 4 = $\boxed{13}$	4 + 8 = $\boxed{12}$
8 + 6 = $\boxed{14}$	5 + 9 = $\boxed{14}$
5 + 7 = $\boxed{12}$	7 + 6 = $\boxed{13}$
3 + 8 = $\boxed{11}$	9 + 3 = $\boxed{12}$
9 + 5 = $\boxed{14}$	6 + 5 = $\boxed{11}$
6 + 7 = $\boxed{13}$	8 + 5 = $\boxed{13}$
9 + 2 = $\boxed{11}$	7 + 7 = $\boxed{14}$

S-85



Find the sums.

$\begin{array}{r}8\\+\ 5\\\hline 13\end{array}$	$\begin{array}{r}6\\+\ 8\\\hline 14\end{array}$	$\begin{array}{r}3\\+\ 8\\\hline 11\end{array}$	$\begin{array}{r}7\\+\ 5\\\hline 12\end{array}$	$\begin{array}{r}5\\+\ 6\\\hline 11\end{array}$
$\begin{array}{r}7\\+\ 4\\\hline 11\end{array}$	$\begin{array}{r}6\\+\ 6\\\hline 12\end{array}$	$\begin{array}{r}9\\+\ 2\\\hline 11\end{array}$	$\begin{array}{r}5\\+\ 9\\\hline 14\end{array}$	$\begin{array}{r}6\\+\ 7\\\hline 13\end{array}$
$\begin{array}{r}7\\+\ 7\\\hline 14\end{array}$	$\begin{array}{r}9\\+\ 4\\\hline 13\end{array}$	$\begin{array}{r}6\\+\ 5\\\hline 11\end{array}$	$\begin{array}{r}5\\+\ 7\\\hline 12\end{array}$	$\begin{array}{r}4\\+\ 7\\\hline 11\end{array}$
$\begin{array}{r}7\\+\ 6\\\hline 13\end{array}$	$\begin{array}{r}9\\+\ 5\\\hline 14\end{array}$	$\begin{array}{r}4\\+\ 8\\\hline 12\end{array}$	$\begin{array}{r}5\\+\ 8\\\hline 13\end{array}$	$\begin{array}{r}8\\+\ 6\\\hline 14\end{array}$
$\begin{array}{r}6\\+\ 5\\\hline 11\end{array}$	$\begin{array}{r}3\\+\ 9\\\hline 12\end{array}$	$\begin{array}{r}7\\+\ 7\\\hline 14\end{array}$	$\begin{array}{r}4\\+\ 9\\\hline 13\end{array}$	$\begin{array}{r}8\\+\ 5\\\hline 13\end{array}$
$\begin{array}{r}8\\+\ 4\\\hline 12\end{array}$	$\begin{array}{r}6\\+\ 8\\\hline 14\end{array}$	$\begin{array}{r}5\\+\ 8\\\hline 13\end{array}$	$\begin{array}{r}8\\+\ 3\\\hline 11\end{array}$	$\begin{array}{r}3\\+\ 8\\\hline 11\end{array}$

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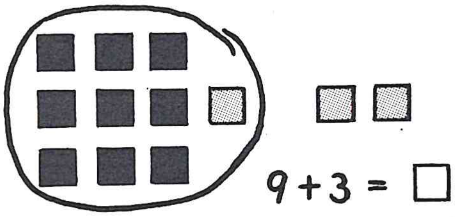
Supplemental Experiences

▶ Draw a set of nine squares and a set of three squares, in another color, on the chalkboard. Have the children tell the numbers of the two sets to be joined. Allow the children to use their plastic numerals, operation and relation symbols, and placeholders to build the equation 9 + 3 = □, which is related to the sets.

Ask a child to take a piece of chalk and circle



ten of the squares, in order to show how a set of nine and a set of three can be regrouped as a set of ten and a set of two.

Tell the children to study the picture of the set activity now. Say that the regrouping in terms of tens and ones will help them think of the number needed to complete the placeholder equation which they have on their desks. Continue in this way to review addition combinations related to sums of eleven through fourteen with the class.

Subtraction—Addition

COMBINATIONS OF THIRTEEN AND FOURTEEN

Work Pages S-87 and S-88

Objectives

To extend the development of subtraction to combinations of thirteen and fourteen.

To emphasize the inverse relation between addition and subtraction.

Fundamentals

The child has had experience adding two numbers represented by one-digit numerals with sums of thirteen and fourteen. He is now ready to examine related subtraction combinations.

The child should be guided to think of ten as holding the key position in subtraction problems just as it did in addition problems. He should be reminded that he must move back to ten and then continue his subtraction from this point.

The teacher should encourage the child to capitalize on what he knows in order to make his learning easier. Whenever an addition fact is developed, the development of the related subtraction fact follows soon after. Being aware that subtraction "undoes" what addition "does," he should have no difficulty determining that 13 − 4 = 9 when he has already learned that 9 + 4 = 13.

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Understanding of the concept of different numerals for the same number.

Ability to subtract a number represented by a one-digit numeral from a number represented by a two-digit numeral.

Mastery of subtraction combinations of ten.

Ability to work with placeholder equations.

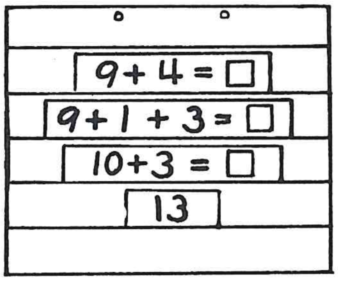
Awareness of the inverse relation between addition and subtraction.

Developmental Experiences

MATERIALS

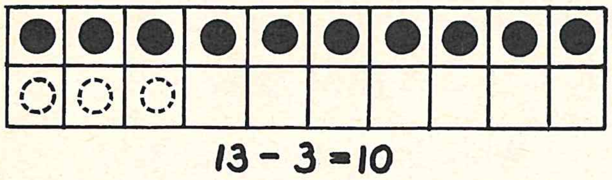
flannel board
latticework
tagboard cards
felt counters
2 Countingmen
pocket chart

▶ Review an addition combination with which the children are familiar: 9 + 4 = □. Use the latticework and felt disks. Have a child join a set of four discs to a set of nine discs. Now place the related numeral cards in the pocket chart. Discuss the thinking steps used in this problem.

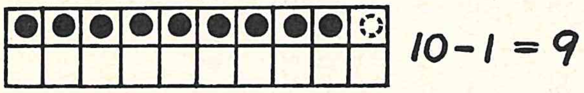


Remind the children that when they add they move forward from nine, step by step, first to ten and then to thirteen. Since subtraction "undoes" what addition "does," ask the children what they think will happen when four is subtracted from thirteen. Guide them to conclude that they will move backward from thirteen, step by step, first to ten and then to nine.

Begin with the set of thirteen disks in the latticework. Put the equation 13 − 4 = □ in the pocket chart. Tell the children that they must remove four disks from the set of thirteen. Tell them that they will do it in steps in order to see what really happens when they undo the addition. Remind them that they will move to ten first—that they must remove as many objects as necessary to have a set of ten. After the removal of the three objects from the frame, place a card in the pocket chart showing what has happened to the set so far: 13 − 3 = 10.

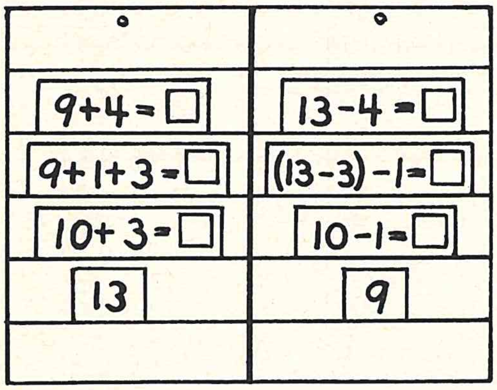


Ask the children to tell how many they were supposed to remove according to the original equation. Ask if any more objects must be removed besides the three. After one more object has been removed, place in the pocket chart an equation related to the action: 10 − 1 = 9.



Remove the two cards 13 − 3 = 10 and 10 − 1 = 9 from the pocket chart. Replace them with two cards showing, respectively, (13 − 3) − 1 = ☐ and 10 − 1 = ☐. Remind the children that the parentheses are there to show which subtraction took place first, to help them think of (13 − 3) as representing the number ten. Finally, place a card bearing the numeral 9 in the pocket chart.

Summarize the thinking steps involved in solving both the addition and the subtraction problems: 9 + 4 = ☐ and 13 − 4 = ☐. Refer to the cards in the pocket chart. Have the children help review the ideas. Ask questions such as:



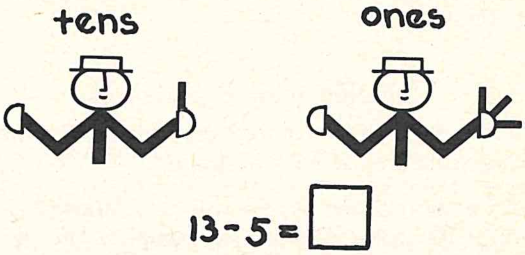
In the problem 9 + 4 = ☐, we added in steps. What two numbers were added to make ten plus three? (One was added and then three was added.)

In the problem 13 − 4 = ☐, we subtracted in steps. What two numbers were subtracted to get back to nine? (Three was subtracted and then one was subtracted.)

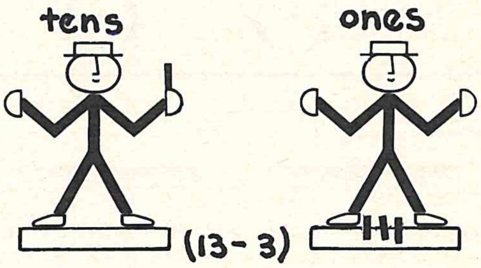
Help children understand that they are moving in reverse in the second problem. Therefore the first number added is the last number subtracted and the last number added is the first number subtracted.

Follow this same procedure to develop other subtraction combinations related to sums of thirteen and fourteen.

▶ Use the Countingmen to help the children work with subtraction combinations of thirteen and fourteen. Place the equation 13 − 5 = ☐ on the chalkboard. Call upon two children to act as operators of the Ones-man and the Tens-man, respectively. Have them show thirteen on the Countingmen.

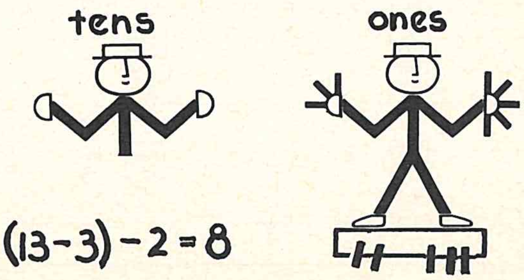


Tell the ones-operator that his job is to remove five fingers. He will remove three fingers and then see that it is impossible to remove five ones from three ones. Have him place the three fingers on the Countingman's base. Write on the chalkboard the symbolism for the action that has taken place up to this point: (13 − 3).



The one ten recorded on the Tens-man will have to be exchanged for ten ones before the removal of five can be completed. As the one finger is removed from the Tens-man, ten fingers are placed on the Ones-man.

The removal of the five fingers can now be continued. It is now possible to take away two more fingers to make a total of five. Complete the symbolism related to this action: (13 − 3) − 2 = 8.



Summarize the thinking steps involved in this problem. First, three fingers were removed, and then two more fingers were removed, making a total of five fingers removed. The subtraction of five from thirteen occurred in two steps: $(13 - 3) - 2 = 8$.

Continue in similar fashion to develop other subtraction combinations of thirteen and fourteen.

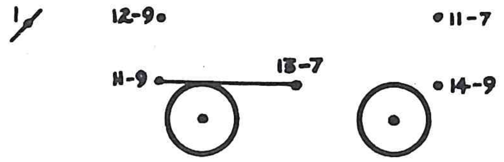
▶ Use work page S-87 for practice in working through the thinking steps involved in solving subtraction equations related to sums of thirteen and fourteen. Work with the children for the entire page. Point out that the parentheses show which number to subtract first.

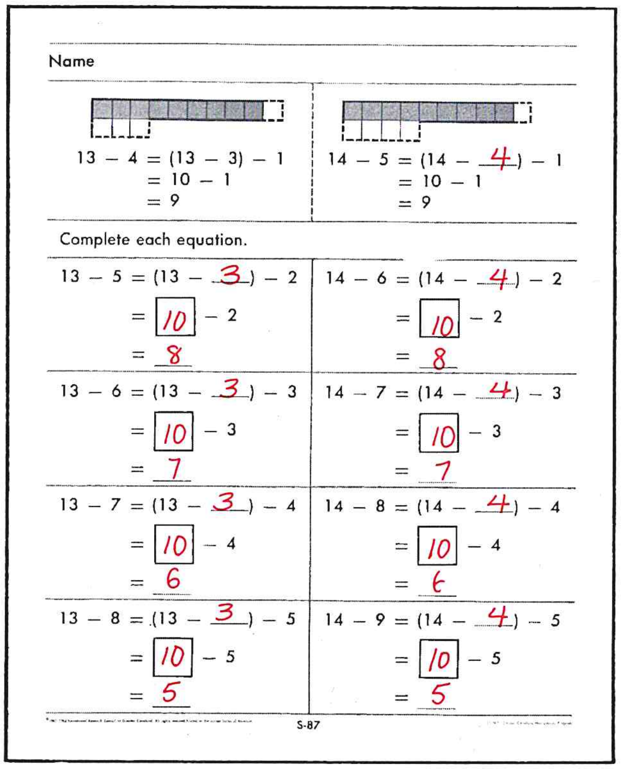
Work page S-88 emphasizes the inverse relation between addition and subtraction. Work the first pair of equations with the children; discuss the inverse concept. Have the class work the rest of the page on their own.

Supplemental Experiences

▶ Give each child a paper on which the structure for a dot picture has been designed.

Have the children begin with the numeral 1 and connect the dots, in order, to complete the picture. Tell them that they must think of the natural order of the counting numbers as they move from one dot to the next. The different numerals for numbers must be carefully related to the counting numbers.




Name

$13 - 4 = (13 - 3) - 1$
$= 10 - 1$
$= 9$

$14 - 5 = (14 - 4) - 1$
$= 10 - 1$
$= 9$

Complete each equation.

$13 - 5 = (13 - 3) - 2$
$= \boxed{10} - 2$
$= 8$

$14 - 6 = (14 - 4) - 2$
$= \boxed{10} - 2$
$= 8$

$13 - 6 = (13 - 3) - 3$
$= \boxed{10} - 3$
$= 7$

$14 - 7 = (14 - 4) - 3$
$= \boxed{10} - 3$
$= 7$

$13 - 7 = (13 - 3) - 4$
$= \boxed{10} - 4$
$= 6$

$14 - 8 = (14 - 4) - 4$
$= \boxed{10} - 4$
$= 6$

$13 - 8 = (13 - 3) - 5$
$= \boxed{10} - 5$
$= 5$

$14 - 9 = (14 - 4) - 5$
$= \boxed{10} - 5$
$= 5$

S-87

Addition—Subtraction

COMBINATIONS

ELEVEN THROUGH FOURTEEN

—Comparison

Work Pages S-89 and S-90

Objectives

To strengthen the children's understanding of addition and subtraction.

To emphasize the inverse relation between addition and subtraction.

To provide practice in completing placeholder equations.

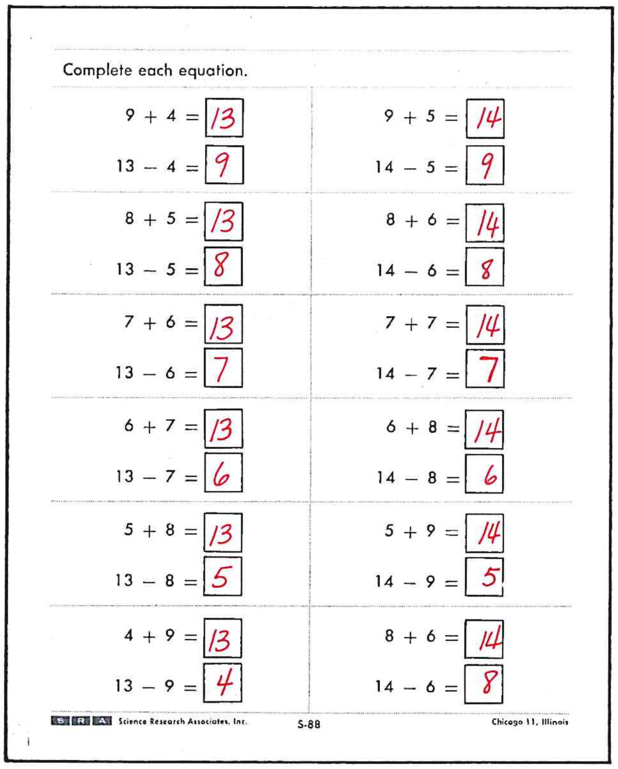
To compare numbers in the environment of known addition and subtraction combinations.

Fundamentals

The child should always be encouraged to use known facts as he faces new problem situations. He knows that $7 + 5 = 12$ and that $12 - 5 = 7$. These facts should help him when he encounters the placeholder equation $\square + 5 = 12$.

The child knows that addition and subtraction are inverse operations. This fact will prove helpful continually. He knows that if he adds two numbers and then subtracts one of these numbers from the sum, the result will be the other number.

The child will make this discovery intuitively. At


Complete each equation.

$9 + 4 = \boxed{13}$
$13 - 4 = \boxed{9}$

$9 + 5 = \boxed{14}$
$14 - 5 = \boxed{9}$

$8 + 5 = \boxed{13}$
$13 - 5 = \boxed{8}$

$8 + 6 = \boxed{14}$
$14 - 6 = \boxed{8}$

$7 + 6 = \boxed{13}$
$13 - 6 = \boxed{7}$

$7 + 7 = \boxed{14}$
$14 - 7 = \boxed{7}$

$6 + 7 = \boxed{13}$
$13 - 7 = \boxed{6}$

$6 + 8 = \boxed{14}$
$14 - 8 = \boxed{6}$

$5 + 8 = \boxed{13}$
$13 - 8 = \boxed{5}$

$5 + 9 = \boxed{14}$
$14 - 9 = \boxed{5}$

$4 + 9 = \boxed{13}$
$13 - 9 = \boxed{4}$

$8 + 6 = \boxed{14}$
$14 - 6 = \boxed{8}$

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this level, he need not concern himself with all the details involved. The teacher, however, will want to be aware of what goes on behind the scenes in problems of this kind.

The teacher should know that when the child subtracts five from thirteen to get his answer in a problem such as $5 + \square = 13$, the commutative property and inverse relation of addition and subtraction are important factors. Since we know by

$5 + \square = 13$
$\square + 5 = 13$ commutative property
$13 - 5 = \square$ inverse relation

subtracting that $13 - 5 = 8$, we know that the placeholder in each of these equations can be replaced by the numeral 8.

The use of the commutative property and the inverse relation of addition and subtraction also permits us to subtract seven from thirteen in the problem $13 - \square = 7$ in order to find the missing addend.

$13 - \square = 7$
$7 + \square = 13$ inverse relation
$\square + 7 = 13$ commutative property
$13 - 7 = \square$ inverse relation

To enrich and vary his contact with sums eleven through fourteen, the child will be asked to compare these sums. In comparing sums, the teacher should encourage the child to be alert for problems such as $7 + 6 \bigcirc 6 + 7$ which can be solved without computation.

Perhaps the more mature child will discover something interesting and challenging in problems such as $5 + 6 \bigcirc 4 + 7$. The first addend on the left, five, is greater by one than the first addend on the right, four. The second addend on the left, six, is less by one than the second addend on the right, seven. The sums, however, are the same. The teacher should encourage the child to make discoveries of this kind whenever possible. Problems such as $13 - 7 \bigcirc 14 - 8$ offer another opportunity to discover a pattern.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Order—Relations—Operations; Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Understanding of the concepts of addition and subtraction.

Awareness of the inverse relation between addition and subtraction.

Ability to work with placeholder equations.

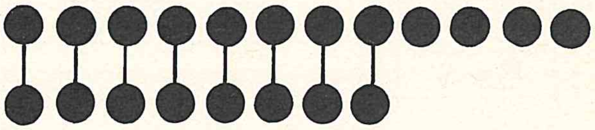
Ability to compare numbers.

Developmental Experiences

MATERIALS

- flannel board
- felt numerals 0 through 9
- felt symbols for operations and relations
- felt placeholders
- felt counters
- pocket chart
- tagboard cards

▶ Place a set of twelve felt cutouts on the flannel board. Directly below this set of twelve, place a set of eight cutouts. Put the placeholder equation $8 + \square = 12$ under the sets. Have children determine how many more objects are needed with the set of eight to make a set of twelve. Ask someone to place the required set directly below the last four disks in the set of twelve. Help the children see that the two sets can now be matched one to one. Discuss the fact that a set of four had to be joined to the set of eight in order to have this one-to-one correspondence. Have the placeholder equation completed.



$$8 + \square = 12$$

$$12 - \square = 8$$

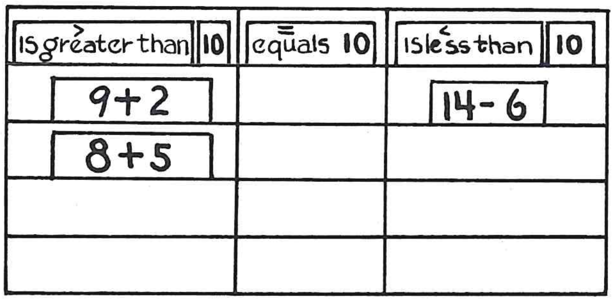
Rearrange the objects on the flannel board into the original grouping—a set of twelve objects and a set of eight objects. Put the placeholder equation $12 - \square = 8$ below the sets. Have the children determine how many objects must be taken away from one of the sets in order to have a set of eight. Ask someone to remove the required number of objects. Help the children see that the two sets can now be matched one to one. Discuss the fact that a set of four had to be taken away from the set of twelve in order to have this one-to-one correspondence. Have the placeholder equation completed.

Help the children conclude that they could have subtracted to get the answer in the addition equation and added to get the answer in the subtraction equation.

▶ Place the equation $4 + \square = 12$ on the chalkboard. Help the children explore possible ways to find the missing addend in this equation. Have them try thinking of the number that is added to four to give twelve. Have them think of the number that is subtracted from twelve to give four. Someone may want to try subtracting four from twelve to see if this gives him the answer.

Place the equation $12 - \square = 4$ on the chalkboard. Help the children explore possible ways to find the missing addend in this problem. Have them try thinking of the number that is added to four to give twelve. Have them think of the number that is subtracted from twelve to give four. Someone may discover that he can subtract four from twelve and find the missing number. Continue in this way to find missing addends in other addition and subtraction equations related to sums eleven through fourteen. Guide children to conclude that answers to addition problems can often be found by subtracting; that answers to subtraction problems can often be found by adding.

▶ Distribute cards showing all possible addition and subtraction combinations of eleven through fourteen. On the left side of the pocket chart, place a card showing the symbol >, the phrase "is greater than," and the numeral 10. On the right side of the chart, place a card showing the symbol <, the phrase "is less than," and the numeral 10. In the center of the chart place a card showing the symbol =, the word "equals," and the numeral 10.



$\overset{>}{\text{is greater than}}$ 10	$\overset{=}{\text{equals}}$ 10	$\overset{<}{\text{is less than}}$ 10
9+2		14-6
8+5		

Call upon various children to come forward, one at a time, and have each child tell the number represented by his numeral. Have him compare his number with ten and then place his numeral under the correct symbol and phrase on the chart. When all the numbers have been compared with ten, redistribute the cards and replace the numeral cards representing the number ten with cards representing the number fourteen. Have all the numbers compared with fourteen.

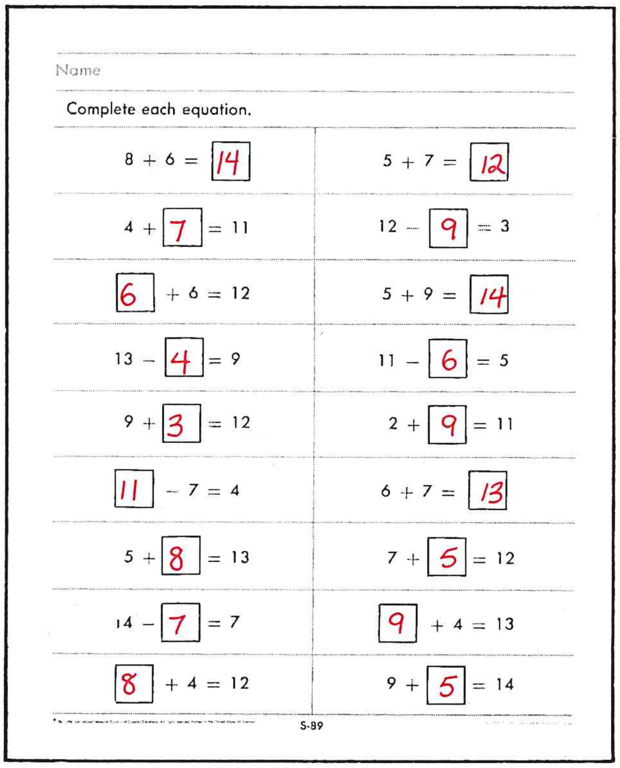
Continue in this way having children compare their numbers with eleven, twelve, and thirteen.

▶ Work page S-89 will give the children the opportunity to test their ability to work with addition and subtraction placeholder equations with the placeholder in different positions. After directions are given, have the children work independently.

Work page S-90 will give the children an opportunity to use their ability to reason. Work the first problem with the group. Encourage them to find answers, without computing whenever possible, then to check their work by adding or subtracting. When the children have completed the work on this page, discuss each problem with the class; have different children explain how they thought out their answers.

Supplemental Experiences

▶ Make four cards, each showing a numeral preceded by a plus or minus sign as shown below, and place them on the flannel board. Make four other cards showing the numerals 6 through 9, and place these cards on the flannel board face down. Call on a child to choose one of the cards that are face down. The numeral on this card indicates his starting number. He must think of this numeral as coming in front of the −2 on the flannel board and then solve the problem that is thus formed. If, for example, the card he chooses is 7, he must solve the problem 7 − 2 = ☐. He then takes his answer (5) and uses it to form the next problem: 5 + 3 = ☐. He continues in this way until he has used all four numerals on the board. The child who chooses 7

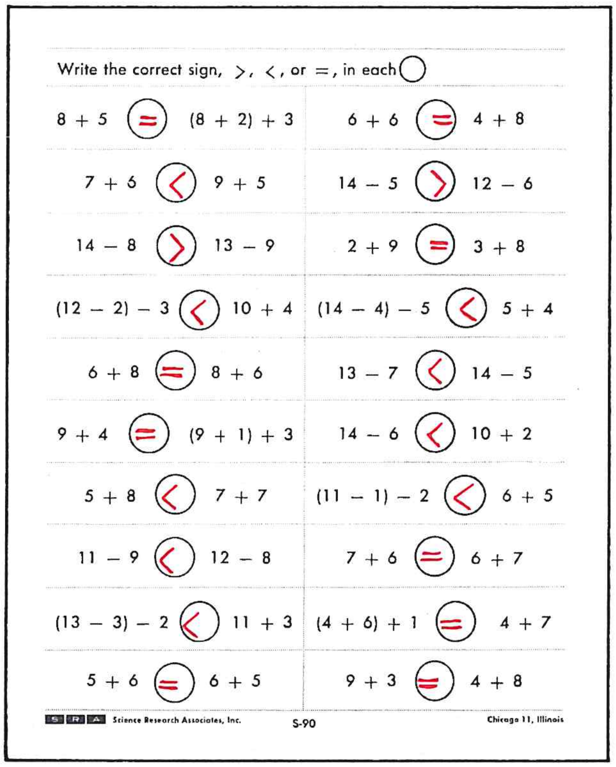


Name

Complete each equation.

$8 + 6 = \boxed{14}$	$5 + 7 = \boxed{12}$
$4 + \boxed{7} = 11$	$12 - \boxed{9} = 3$
$\boxed{6} + 6 = 12$	$5 + 9 = \boxed{14}$
$13 - \boxed{4} = 9$	$11 - \boxed{6} = 5$
$9 + \boxed{3} = 12$	$2 + \boxed{9} = 11$
$\boxed{11} - 7 = 4$	$6 + 7 = \boxed{13}$
$5 + \boxed{8} = 13$	$7 + \boxed{5} = 12$
$14 - \boxed{7} = 7$	$\boxed{9} + 4 = 13$
$\boxed{8} + 4 = 12$	$9 + \boxed{5} = 14$

S-89



Write the correct sign, >, <, or =, in each ○

8 + 5 = (8 + 2) + 3	6 + 6 = 4 + 8
7 + 6 < 9 + 5	14 − 5 > 12 − 6
14 − 8 > 13 − 9	2 + 9 = 3 + 8
(12 − 2) − 3 < 10 + 4	(14 − 4) − 5 < 5 + 4
6 + 8 = 8 + 6	13 − 7 < 14 − 5
9 + 4 = (9 + 1) + 3	14 − 6 < 10 + 2
5 + 8 < 7 + 7	(11 − 1) − 2 < 6 + 5
11 − 9 < 12 − 8	7 + 6 = 6 + 7
(13 − 3) − 2 < 11 + 3	(4 + 6) + 1 = 4 + 7
5 + 6 = 6 + 5	9 + 3 = 4 + 8

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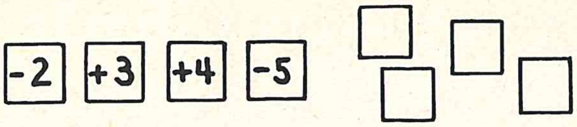


will set up the following equations: 7 − 2 = 5; 5 + 3 = 8; 8 + 4 = 12; 12 − 5 = 7. (Note that the last number is always the number the child started with.)

When the child has completed his assignment, have his card placed on one side of the board and continue with the game.

Other numeral cards may be used in place of those suggested. The only requirement is that the children be able to work with the number combinations used.

▶ Play a riddle game.

> If I subtract nine from this number, I get two. What is my number?
>
> If I subtract eight from thirteen, what number do I have?
>
> If you subtract three from me, you will have eight. What number am I?
>
> One part of a set of fourteen is a set of six. I am the other part. I am a set of how many objects?

Addition

SUMS OF FIFTEEN AND SIXTEEN

Work Pages S-91 and S-92

Objectives

To extend the knowledge of addition of two numbers, each less than ten, to sums of fifteen and sixteen.

To help children apply their understanding of the associative property.

Fundamentals

Learning only the answers to problems involving the harder combinations is apt to reduce classroom activity to rote learning. A child can be drilled over and over again to come up with the answer to a cue such as 9 + 6. The thoughtful teacher will want to give the child more than this. She will want to help him *understand* his work with numbers—to know *why* a particular rule works.

When the child meets a problem involving two addends each of which is less than ten and whose sum is greater than ten, he should approach the answer in terms of the base of our numeration system. He should think of 9 + 6, for example, as 10 + 5.

For efficient computation, the child must eventually learn to quickly respond "15" to the problem 9 + 6 = ☐. If however, he learns the fundamental ideas rather than just rules, he will be better equipped for his future work with numbers!

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers.

Readiness for Understanding

Understanding of the associative property.

Understanding of the concept of different numerals for the same number.

Understanding of the concept of place value.

Mastery of the addition combinations to ten.

Ability to work with placeholder equations.

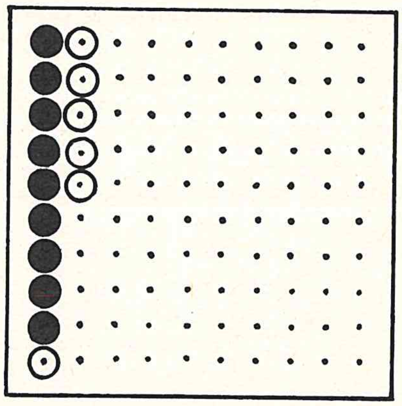
Developmental Experiences

MATERIALS

pegboards
pegs
a hundred-board
colored spools

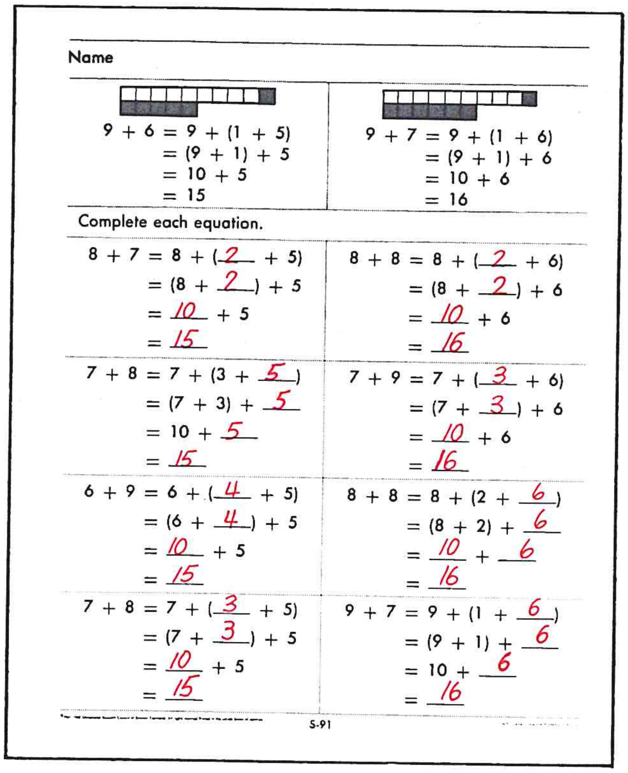
▶ Construct a hundred-board from a 2 x 2-foot piece of half-inch plywood. Rule the board off into one hundred two-inch squares, leaving a two-inch border round the outside. Drive a finishing nail through the center of the top line of each small square. Use oak tag labels, poker chips punched for hanging, key tags, curtain rings, rubber washers, or spools on the board.

Provide each child with a pegboard and pegs. Write the problem 9 + 6 = ☐ on the chalkboard. Have the children put the correct number of pegs in their pegboards to represent the first addend. Tell the children to join a set of six pegs to the set of nine pegs. Have them use different-colored pegs for the set of six. Guide the activity by placing colored spools on the hundred-board.



Have the children tell how they will join six to nine ("Join a set of one and then a set of five"). The children should be guided to think of sets of nine and six regrouped as sets of ten and five. Write 9 + 6 = 10 + 5 on the chalkboard. Now have the placeholder equation completed. Continue in this way to develop other addition combinations of fifteen and sixteen.

▶ Use work page S-91 for practice in working through the thinking steps involved in addition combinations of fifteen and sixteen. Work the first two problems with the children; discuss the block pictures and the use of parentheses. Encourage the children to complete the rest of the page independently; give help only if needed.



Name

9 + 6 = 9 + (1 + 5)
= (9 + 1) + 5
= 10 + 5
= 15

9 + 7 = 9 + (1 + 6)
= (9 + 1) + 6
= 10 + 6
= 16

Complete each equation.

8 + 7 = 8 + (2 + 5)
= (8 + 2) + 5
= 10 + 5
= 15

8 + 8 = 8 + (2 + 6)
= (8 + 2) + 6
= 10 + 6
= 16

7 + 8 = 7 + (3 + 5)
= (7 + 3) + 5
= 10 + 5
= 15

7 + 9 = 7 + (3 + 6)
= (7 + 3) + 6
= 10 + 6
= 16

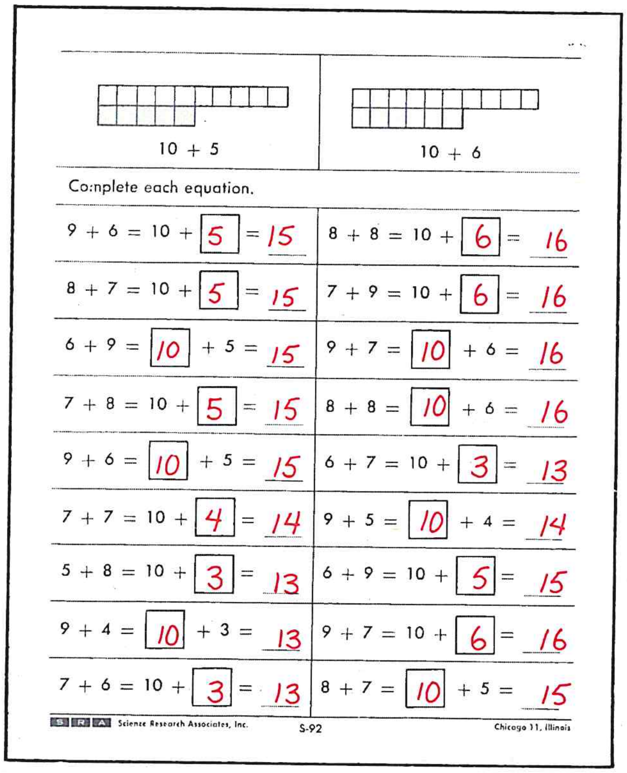
6 + 9 = 6 + (4 + 5)
= (6 + 4) + 5
= 10 + 5
= 15

8 + 8 = 8 + (2 + 6)
= (8 + 2) + 6
= 10 + 6
= 16

7 + 8 = 7 + (3 + 5)
= (7 + 3) + 5
= 10 + 5
= 15

9 + 7 = 9 + (1 + 6)
= (9 + 1) + 6
= 10 + 6
= 16

S-91



10 + 5

10 + 6

Complete each equation.

$9 + 6 = 10 + \boxed{5} = 15$	$8 + 8 = 10 + \boxed{6} = 16$
$8 + 7 = 10 + \boxed{5} = 15$	$7 + 9 = 10 + \boxed{6} = 16$
$6 + 9 = \boxed{10} + 5 = 15$	$9 + 7 = \boxed{10} + 6 = 16$
$7 + 8 = 10 + \boxed{5} = 15$	$8 + 8 = \boxed{10} + 6 = 16$
$9 + 6 = \boxed{10} + 5 = 15$	$6 + 7 = 10 + \boxed{3} = 13$
$7 + 7 = 10 + \boxed{4} = 14$	$9 + 5 = \boxed{10} + 4 = 14$
$5 + 8 = 10 + \boxed{3} = 13$	$6 + 9 = 10 + \boxed{5} = 15$
$9 + 4 = \boxed{10} + 3 = 13$	$9 + 7 = 10 + \boxed{6} = 16$
$7 + 6 = 10 + \boxed{3} = 13$	$8 + 7 = \boxed{10} + 5 = 15$

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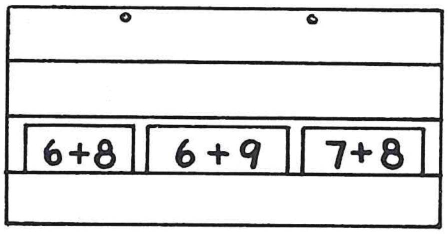
Work page S-92 condenses the thinking steps involved in solving such problems as 9 + 6 = ☐. Encourage the children to rethink each problem in terms of moving to ten and then beyond. Have them use the pictures at the top of the page if they need a visual clue in their work.

Supplemental Experiences

▶ Place three numeral cards in the pocket chart. Have two of the cards represent the number fifteen, and the third a different number. Call upon a child to pull out any card that does not belong to the "fifteen family." Continue in this way using numbers from eleven through sixteen.



▶ Play the game "Who Can Be My Partner?" Distribute addition placeholder-equation cards with all possible combinations of eleven through fifteen. Call upon someone to place his card, for example 8 + 7 = ☐, on the chalk tray. He must read his placeholder equation and say, "Who Can Be My Partner?" The child with the commuted form for this addition placeholder-equation, 7 + 8 = ☐, must come forward and put his card on the chalk tray next to the first card and read his equation. Have the class read the two equations and give the answers. Continue the game with another search for partners.

Subtraction—Addition

COMBINATIONS OF FIFTEEN AND SIXTEEN

Work Pages S-93 and S-94

Objectives

To extend the development of subtraction to combinations of fifteen and sixteen.

To emphasize the inverse relation between addition and subtraction.

Fundamentals

The child has worked with addition problems, involving two single-digit numerals, with sums greater than ten. He is now ready to examine subtraction equations related to sums of fifteen and sixteen. As he subtracts a number from fifteen, for example, he should be encouraged to think first of moving back to ten and then to the number which is the difference. The teacher will help the child apply his knowledge of the inverse relation between addition and subtraction to combinations of fifteen and sixteen.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition; Subtraction.

Readiness for Understanding

Understanding of the concept of different numerals for a number.

Ability to subtract a number represented by a one-digit numeral from a number represented by a two-digit numeral.

Mastery of subtraction combinations to ten.

Awareness of the inverse relation between addition and subtraction.

Ability to work with placeholder equations.

Developmental Experiences

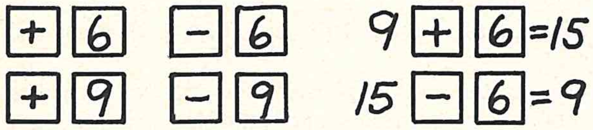
MATERIALS

pegboards
pegs
hundred-board
colored spools
tagboard cards

▶ Have children use peg boards to work out answers to subtraction problems related to sums of fifteen and sixteen. Use the large hundred-board with colored spools to guide the activity.

Place the equation $15 - 6 = \square$ on the chalkboard. Direct children to place fifteen pegs in the first two rows of their boards. Ask them to tell how many pegs they must remove from the set of fifteen according to the equation. Ask someone to tell how he plans to remove the set of six: he should remove the set in two parts, as a set of five and a set of one. Have the children remove the necessary pegs and then tell how many remain. Have the placeholder equation completed. Continue in this way to develop other subtraction combinations related to sums of fifteen and sixteen.

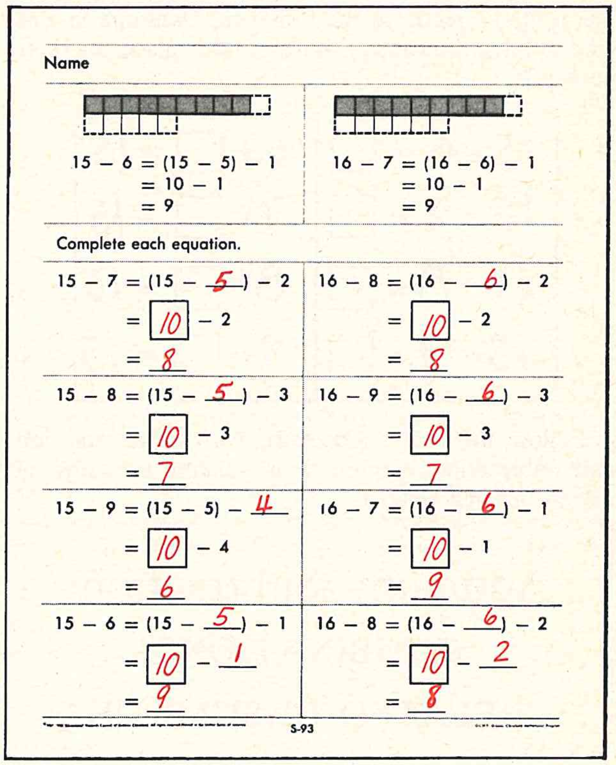
▶ Distribute cards bearing plus signs, minus signs, and numerals 6 through 9, as above. Write several incomplete equations on the chalkboard. Ask the children who have the needed operation symbol and numeral to bring their cards forward, one at a time, and place them in the incomplete statements to make true statements. Have the completed statements read. Continue in this way with other combinations of fifteen and sixteen.



▶ Use work page S-93 to provide practice in working through the thinking steps involved in subtraction problems related to sums of fifteen and sixteen.

After working the first four problems with the class, encourage them to complete the remainder of the page independently.

Work page S-94 emphasizes the inverse relation between addition and subtraction. Work the first pair of equations with the children, discussing the inverse concept. Have the class work the remainder of the page on their own.



Name

$15 - 6 = (15 - 5) - 1$
$= 10 - 1$
$= 9$

$16 - 7 = (16 - 6) - 1$
$= 10 - 1$
$= 9$

Complete each equation.

$15 - 7 = (15 - \underline{5}) - 2$
$= \boxed{10} - 2$
$= \underline{8}$

$16 - 8 = (16 - \underline{6}) - 2$
$= \boxed{10} - 2$
$= \underline{8}$

$15 - 8 = (15 - \underline{5}) - 3$
$= \boxed{10} - 3$
$= \underline{7}$

$16 - 9 = (16 - \underline{6}) - 3$
$= \boxed{10} - 3$
$= \underline{7}$

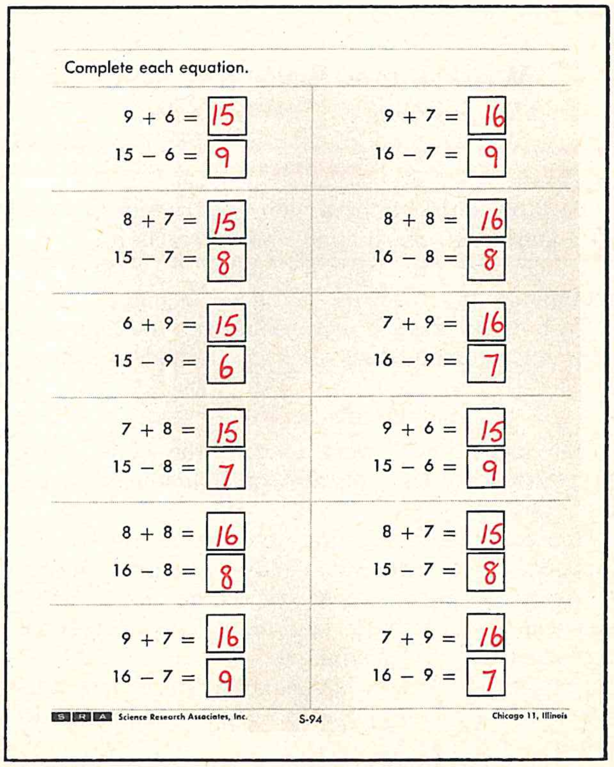
$15 - 9 = (15 - 5) - \underline{4}$
$= \boxed{10} - 4$
$= \underline{6}$

$16 - 7 = (16 - \underline{6}) - 1$
$= \boxed{10} - 1$
$= \underline{9}$

$15 - 6 = (15 - \underline{5}) - 1$
$= \boxed{10} - \underline{1}$
$= \underline{9}$

$16 - 8 = (16 - \underline{6}) - 2$
$= \boxed{10} - \underline{2}$
$= \underline{8}$

S-93



Complete each equation.

$9 + 6 = \boxed{15}$
$15 - 6 = \boxed{9}$

$9 + 7 = \boxed{16}$
$16 - 7 = \boxed{9}$

$8 + 7 = \boxed{15}$
$15 - 7 = \boxed{8}$

$8 + 8 = \boxed{16}$
$16 - 8 = \boxed{8}$

$6 + 9 = \boxed{15}$
$15 - 9 = \boxed{6}$

$7 + 9 = \boxed{16}$
$16 - 9 = \boxed{7}$

$7 + 8 = \boxed{15}$
$15 - 8 = \boxed{7}$

$9 + 6 = \boxed{15}$
$15 - 6 = \boxed{9}$

$8 + 8 = \boxed{16}$
$16 - 8 = \boxed{8}$

$8 + 7 = \boxed{15}$
$15 - 7 = \boxed{8}$

$9 + 7 = \boxed{16}$
$16 - 7 = \boxed{9}$

$7 + 9 = \boxed{16}$
$16 - 9 = \boxed{7}$

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Supplemental Experiences

▶ Arrange placeholder equations related to addition and subtraction combinations of fifteen on the flannel board. Distribute two sets of felt numerals 6 through 9 to the children. Call upon a child to come forward and place his numeral in one of the placeholders in order to make a true statement. It makes no difference whether he chooses to complete an addition or subtraction equation as long as the numeral fits correctly in the sentence. Continue in this way to have the other children find a place for their numerals.

15 − 6 = □	6 + □ = 15
15 − 7 = □	7 + □ = 15
15 − 8 = □	8 + □ = 15
15 − 9 = □	9 + □ = 15

Follow the same procedure with other addition and subtraction combinations related to sums of eleven through sixteen.

Addition—Subtraction
COMBINATIONS
RELATED TO SUMS OF
ELEVEN THROUGH SIXTEEN

Work Pages S-95 and S-96

Objectives

To provide the children with opportunities to use their knowledge of addition and subtraction.

To develop the children's ability to choose the operational sign required in a given equation.

To provide practice in working with addition and subtraction combinations of eleven through sixteen.

Fundamentals

The teacher will want to give the child every chance to apply his skills and understandings to new situations.

The concepts of addition and subtraction will be expanded. Once again, the child will be given an opportunity to apply what he has learned about addition and subtraction to determining the operational sign needed in a given equation.

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concepts of addition and subtraction.

Ability to determine the operational symbol required in a given equation.

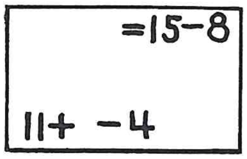
Ability to work with addition and subtraction combinations of eleven through sixteen.

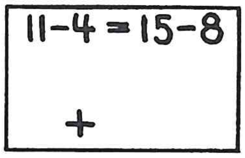
Developmental Experiences

MATERIALS

- flannel board
- felt numerals 0 through 9
- felt symbols for operations and relations
- felt triangle
- individual sets of plastic numerals
- plastic symbols for operations and relations
- triangles
- numeral cards 0 through 19
- tagboard folding cards
- counting sticks
- rubber bands

▶ Place the expression = 15 − 8 in the center of the flannel board. Scatter a plus sign, a minus sign, and numeral cards 11 and 4 below these symbols. Call on a child to use these scattered symbols to form a numeral representing the same number that 15 − 8 represents. Have him place it to the left of the equal sign and read his equation.





Place the expression = 6 + 5 on the flannel board. Scatter a plus sign, a minus sign, and numeral cards 14 and 3 below these symbols. Call on a child to use these scattered symbols to form a numeral representing the same number that 6 + 5 represents.

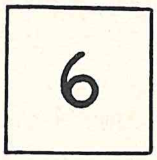
Continue in this way, having children construct other equations related to sums of five through sixteen.

▶ Have the children use their plastic numerals, symbols for operations and relations, triangle, and parentheses at their desks to build equations requiring operational signs. Give directions for the equation you wish built: 9 + 7 = 7 △ 9 or (12 − 2) − 4 = 12 △ 6. After the children have replaced the triangle with the correct operational sign, have the equation read. Have the children tell the name of the operational sign they chose, the name of the operation it represents, and the reason they chose it.

▶ Construct a series of folding cards bearing numerals that represent the thinking steps in harder addition and subtraction combinations.

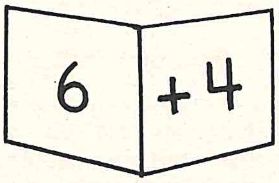
Place the equation 6 + 7 = ☐ on the chalkboard. Have the children place a set of six sticks and a set of seven sticks on their desks.

Call on a child to name the first addend in the problem. Show just this part of the folding card.

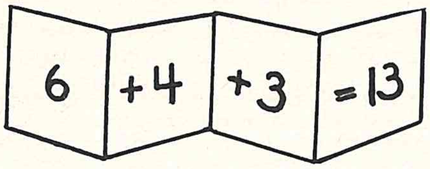


Have the child tell the next thinking step in the problem: he must get to ten.

Tell the children to join part of the set of seven sticks to the set of six sticks so that a set of ten is made. Have them place a rubber band around the set of ten sticks in order to make a bundle of one ten. Ask them to tell how many they joined to the set of six to make ten. When they answer "four," unfold the second section of the card.



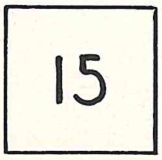
Remind the children that they still have a job to do with the other part of the set of seven: they still have one more number to add in their problem. When they look at their sets and discover that three more must be joined to ten, unfold the third section of the card. As the number of the union is given, unfold the last section of the card and complete the equation.



Follow this procedure with other addition combinations related to sums of eleven through sixteen.

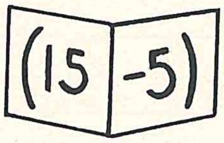
▶ Place the equation 15 − 8 = ☐ on the chalkboard. Have the children place a bundle of ten sticks and five loose sticks on their desks.

Call on a child to name the first number in the problem. Show just this part of the folding card.

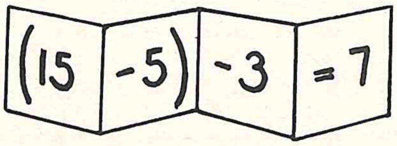


Have the child tell the next thinking step in the problem: he must get back to ten.

Tell the children to remove as many of the fifteen sticks as they need to so that a set of ten remains. Ask them to tell how many they removed from fifteen to make ten. When they answer "five," unfold the second section of the card.

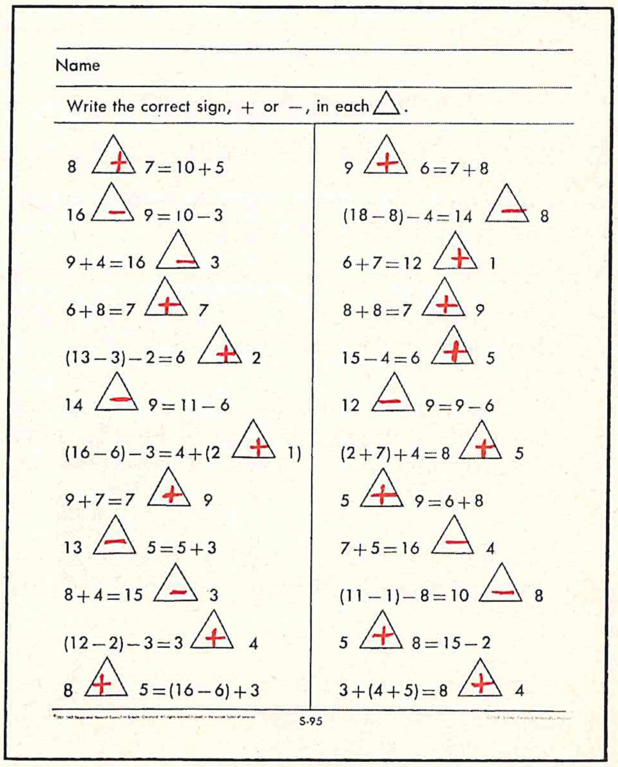


Remind the children that they still have a job to do. They have more objects to remove; they have one more number to subtract in the problem. When they discover that three more must be removed from ten to make a total of eight objects removed, have them take apart their set of ten sticks so that they can remove three. As they do this, unfold the third section of the card. When the number of the set that remains is given, unfold the last section of the card to complete the equation.



Follow this procedure with other subtraction combinations of eleven through sixteen.

▶ Use work page S-95 to give the children an additional opportunity to select the appropriate operational sign to complete equations. Work the first equation with the group, and then have them complete the page independently.



Name

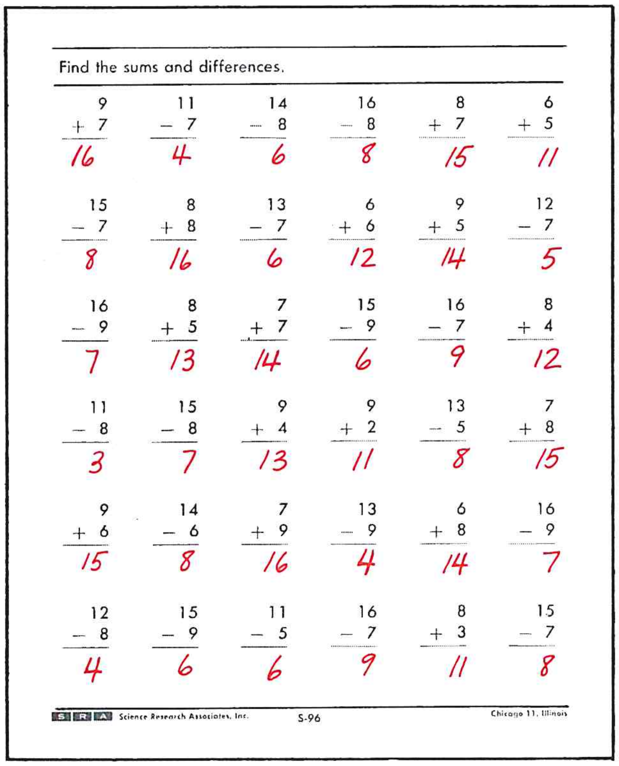
Write the correct sign, + or −, in each △.

8 △+ 7 = 10 + 5
16 △− 9 = 10 − 3
9 + 4 = 16 △− 3
6 + 8 = 7 △+ 7
(13 − 3) − 2 = 6 △+ 2
14 △− 9 = 11 − 6
(16 − 6) − 3 = 4 + (2 △+ 1)
9 + 7 = 7 △+ 9
13 △− 5 = 5 + 3
8 + 4 = 15 △− 3
(12 − 2) − 3 = 3 △+ 4
8 △+ 5 = (16 − 6) + 3

9 △+ 6 = 7 + 8
(18 − 8) − 4 = 14 △− 8
6 + 7 = 12 △+ 1
8 + 8 = 7 △+ 9
15 − 4 = 6 △+ 5
12 △− 9 = 9 − 6
(2 + 7) + 4 = 8 △+ 5
5 △+ 9 = 6 + 8
7 + 5 = 16 △− 4
(11 − 1) − 8 = 10 △− 8
5 △+ 8 = 15 − 2
3 + (4 + 5) = 8 △+ 4

S-95

Work page S-96 provides a review of known addition and subtraction combinations of eleven through sixteen. Tell the children to find the sums or differences. After giving directions, have them do the entire work page on their own.



Find the sums and differences.

9 + 7 16	11 − 7 4	14 − 8 6	16 − 8 8	8 + 7 15	6 + 5 11
15 − 7 8	8 + 8 16	13 − 7 6	6 + 6 12	9 + 5 14	12 − 7 5
16 − 9 7	8 + 5 13	7 + 7 14	15 − 9 6	16 − 7 9	8 + 4 12
11 − 8 3	15 − 8 7	9 + 4 13	9 + 2 11	13 − 5 8	7 + 8 15
9 + 6 15	14 − 6 8	7 + 9 16	13 − 9 4	6 + 8 14	16 − 9 7
12 − 8 4	15 − 9 6	11 − 5 6	16 − 7 9	8 + 3 11	15 − 7 8

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Supplemental Experiences

▶ Present oral story problems. First, have the children tell what operation is needed to get the answer to the problem. Next, have them tell the number combination involved. For example:

> Janet had fourteen cookies. On the way to school she gave six of the cookies to her friends. How many cookies does Janet have now?
>
> Tom put seven books on the top shelf of the cupboard. Bill put nine books on the same shelf. How many books did the boys place on the top shelf?
>
> Don gave five of his marbles to Jim. Then Don had eight marbles. How many marbles did Don have to begin with?
>
> Molly has eleven blue pencils and eight red pencils. How many more blue pencils does Molly have than red pencils?

Addition—Subtraction

COMBINATIONS OF SEVENTEEN AND EIGHTEEN

Work Pages S-97 and S-98

Objectives

To extend the children's knowledge of addition involving two numbers, each less than ten, to sums of seventeen and eighteen.

To help the children apply the associative property.

To re-emphasize the inverse relation between addition and subtraction.

To review known addition and subtraction combinations of eleven through sixteen.

Fundamentals

The child will now apply the associative property to addition combinations of seventeen and eighteen and related subtractions. When he meets a problem with two single-digit addends whose sum is greater than ten, he should think of the solution in terms of ten. He should think of the sum $9 + 9$, for example, as $10 + 8$.

Using known ideas to explore new situations can be challenging. The more mature child is especially intrigued by this kind of activity. The teacher will need to give more time to the less mature child so that he, too, will be encouraged to explore and to achieve maximum benefit from new materials.

In working problems such as $9 + 8 = \square$, the child adds eight to nine with a result of seventeen. If he is aware of the inverse relation between addition and subtraction, he knows that he can take the sum seventeen, subtract eight, and be right back where he started: $17 - 8 = 9$. He also knows how to solve such problems as $9 + \square = 17$, $17 - \square = 9$, and $\square + 8 = 17$.

The child is learning that his ideas about numbers and operations are definitely related to each other. As he acquires and applies these ideas, it becomes easier for him to find answers to new problems.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the associative property.

An understanding of the concept of different numerals for the same number.

An understanding of the concept of place value.

Mastery of the addition combinations through ten.

Awareness of the inverse relation between addition and subtraction.

Ability to work with addition and subtraction combinations of eleven through eighteen.

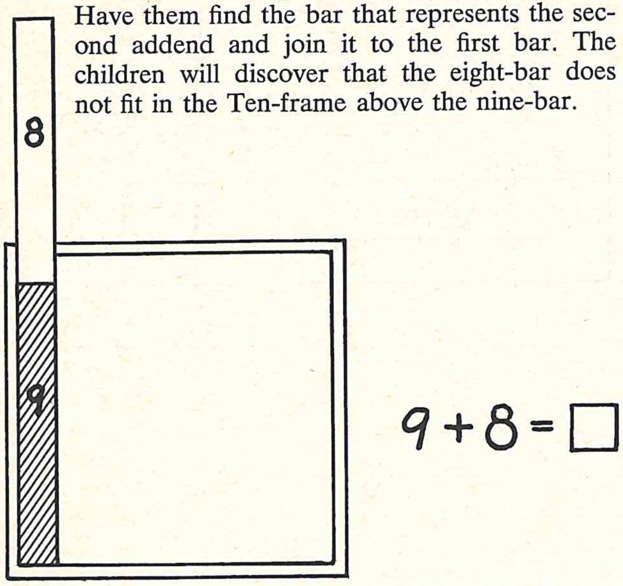
Ability to work with placeholder equations.

Developmental Experiences

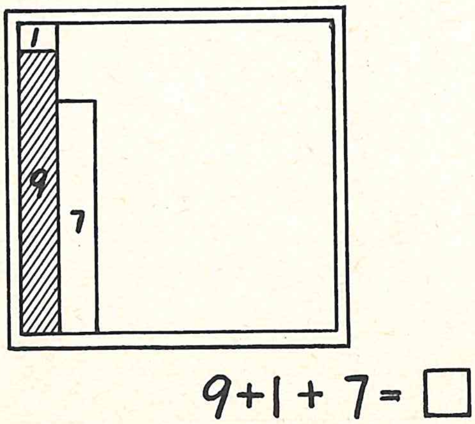
MATERIALS

individual Ten-frames
tagboard cards
pocket chart

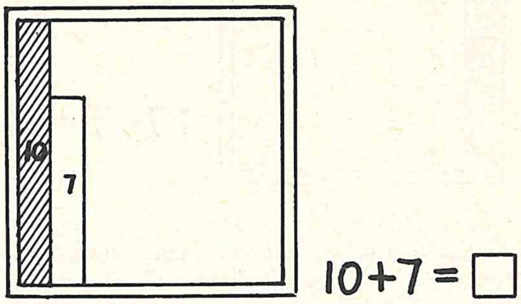
▶ The use of Ten-frames will prove helpful in the development of harder addition and subtraction combinations. Place a card bearing the equation 9 + 8 = □ in the pocket chart. Have the children place the bar which represents the first addend in the equation in their Ten-frames. Have them find the bar that represents the second addend and join it to the first bar. The children will discover that the eight-bar does not fit in the Ten-frame above the nine-bar.



Ask the children what bar is needed with nine to fit into the Ten-frame. Have them think what two bars could be used in place of eight, if one of the bars is the one-bar. Have them replace the eight-bar with the one-bar and the seven-bar. Have the one-bar fitted into place above the nine-bar and the seven-bar next to the nine-bar. From an assortment of tagboard cards, have a child select the one that relates to this phase of the activity and place it in the pocket chart under the original equation: 9 + 1 + 7 = □.



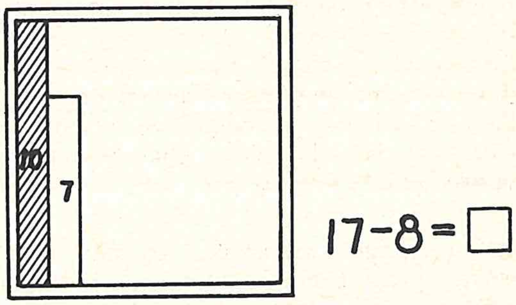
Ask the children whether there is a single bar that can replace the nine-bar and the one-bar. Have the children remove these bars and put the ten-bar in the frame in their place. From an assortment of tagboard cards, have a child select the one that relates to this phase of the activity. Have him place it in the pocket chart: 10 + 7 = □.



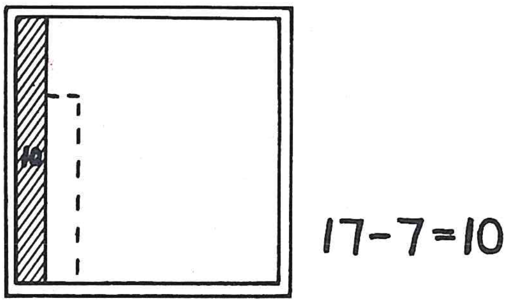
Have the standard place-value numeral for seventeen placed below the equation 10 + 7 = □ in the chart. Have the original placeholder equation read and the answer given. Guide the children as they summarize the thinking steps in this problem.

Continue in this way, developing other addition combinations related to sums of seventeen and eighteen. Use the Ten-frame to review addition combinations of eleven through eighteen.

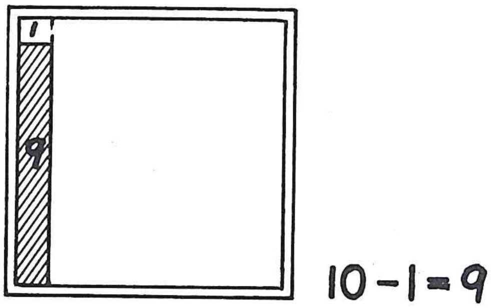
▶ Use the Ten-frame to aid in the development of subtraction combinations of seventeen and eighteen. In the pocket chart, place a card on which the equation 17 − 8 = □ is written. Have the children place the bars that represent seventeen in the Ten-frame. Tell them to use the two bars that show what seventeen really means in place value: a ten-bar and a seven-bar.



From seventeen, or ten and seven as it is seen in the Ten-frame, tell the children to remove a set of eight. Have them start by removing as many as necessary in order to move back to ten. From an assortment of tagboard cards, have a child select the one that relates to this phase of the activity and place it in the pocket chart under the original equation: 17 − 7 = 10.



Ask the children to tell how many more they must remove to solve the given problem, 17 − 8 = ☐. Have them tell how they can remove one from the ten-bar. Ask whether they can replace the ten-bar with any other bars in order to have a one-bar ready to remove. After the ten-bar has been replaced with the nine-bar and the one-bar, have the children take away as many as necessary to make a total of eight removed.



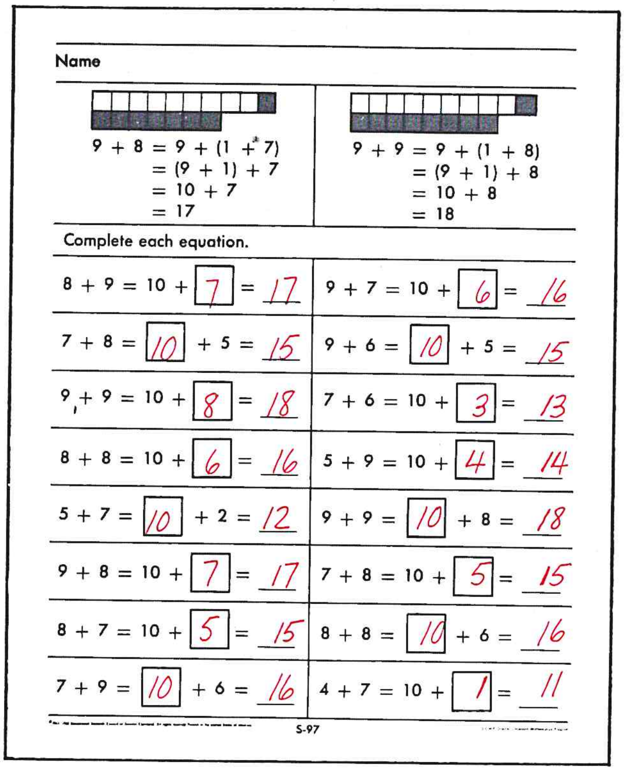
Have a child select a card that relates to this last take-away action and place it in the pocket chart: 10 − 1 = 9.

Have someone now choose a card that has both of the subtraction steps compressed into one. Have this card put into the pocket chart to replace the other two: (17 − 7) − 1 = 9.

Have the original placeholder equation read and the answer given. Guide children as they summarize the thinking steps in this problem.

Continue in this way, developing other subtraction combinations related to sums of seventeen and eighteen. Use the Ten-frame to review subtraction combinations of eleven through eighteen.

▶ Using work page S-97, encourage the children to think through their additions in terms of tens and ones. Work the two problems at the top of the page with the children and then encourage them to complete the remaining equations independently.



Name

$9 + 8 = 9 + (1 + 7)$
$= (9 + 1) + 7$
$= 10 + 7$
$= 17$

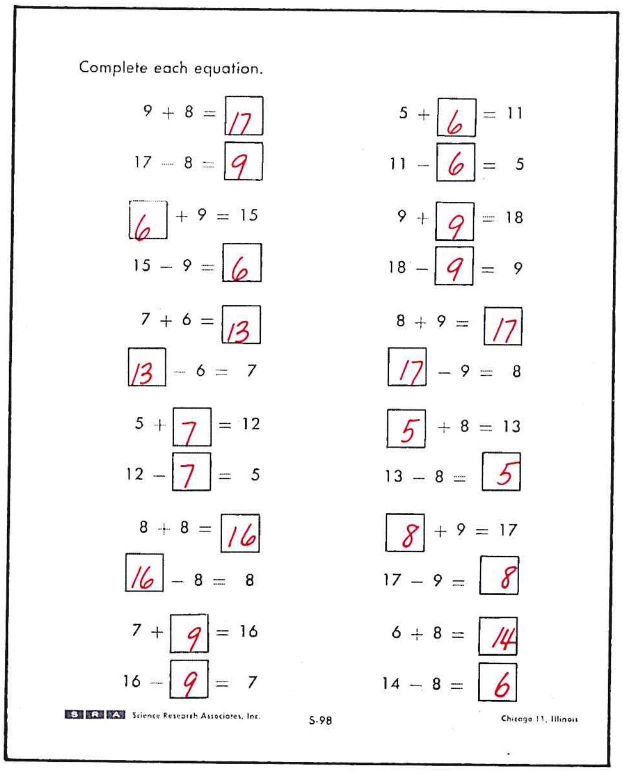
$9 + 9 = 9 + (1 + 8)$
$= (9 + 1) + 8$
$= 10 + 8$
$= 18$

Complete each equation.

$8 + 9 = 10 + \boxed{7} = \underline{17}$
$7 + 8 = \boxed{10} + 5 = \underline{15}$
$9 + 9 = 10 + \boxed{8} = \underline{18}$
$8 + 8 = 10 + \boxed{6} = \underline{16}$
$5 + 7 = \boxed{10} + 2 = \underline{12}$
$9 + 8 = 10 + \boxed{7} = \underline{17}$
$8 + 7 = 10 + \boxed{5} = \underline{15}$
$7 + 9 = \boxed{10} + 6 = \underline{16}$

$9 + 7 = 10 + \boxed{6} = \underline{16}$
$9 + 6 = \boxed{10} + 5 = \underline{15}$
$7 + 6 = 10 + \boxed{3} = \underline{13}$
$5 + 9 = 10 + \boxed{4} = \underline{14}$
$9 + 9 = \boxed{10} + 8 = \underline{18}$
$7 + 8 = 10 + \boxed{5} = \underline{15}$
$8 + 8 = \boxed{10} + 6 = \underline{16}$
$4 + 7 = 10 + \boxed{1} = \underline{11}$

S-97



Complete each equation.

$9 + 8 = \boxed{17}$
$17 - 8 = \boxed{9}$

$\boxed{6} + 9 = 15$
$15 - 9 = \boxed{6}$

$7 + 6 = \boxed{13}$
$\boxed{13} - 6 = 7$

$5 + \boxed{7} = 12$
$12 - \boxed{7} = 5$

$8 + 8 = \boxed{16}$
$\boxed{16} - 8 = 8$

$7 + \boxed{9} = 16$
$16 - \boxed{9} = 7$

$5 + \boxed{6} = 11$
$11 - \boxed{6} = 5$

$9 + \boxed{9} = 18$
$18 - \boxed{9} = 9$

$8 + 9 = \boxed{17}$
$\boxed{17} - 9 = 8$

$\boxed{5} + 8 = 13$
$13 - 8 = \boxed{5}$

$\boxed{8} + 9 = 17$
$17 - 9 = \boxed{8}$

$6 + 8 = \boxed{14}$
$14 - 8 = \boxed{6}$

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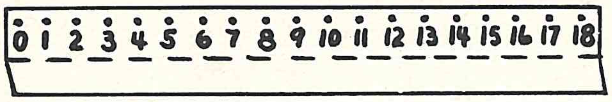
Work page S-98 will re-emphasize the inverse relation between addition and subtraction. It will provide a review of the addition and subtraction combinations of eleven through eighteen. After giving directions, have the children work the entire page on their own.

Supplemental Experiences

▶ Place a number-strip on the flannel board. Make the strip so that it is possible to fold the lower section up over the numerals. In this way the symbols on the line can be covered and uncovered at will.

Begin the activity by covering the numerals, leaving only the dots on the line exposed. Place your finger on the dot that represents the ninth step from zero. "Walk" your fingers six steps to the right of 9. Have the class count these steps as you move your fingers: "one, two, three, four, five, six." Hold your index finger on the stopping point. Ask the children to tell where you are on the strip now. After the number has been named, uncover the numerals on the strip and have children check their answers. Show them "the path you walked" by starting again at 9 and "rewalking" the six steps. Have the children watch the action and relate it to the numbers they see represented on the strip. Continue in this way, using the number-strip with other addition combinations of eleven through eighteen.



▶ Use the number-strip for subtraction combinations. Begin the activity by covering the numerals on the strip, leaving only the dots on the line exposed. Place your finger on the dot that represents the fifteenth step from zero. "Walk" your fingers six steps backward from 15. Have the children count these steps as you move your fingers. Hold your index finger on the stopping point and ask the children to tell where you are now. After the location has been given, uncover the numerals on the strip and have the children check their answers. Show them "the path you walked" by starting again at 15 and "rewalking" the six steps. Have the children watch the action and relate it to the numbers they see represented on the strip. Continue in this way with other subtraction combinations related to sums of eleven through eighteen.

Addition—Subtraction

COMBINATIONS OF

ZERO THROUGH EIGHTEEN

Work Pages S-99 Through S-102

Objectives

To help the children record known addition combinations in the addition table.

To review known addition and subtraction combinations of zero through eighteen.

To re-emphasize the inverse relation between addition and subtraction.

To re-emphasize the number patterns.

Fundamentals

In the development of any concept, the child may benefit by stopping at various points along the way and taking a look at what has gone on up to that point.

The skillful teacher will want to take time out to pull together ideas related to harder combinations to which she has exposed the second-grade child. She will want to give the child an opportunity to test himself on what he has learned about addition and subtraction combinations related to sums of eleven through eighteen, so that he may feel a sense of accomplishment as he demonstrates his growing ability to work with these combinations.

The second-grade teacher must continue to provide a variety of experiences, so that the child will have every opportunity to gain deeper insight into the fundamental ideas in arithmetic to which he is being exposed. Often a seemingly simple activity that merely provides review for some children will serve to clarify an idea for others.

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concept of addition and subtraction.

Awareness of the inverse relation between addition and subtraction.

Ability to work with addition and subtraction combinations related to sums of zero through eighteen.

Ability to work with placeholder equations and vertical notation.

Ability to write numerals for missing numbers in placeholder equations and vertical notation.

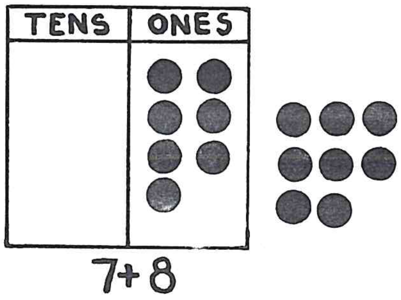
Developmental Experiences

MATERIALS

- bottle caps
- construction paper
- crayons
- chart paper
- 2 pointers

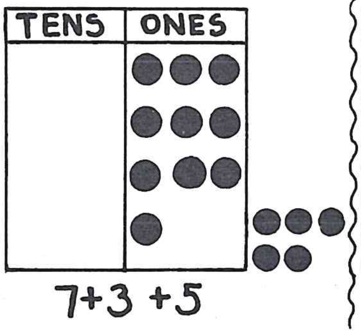
▶ Give a 9 x 12-inch sheet of heavy construction paper to each child and have him use a black crayon to draw a vertical line marking off two areas of the same size on the paper. Tell him to label the area on the left "tens" and the area on the right "ones," and to draw a line under the two words.

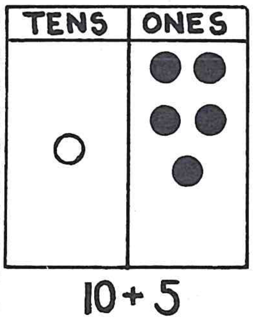
Provide each child with eighteen gold-colored bottle caps (or disks of heavy yellow construction paper that are comparable in size) and one silver-colored bottle cap (or a disk of comparable size made of heavy white construction paper). Have the children use their black crayons to label the gold caps or disks with the numeral 1 to simulate pennies, and the silver caps or disks with the numeral 10 to simulate dimes.



Place the equation 7 + 8 = ☐ on the chalkboard. Have the children place seven ones (or pennies) in the ones column on their paper. Tell them to join a set of eight pennies to the set of seven. Have them tell how they plan to do this.

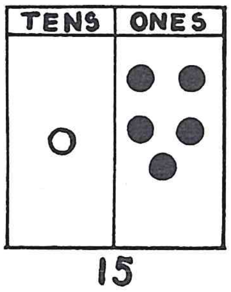
The children should first put three pennies with the seven pennies, then remove the ten ones (pennies) that they have in the ones column and replace them with one ten (dime) in the tens column. Next have them place the other part of the set of eight (five pennies) in the ones column. The final result will be viewed as one ten and five ones, or fifteen.



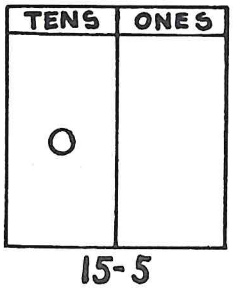


Have the placeholder equation completed and the thinking steps in the problem summarized.

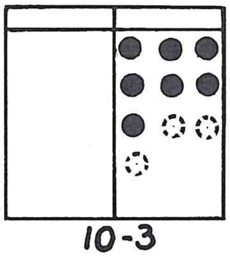
Place the inverse equation, 15 − 8 = ☐, on the chalkboard. Have children "undo" the addition problem they just completed. Have them tell what they will do to get back to the seven pennies they started with.



They should first remove five pennies to get back to ten.



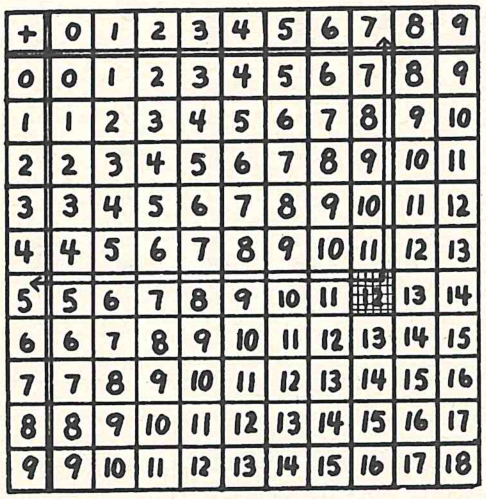
So that they can remove a total of eight pennies, they must exchange the one ten, or dime, for ten ones (pennies) and put these in the ones column. Three pennies will now be removed from the set of ten ones, making a total of eight ones taken away. The final result will be seven ones. Have the placeholder equation completed and the thinking steps summarized.



Follow the same procedure to develop other addition and subtraction combinations related to sums of eleven through eighteen.

▶ Use either or both of the two procedures suggested under "Developmental Experiences" on work page S-5 as children help to fill in a chart-size addition table for sums of zero through eighteen. When the table is completed, have the children choose number combinations and find their sums in the table. Have them place one pointer on the first addend, in the first column, and another pointer on the second addend, in the top row. Have them move the pointers, one down and one across, until they meet at the box that contains the sum. Have the equations read.

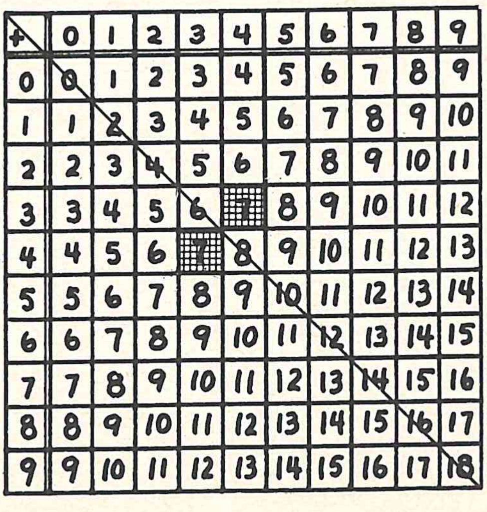
Have the children show the inverse subtraction equation related to the sums they have just pointed out. After a child gives a particular addition combination, have him reverse his steps with the pointers and give the related subtraction combination. The one pointer should be moved first to the numeral in the top row which represents the number to be subtracted. The other pointer should be moved to the numeral in the first column which represents the difference.


12−7= 5

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

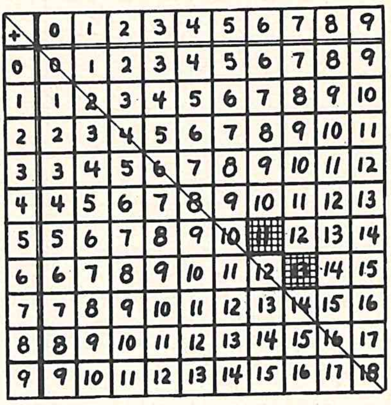
Depending on the maturity level of the group, children may find interesting patterns in the addition table. Encourage them to be observant.

The main diagonal of the table is the one from the upper left to the lower right. Some child may observe that entries on either side of the main diagonal are identical and illustrate the commutative property of addition.


3+4=7→
4+3=7→

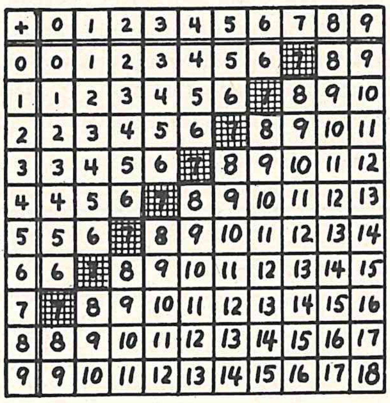
+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Another child may become aware that adjacent entries along the main diagonal differ by two, and he may associate this with a fact about addition: if each of two addends is increased (or decreased) by one, the sum is increased (or decreased) by two.


5+6=11
(5+1)+(6+1)=13
6+7=13

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

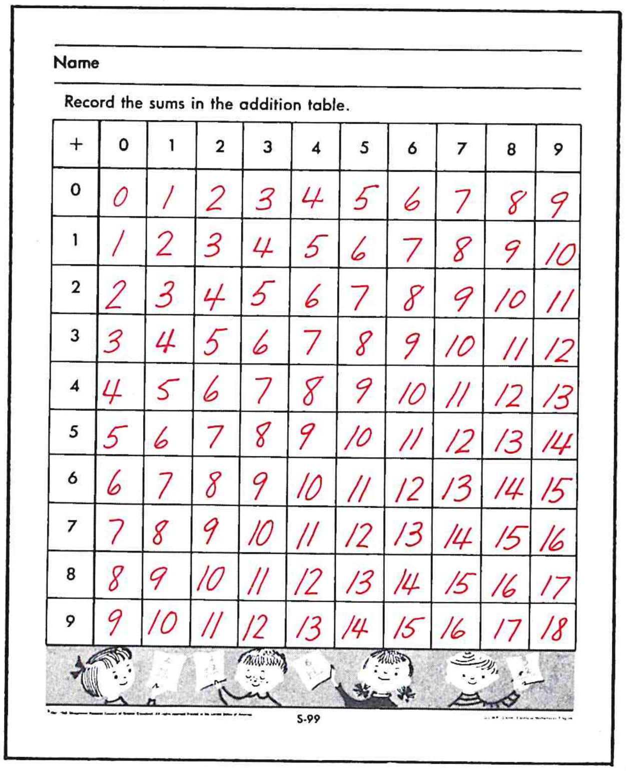
Someone else may discover that entries along any diagonal from the lower left to the upper right are identical, showing that in addition, if one addend is decreased by some number and the other addend is increased by the same number, the sums remain unchanged.


(5-2)+(2+2)=7
5+2=7
3+4=7

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

One of the easiest patterns for children to recognize is that of corresponding rows and columns which are identical and could be completed by counting by ones. The fourth row, for example, is the same as the fourth column: 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. This is the sequence for counting by ones, starting with three.

▶ Use work page S-99 to provide children with an opportunity to record known addition facts. Place a duplicate table on the chalkboard. Work with the children as they fill in two or three columns (or rows) in the addition table. Point to each addend as it is given, first the one in the column on the left and then the one in the top row. Point to the box which is to hold the answer. Have the children fill in the appropriate answer box on their own papers with the sum of the first two addends. Encourage

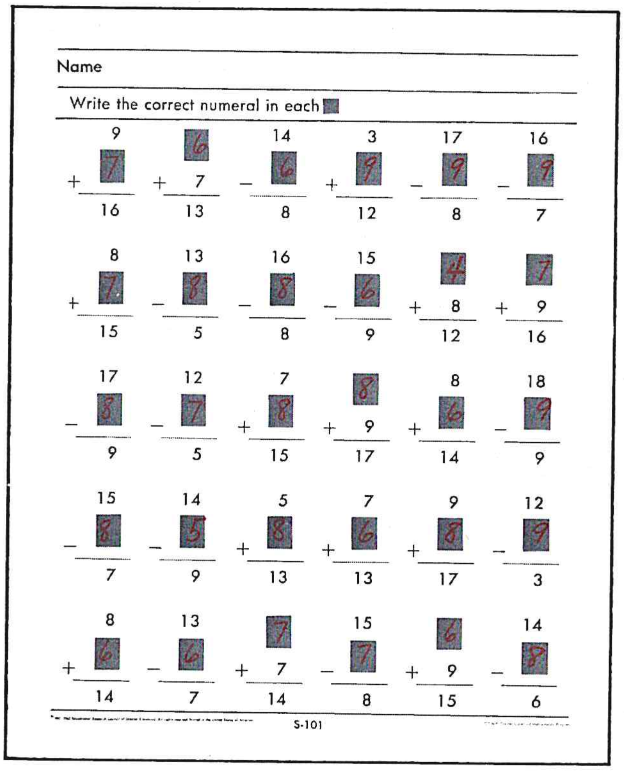


Name

Record the sums in the addition table.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

S-99



Name

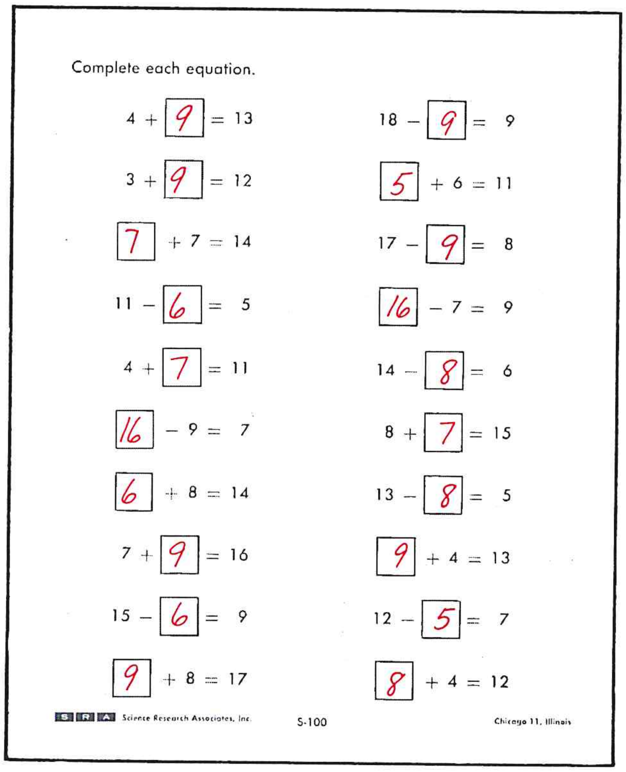
Write the correct numeral in each ▨

9	6	14	3	17	16
+ 7	+ 7	− 6	+ 9	− 9	− 9
16	13	8	12	8	7
8	13	16	15	4	7
+ 7	− 8	− 8	− 6	+ 8	+ 9
15	5	8	9	12	16
17	12	7	8	8	18
− 8	− 7	+ 8	+ 9	+ 6	− 9
9	5	15	17	14	9
15	14	5	7	9	12
− 8	− 5	+ 8	+ 6	+ 8	− 9
7	9	13	13	17	3
8	13	7	15	6	14
+ 6	− 6	+ 7	− 7	+ 9	− 8
14	7	14	8	15	6

S-101

them to fill in the remainder of the addition table on their own. Give help only if necessary.

Work page S-100 provides the children with an opportunity to work with placeholders in various positions in addition and subtraction equations. An understanding of the relation between addition and subtraction will be a distinct advantage to the children in working these problems. Work the first problem with the children, and then have them complete the page independently. When they have finished, discuss each problem with the class. Have them explain how they thought out their answers.



Complete each equation.

4 + $\boxed{9}$ = 13

3 + $\boxed{9}$ = 12

$\boxed{7}$ + 7 = 14

11 − $\boxed{6}$ = 5

4 + $\boxed{7}$ = 11

$\boxed{16}$ − 9 = 7

$\boxed{6}$ + 8 = 14

7 + $\boxed{9}$ = 16

15 − $\boxed{6}$ = 9

$\boxed{9}$ + 8 = 17

18 − $\boxed{9}$ = 9

$\boxed{5}$ + 6 = 11

17 − $\boxed{9}$ = 8

$\boxed{16}$ − 7 = 9

14 − $\boxed{8}$ = 6

8 + $\boxed{7}$ = 15

13 − $\boxed{8}$ = 5

$\boxed{9}$ + 4 = 13

12 − $\boxed{5}$ = 7

$\boxed{8}$ + 4 = 12

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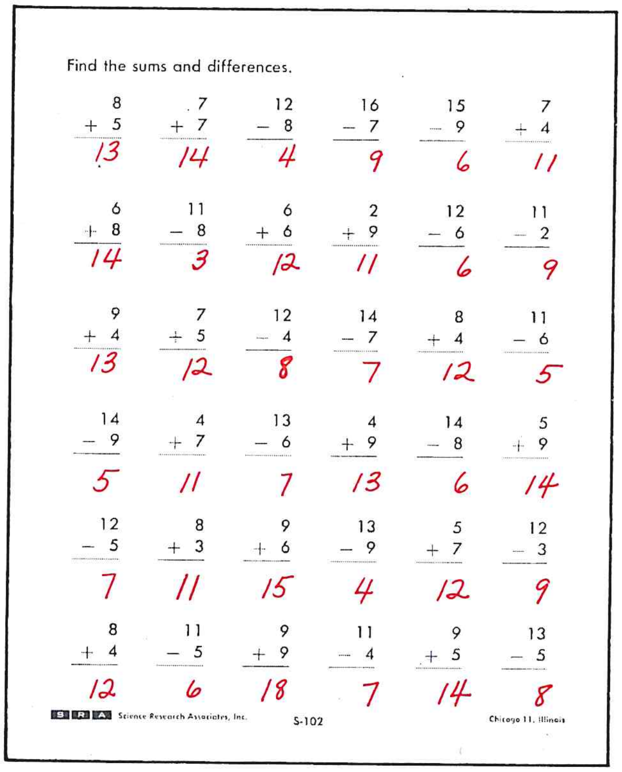

Find the sums and differences.

8	7	12	16	15	7
+ 5	+ 7	− 8	− 7	− 9	+ 4
13	14	4	9	6	11
6	11	6	2	12	11
+ 8	− 8	+ 6	+ 9	− 6	− 2
14	3	12	11	6	9
9	7	12	14	8	11
+ 4	+ 5	− 4	− 7	+ 4	− 6
13	12	8	7	12	5
14	4	13	4	14	5
− 9	+ 7	− 6	+ 9	− 8	+ 9
5	11	7	13	6	14
12	8	9	13	5	12
− 5	+ 3	+ 6	− 9	+ 7	− 3
7	11	15	4	12	9
8	11	9	11	9	13
+ 4	− 5	+ 9	− 4	+ 5	− 5
12	6	18	7	14	8

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▶ Use work page S-101 to provide practice in finding the numeral for a missing number in the vertical-notation form for addition and subtraction combinations involving one two-digit numeral. Work the first problem with the children; then have them complete the remainder of the page independently. When work on this page is completed, discuss each problem in turn with the class. Have them explain their thinking as they solved these problems.

Work page S-102 provides a review of harder combinations written in vertical notation. Have the children find the sums or differences. Remind them to keep the numerals in the ones and the tens positions lined up as carefully as possible.

Supplemental Experiences

▶ Plan a "wishing box" activity for the children. Place nine wooden beads in a box. Tell the children that nine beads are in your wishing box now, but that you wish there were fourteen.

Place the equation $9 + \square = 14$ on the chalkboard. Ask who would like to make your wish come true by placing as many more beads in the box as are needed to make fourteen. Have a child tell how many more beads he plans to place in the box. Have him drop them in, in two phases—first enough to make ten, and then the others. Have him describe his thinking steps as he makes your wish come true. Have the placeholder equation completed.

Continue in this way with other addition combinations related to sums of eleven through eighteen.

Addition

ASSOCIATIVE OR GROUPING PROPERTY

Work Pages S-103 and S-104

Objectives

To strengthen the children's understanding of the associative property.

To provide practice in using the scrambling process.

To provide practice with column addition.

Fundamentals

By combining the properties of commutativity and associativity, the child discovered that he could order and group the addends of an addition problem any way he pleased. In column addition involving sums greater than ten it is quicker to group addends which give the sum of, or multiples of, ten. The teacher will want to help the child become aware of the advantages in using this grouping.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

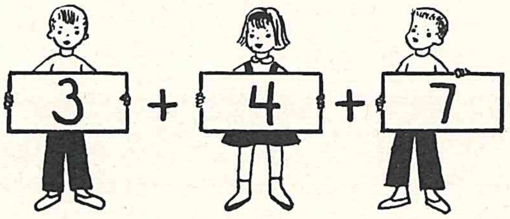
Ability to add two numbers.
Ability to work with placeholder equations.
Ability to work with vertical notation.

Developmental Experiences

MATERIALS

pegboards
pegs
hundred-board
spools
tagboard cards

▶ Give three children cards with numerals that represent the numbers whose sums are eleven through eighteen. Have them stand in a row in front of the chalkboard where there are two large plus signs. Call on a child to read the addition equation and to give the sum. Have the class decide on a different grouping or order for these numbers, and ask the children to change places accordingly. Have the combinations related to this new grouping read and the sum given.



Continue this activity until the class discovers all the possible combinations for the three addends. Ask the children if they think any one grouping is better than any other. Perhaps some of the children will have discovered for themselves that it is easier to add the numbers if a ten combination is selected first.

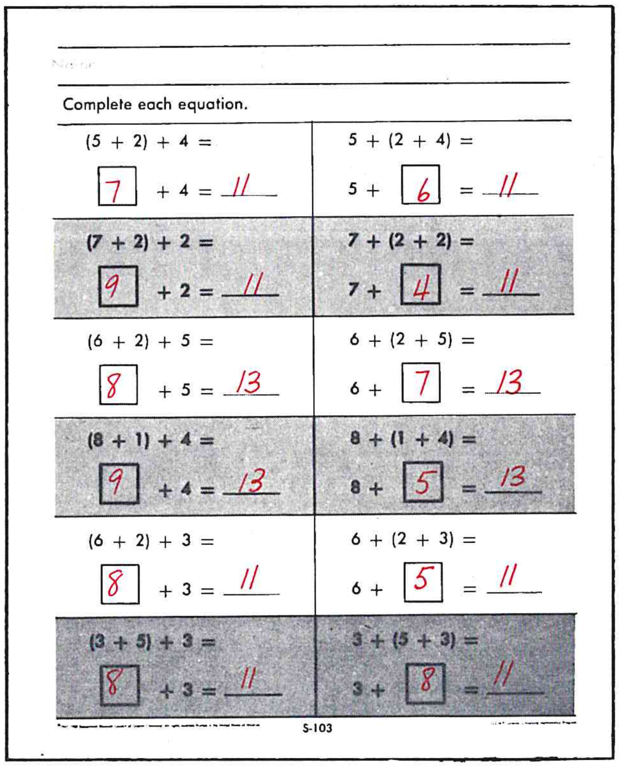
Change the numeral cards and call on three other children to hold them, repeating the activity several times. Control the numbers used in such a way that the sum of the first two is ten or less, no matter how they are grouped.

▶ Use pegboards to reinforce the children's concept of scrambling and to make them aware of the advantage in grouping addends whose sum is ten. Begin with a set of six blue, a set of three yellow, and a set of four red pegs. Direct the children to place these sets of colored pegs at the top of their desks.

Write the equation $6 + 3 + 4 = \square$ on the chalkboard. Tell the children that they may join the sets in any order and make any groupings they wish. When the sets of pegs have been joined and the result noted, complete the placeholder equation on the chalkboard.

Talk about the various groupings the children have made, using the hundred-board to visualize certain points in the discussion. Write the equation for each grouping on the chalkboard, and discuss the particular break that each child had to make when he reached ten. Express amazement that the final result was the same no matter which grouping was chosen.

Have children continue this activity and use sets with different numbers. Keep the numbers of the sets controlled in such a way that the cardinal number of the union of the first two sets is ten or less, no matter how they are grouped.

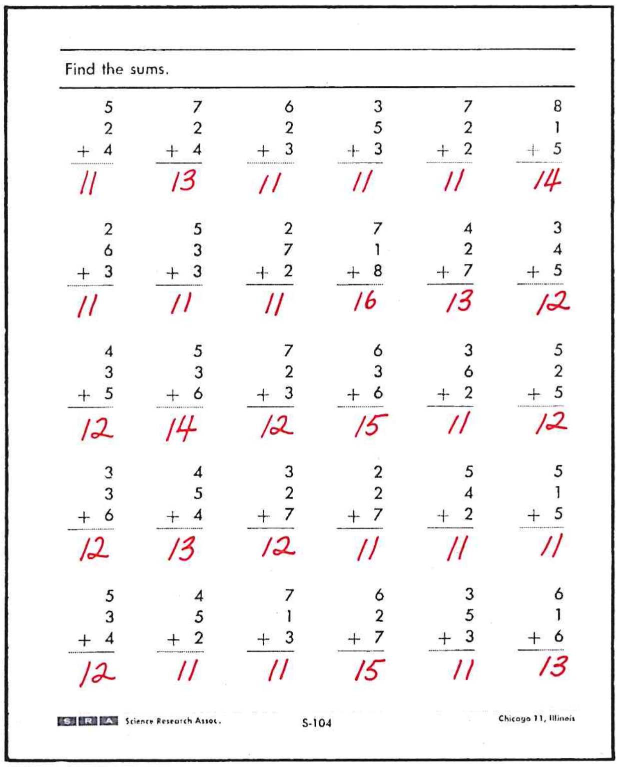


Name

Complete each equation.

$(5 + 2) + 4 =$ $\boxed{7} + 4 = 11$	$5 + (2 + 4) =$ $5 + \boxed{6} = 11$
$(7 + 2) + 2 =$ $\boxed{9} + 2 = 11$	$7 + (2 + 2) =$ $7 + \boxed{4} = 11$
$(6 + 2) + 5 =$ $\boxed{8} + 5 = 13$	$6 + (2 + 5) =$ $6 + \boxed{7} = 13$
$(8 + 1) + 4 =$ $\boxed{9} + 4 = 13$	$8 + (1 + 4) =$ $8 + \boxed{5} = 13$
$(6 + 2) + 3 =$ $\boxed{8} + 3 = 11$	$6 + (2 + 3) =$ $6 + \boxed{5} = 11$
$(3 + 5) + 3 =$ $\boxed{8} + 3 = 11$	$3 + (5 + 3) =$ $3 + \boxed{8} = 11$

S-103



Find the sums.

5 2 + 4 11	7 2 + 4 13	6 2 + 3 11	3 5 + 3 11	7 2 + 2 11	8 1 + 5 14
2 6 + 3 11	5 3 + 3 11	2 7 + 2 11	7 1 + 8 16	4 2 + 7 13	3 4 + 5 12
4 3 + 5 12	5 3 + 6 14	7 2 + 3 12	6 3 + 6 15	3 6 + 2 11	5 2 + 5 12
3 3 + 6 12	4 5 + 4 13	3 2 + 7 12	2 2 + 7 11	5 4 + 2 11	5 1 + 5 11
5 3 + 4 12	4 5 + 2 11	7 1 + 3 11	6 2 + 7 15	3 5 + 3 11	6 1 + 6 13

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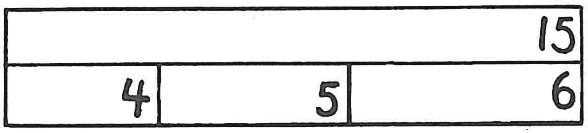
▶ Use work page S-103 to provide practice in working with the associative or grouping property. Work the first pair of problems across the top of the page with the children. Encourage them to complete the page on their own, and give help only if needed.

Work page S-104 will provide practice in column addition involving sums not greater than eighteen. After you have given directions, the children can work the problems on their own.

Supplemental Experiences

▶ Design number-strips to represent the sums eleven through eighteen. Fix them so they will adhere to the flannel board. On a table next to the flannel board, place an assortment of number-strips representing numbers one through nine. Place the fifteen-strip on the flannel board. Have children take turns finding three strips that when joined will have the same number of squares as the fifteen-strip. Have them write the equation related to their grouping on the chalkboard and read it to the class. Place other strips representing the sums eleven through eighteen on the flannel board, and continue the activity.



Story Problems

Work Pages S-105 Through S-108

Objectives

To help children reason through and find the correct solution to story problems.

To provide practice in the use of known addition and subtraction combinations for solving story problems.

Fundamentals

Multiple experiences with joining, separating, and comparing sets bring meaning to story problems. The children have made, solved, and illustrated problems of their own. They have followed the step-by-step procedure found on work pages S-17 and S-18.

Readiness for Understanding

Ability to find the sums and differences of numbers represented by one- and two-digit numerals.

Ability to read and comprehend story problems.

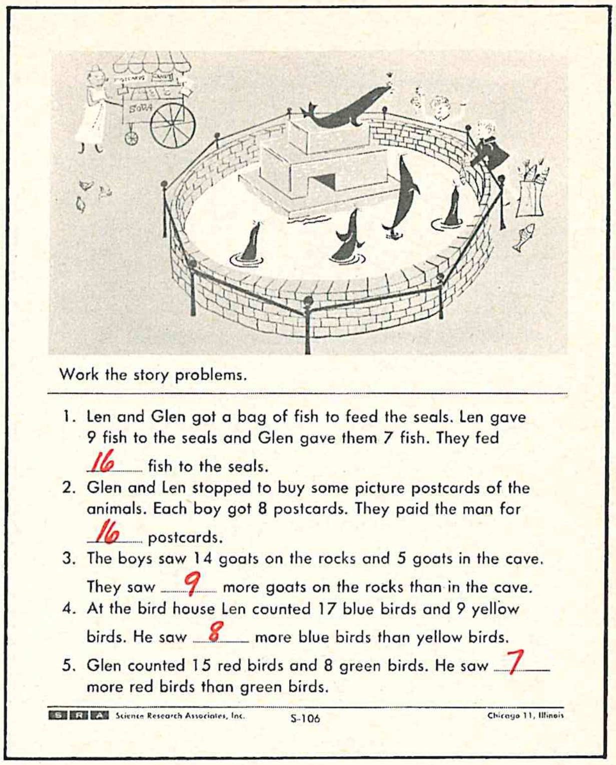
Developmental Experiences

MATERIALS

animal pictures
addition and subtraction combination cards

▶ Display pictures of zoo animals and talk about them with the class, asking several children to tell of their experiences at the zoo. Distribute work pages S-105 and S-106 then discuss the pictures at the top of the page, telling the children they will read about two boys and their trip to the zoo. Ask the children to think of the animals in groups, or in cages, as they have seen them at the zoo or in pictures, in order to visualize the groups in each story.

Review the four steps in thinking through a problem, and have the children solve the story problems found on both pages. Encourage them to evaluate their own work before group checking, since they should be aware of the need for deciding if an answer is reasonable and credible.



Work the story problems.

1. Len and Glen got a bag of fish to feed the seals. Len gave 9 fish to the seals and Glen gave them 7 fish. They fed 16 fish to the seals.
2. Glen and Len stopped to buy some picture postcards of the animals. Each boy got 8 postcards. They paid the man for 16 postcards.
3. The boys saw 14 goats on the rocks and 5 goats in the cave. They saw 9 more goats on the rocks than in the cave.
4. At the bird house Len counted 17 blue birds and 9 yellow birds. He saw 8 more blue birds than yellow birds.
5. Glen counted 15 red birds and 8 green birds. He saw 7 more red birds than green birds.

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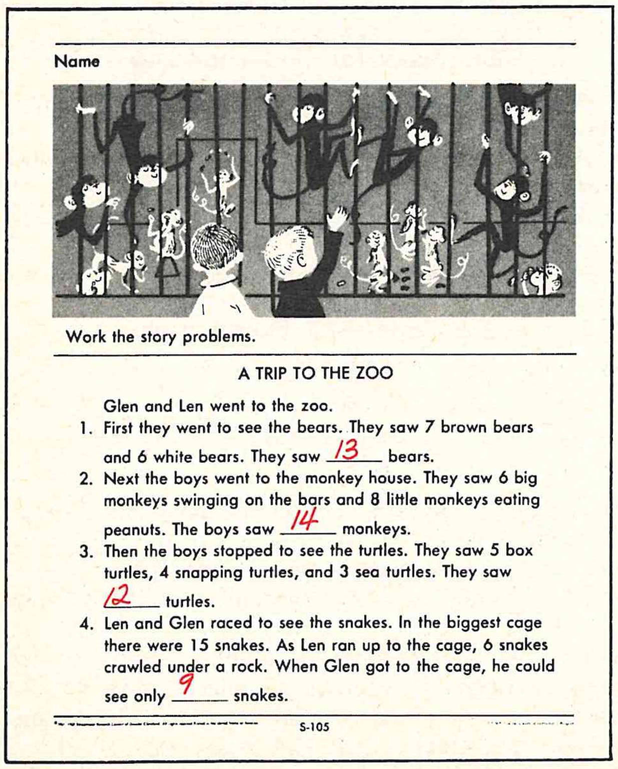
▶ Dramatize story-problem situations that occur in the classroom. Tell a story, pose a question, and have the children work out the answer—first by thinking through the situation, and then by acting it out. For example, you might say, "During the arithmetic lesson, seven children worked problems at the chalkboard. When they sat down, six more children went to the chalkboard. How many children worked at the chalkboard?" After a child has given the answer, let him play the part of a stage director and choose actors to show how he thought out his answer.

▶ Use work pages S-107 and S-108 to give the class practice in solving story problems. Have one child read the first problem aloud, while another



Name

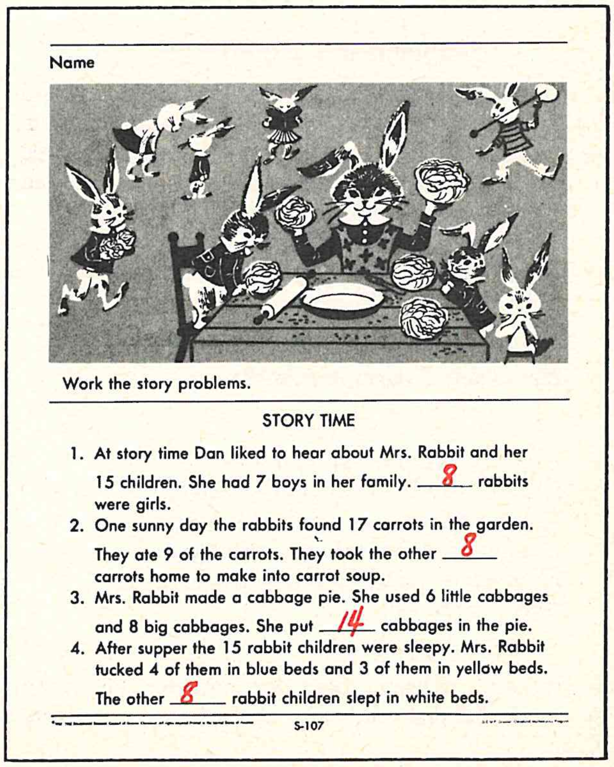
Work the story problems.

A TRIP TO THE ZOO

Glen and Len went to the zoo.

1. First they went to see the bears. They saw 7 brown bears and 6 white bears. They saw 13 bears.
2. Next the boys went to the monkey house. They saw 6 big monkeys swinging on the bars and 8 little monkeys eating peanuts. The boys saw 14 monkeys.
3. Then the boys stopped to see the turtles. They saw 5 box turtles, 4 snapping turtles, and 3 sea turtles. They saw 12 turtles.
4. Len and Glen raced to see the snakes. In the biggest cage there were 15 snakes. As Len ran up to the cage, 6 snakes crawled under a rock. When Glen got to the cage, he could see only 9 snakes.

S-105



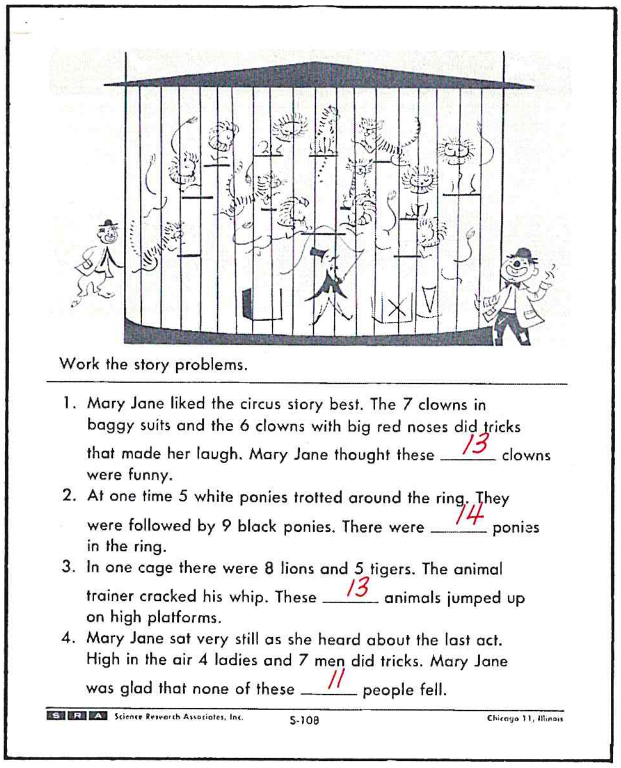
Name

Work the story problems.

STORY TIME

1. At story time Dan liked to hear about Mrs. Rabbit and her 15 children. She had 7 boys in her family. 8 rabbits were girls.
2. One sunny day the rabbits found 17 carrots in the garden. They ate 9 of the carrots. They took the other 8 carrots home to make into carrot soup.
3. Mrs. Rabbit made a cabbage pie. She used 6 little cabbages and 8 big cabbages. She put 14 cabbages in the pie.
4. After supper the 15 rabbit children were sleepy. Mrs. Rabbit tucked 4 of them in blue beds and 3 of them in yellow beds. The other 8 rabbit children slept in white beds.

S-107



Work the story problems.

1. Mary Jane liked the circus story best. The 7 clowns in baggy suits and the 6 clowns with big red noses did tricks that made her laugh. Mary Jane thought these 13 clowns were funny.
2. At one time 5 white ponies trotted around the ring. They were followed by 9 black ponies. There were 14 ponies in the ring.
3. In one cage there were 8 lions and 5 tigers. The animal trainer cracked his whip. These 13 animals jumped up on high platforms.
4. Mary Jane sat very still as she heard about the last act. High in the air 4 ladies and 7 men did tricks. Mary Jane was glad that none of these 11 people fell.

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child works out the problem at the chalkboard, explaining each step. The class can then solve the remaining problems without assistance.

Supplemental Experiences

▶ Ask the children to think of nursery rhymes, and help them make the necessary changes in wording, and substitute numerals in order to change the rhyme into a story problem. Each child may say or read his rhyme and call on others to tell the answer.

Mary, Mary, quite contrary
How does your garden grow?
With ten silver bells and eight cockleshells
How many flowers in a row?

Humpty-Dumpty sat on a wall
Humpty-Dumpty had a great fall
Eight king's horses and seven king's men
Couldn't put Humpty together again.
How many tried to put Humpty
Back on the wall?

▶ Problems written about the students and their pets may interest the less enthusiastic problem-solver.

Place Value

Work Pages S-109 Through S-114

Objectives

To extend the children's understanding of expanded notation.

To stress the idea of different numerals for a number.

To strengthen the understanding of place-value concepts.

Fundamentals

Before the child is introduced to addition problems involving pairs of two-digit addends, the teacher will want to review the meaning of the digit in the tens place. In readiness for the addition of multiples of ten, the child must grasp completely the idea that a numeral such as 60 represents six tens and no ones, or simply six tens.

It is important that the child have a clear understanding of expanded notation so that later, in working with higher-decade additions and subtractions, he will understand their meaning. The teacher will want to help the child recall that in the numeral 86, for instance, the digit 8 stands for eight tens, or eighty, while the digit 6 stands for six ones, or six. The number eighty-six, which this numeral represents, can also be named by the expanded-notation numeral 80 + 6.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Systems of Numeration.

Readiness for Understanding

An understanding of the numbers zero through ninety-nine.

Ability to recognize the standard place-value numerals 0 through 99.

Awareness that different numerals may represent the same number.

Understanding of place-value concept.

Developmental Experiences

MATERIALS

2 Countingmen
2 coffee cans
sticks

▶ Write the numeral 36 on the chalkboard, and have the children tell the meaning of the digit 3 in the tens place and the 6 in the ones place. Write other numerals and have the children review their place-value meaning.

▶ Ask a child to show the number eighty-five on the Countingmen. After he has placed eight fingers on the Tens-man and five fingers on the Ones-man, ask another child to tell how many tens and how many ones are represented in the number eighty-five. Call on another child to write the standard place-value numeral for eighty-five on the chalkboard. After he writes the numeral 85, ask if someone can think of a different way to write a numeral for the number eighty-five. Have a child write the expanded-notation numeral 80 + 5 to the right of the numeral 85.

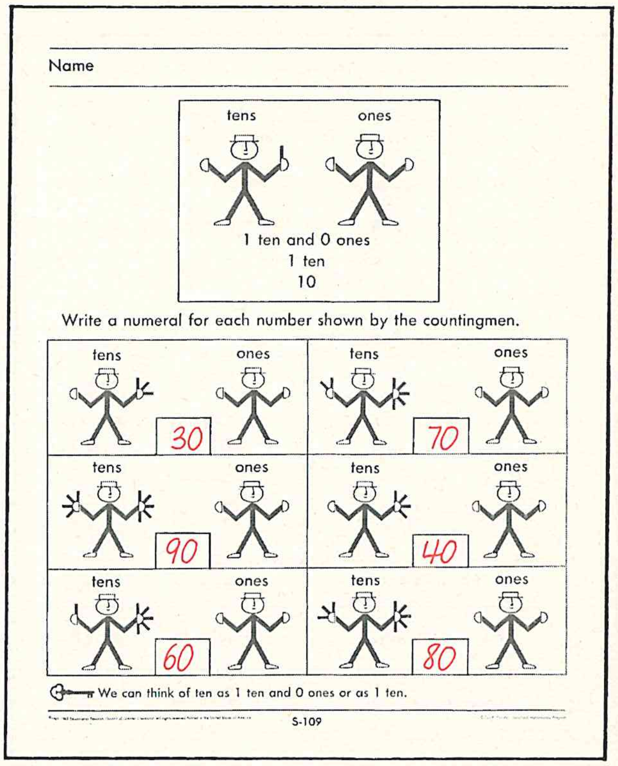
Now ask a child to show the number fifty-eight on the Countingmen. Have another child write the numeral 58 beneath the 85 and explain it in terms of place-value concepts. Have another child come to the chalkboard and write 58 in another way, as 50 + 8, beneath the 80 + 5. Explain to the class once again that this way of writing the numeral is called expanded notation or expanded form, and that it reminds them of what the numeral really means. Discuss the numerals 85 and 58, helping the children to understand that in one case the 5 stands for the number five and in the other it stands for the number fifty. Talk about the meaning of the digit 8 in each of the numerals. Repeat the activity with other pairs of such numerals as 37 and 73, 96 and 69, and 41 and 14.

▶ Label one coffee can as ones and another as tens. Provide wooden tongue depressors, some of which have been bundled into groups of ten. Ask a child to count out fifty-two sticks; help him to see that it will be easier for him to choose five bundles of ten and two individual sticks rather than start with the single sticks and count to fifty-two. Have the child place the bundles in the tens-can and the single sticks in the ones-can. Ask another child to write on the chalkboard the standard place-value numeral (52) which tells how many sticks he sees. Have a second child write the expanded-notation numeral 50 + 2. Repeat the activity, calling for numbers in which only bundles of tens are needed: thirty, seventy, or ninety.

To vary the activity, place eight bundles of ten sticks in the tens-can and nine single sticks in the ones-can. Call on a child to tell how many sticks have been displayed and write two different numerals to represent this number. If he is able to do this correctly, he may place a different number of sticks in the cans and call on a second child to tell the number and write the numerals.

▶ Use work page S-109 to provide a quick review of the meaning of numerals representing multiples of ten. Discuss the pictures at the top of the page, pointing out that the number ten is described here in four ways: by the Countingmen, by the phrase "one ten and zero ones," by the shortened form "one ten," and by the place-value numeral 10. Ask the children to determine the number represented by the Countingmen in each of the remaining boxes and then write the corresponding place-value numeral.

The first column on work page S-110 provides an opportunity for the children to compare the meaning of a digit which appears in the ones place and the same digit as it appears in the tens place. Have the children fill the blanks to show how many tens are represented by each numeral. In the second column have them write the standard place-value numeral for each of the numbers described.


Name

tens ones

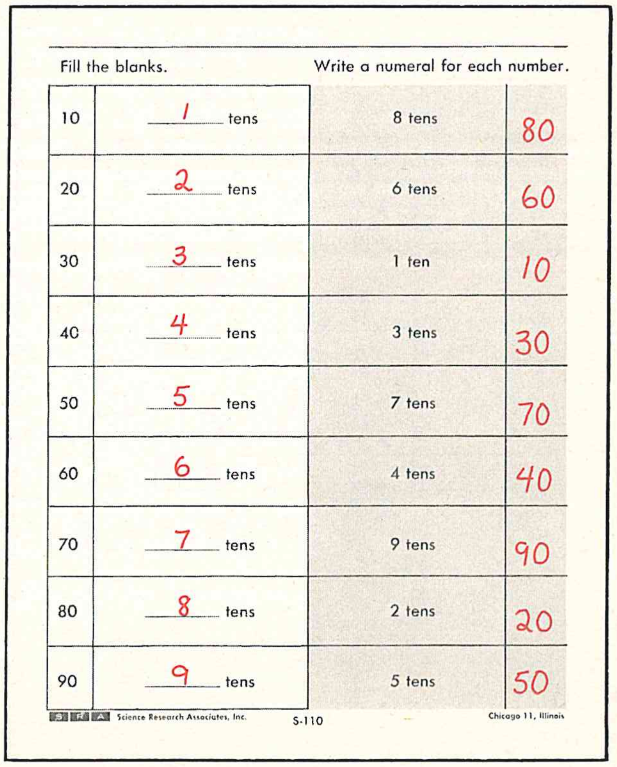
1 ten and 0 ones
1 ten
10

Write a numeral for each number shown by the countingmen.

tens	ones	tens	ones
30		70	
90		40	
60		80	

We can think of ten as 1 ten and 0 ones or as 1 ten.

S-109



Fill the blanks.		Write a numeral for each number.	
10	1 tens	8 tens	80
20	2 tens	6 tens	60
30	3 tens	1 ten	10
40	4 tens	3 tens	30
50	5 tens	7 tens	70
60	6 tens	4 tens	40
70	7 tens	9 tens	90
80	8 tens	2 tens	20
90	9 tens	5 tens	50

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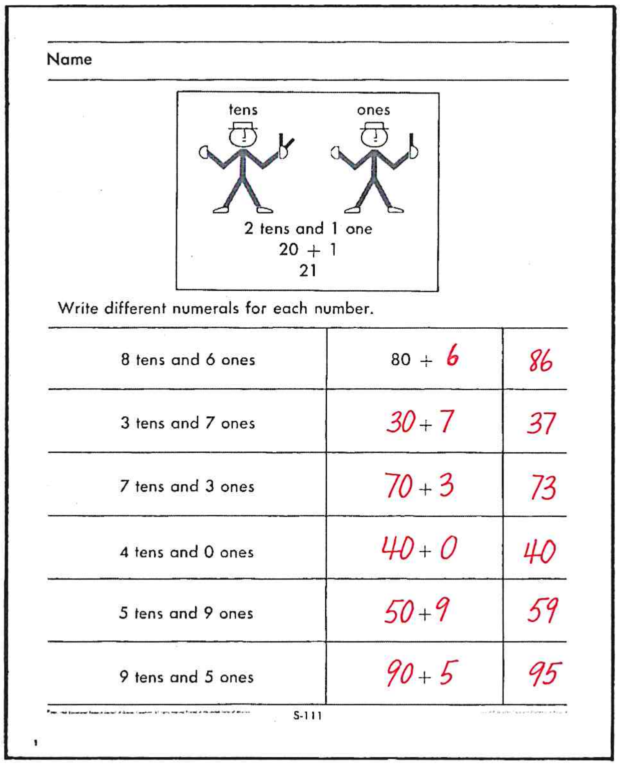
▶ Work page S-111 will give the children an opportunity to evaluate their understanding of different numerals for the same number. Discuss the number twenty-one as it is represented in four ways in the first box. Then direct the children to write different numerals for each number described. In each row have them read the phrase which describes the number, then write the expanded-form and the standard place-value numerals for the number.

Use work page S-112 to evaluate the children's understanding of expanded notation. Ask the children to look at the numeral in each box and then draw the correct number of fingers on the Countingmen.



Name

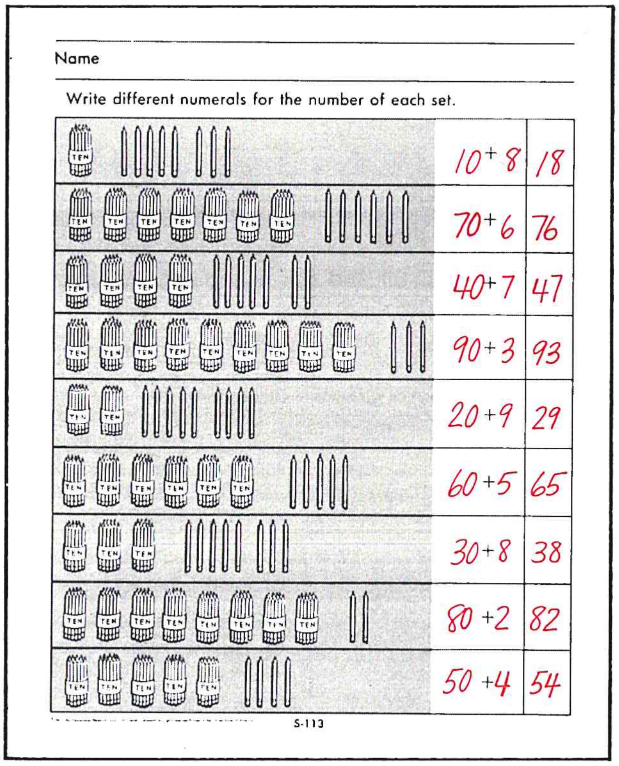
tens ones

2 tens and 1 one
20 + 1
21

Write different numerals for each number.

8 tens and 6 ones	80 + 6	86
3 tens and 7 ones	30 + 7	37
7 tens and 3 ones	70 + 3	73
4 tens and 0 ones	40 + 0	40
5 tens and 9 ones	50 + 9	59
9 tens and 5 ones	90 + 5	95

S-111

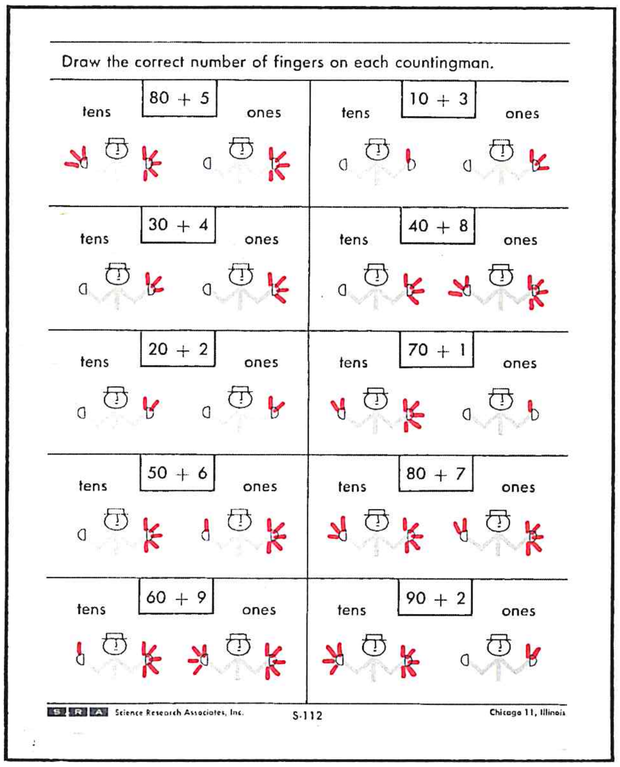


Name

Write different numerals for the number of each set.

	10 + 8	18
	70 + 6	76
	40 + 7	47
	90 + 3	93
	20 + 9	29
	60 + 5	65
	30 + 8	38
	80 + 2	82
	50 + 4	54

S-113



Draw the correct number of fingers on each countingman.

tens 80 + 5 ones	tens 10 + 3 ones
tens 30 + 4 ones	tens 40 + 8 ones
tens 20 + 2 ones	tens 70 + 1 ones
tens 50 + 6 ones	tens 80 + 7 ones
tens 60 + 9 ones	tens 90 + 2 ones

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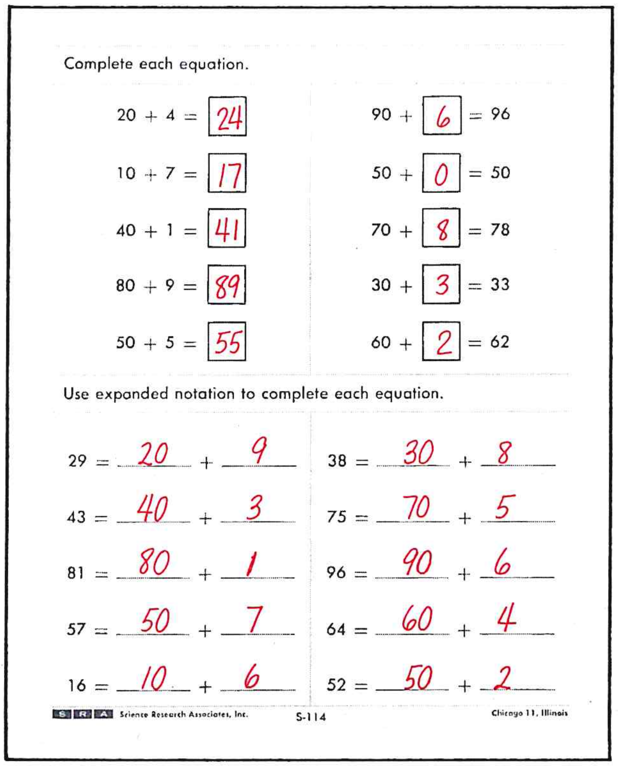

Complete each equation.

20 + 4 = $\boxed{24}$	90 + $\boxed{6}$ = 96
10 + 7 = $\boxed{17}$	50 + $\boxed{0}$ = 50
40 + 1 = $\boxed{41}$	70 + $\boxed{8}$ = 78
80 + 9 = $\boxed{89}$	30 + $\boxed{3}$ = 33
50 + 5 = $\boxed{55}$	60 + $\boxed{2}$ = 62

Use expanded notation to complete each equation.

29 = 20 + 9	38 = 30 + 8
43 = 40 + 3	75 = 70 + 5
81 = 80 + 1	96 = 90 + 6
57 = 50 + 7	64 = 60 + 4
16 = 10 + 6	52 = 50 + 2

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▶ Direct the children to determine the number of the set of sticks in each row on work page S-113, and to write both an expanded-notation numeral and a standard place-value numeral for the number of the set.

On work page S-114 have the children write the standard place-value numeral to complete each equation in the first box on the page. In the second box they must read the place-value numeral on the right of each equal sign and complete the expanded form for the number on the left. The children will use expanded notation to complete each equation on the lower portion of the page. Work through the first problem in each section with them, and then encourage independent work on the other problems.

Supplemental Experiences

▶ Continue to develop an understanding of the order of the numbers from zero through ninety-nine. Help the children see that in comparing numbers which are represented by two-digit numerals, the digit in the ones place does not always give an indication of which number is greater. In comparing the numbers named by 68 and 86, for instance, they must refer to the digit in the tens place before they can see which number is greater. When two numbers to be compared are both represented by two-digit numerals which have the same digit in the tens place, as in 43 and 47, it becomes important to examine the digit in the ones place in order to make a decision about which number is greater.

▶ Write a series of incomplete equations and inequalities on the chalkboard. Ask the children to write the correct sign (>, <, or =) in each circle to make a true statement. Encourage them to think about the place-value meaning of the numerals on either side of the circle in order to make a quick judgment about the comparative value of the numbers represented.

30+6 ○ 6+3	43 ○ 47
72 ○ 20+7	74 ○ 73
90+3 ○ 3+90	80+5 ○ 80+8
49 ○ 94	50+8 ○ 8+5

▶ Construct two sets of numeral cards for use in helping to strengthen the child's understanding of place-value concepts. On nine 4″ x 8″ cards write a numeral, 10 through 90; and on ten 4″ x 4″ cards write a single digit 0 through 9. Distribute the cards to the children, saying that you would like two children to show you a numeral for thirty-seven. Ask the children to think of the place-value meaning of the numeral 37 and to come forward if their cards can be used to build this numeral. The child with the 30 and the child with the 7 should come forward and hold their cards up on either side of a plus sign on the chalkboard. Now have the 7 card held over the 0 of the 30 card to show the numeral 37. Explain to the children that 37 is just a short way of writing the numeral 30 + 7; once again, display the numerals in expanded form.

$$\boxed{30} + \boxed{7}$$

$$\boxed{3}\boxed{7}$$

▶ As a warm-up activity for the arithmetic period, or as a fun game for the end of the day, play an elimination game. Divide the class into two teams, have them line up facing the chalkboard, and provide the first member of each team with a piece of chalk. Give the direction, "Write the place-value numeral that means nine tens and three ones." The leaders must hurry to the chalkboard and write the numeral 93. The first child to complete this task correctly passes the chalk to the second child on his team and returns to the end of the line; the other child is eliminated from the game.

Vary the directions:

"Write the numeral 27 in expanded form."

"Write a numeral for the number which is one more than thirty-two."

"Write numerals to show the order of numbers from seventeen to twenty-one."

"Write a numeral to show the number which is one less than seventy-six."

"Write a numeral for the number that comes between fifty-four and fifty-six."

The team which has the most children standing at the end of a given period of time wins the game.

Two-Digit Addition

NO CARRYING

Comparison

Work Pages S-115 Through S-120

Objectives

To extend the development of two-digit addition without carrying.

To introduce vertical notation for work with expanded notation.

To compare numbers in the environment of two-digit addition.

Fundamentals

As known ideas are applied in solving problems like 23 + 2 = □, the child should be able to think of the place-value description of 23. He should think of 20 + 3 as a different numeral for 23, and the problem now becomes (20 + 3) + 2 = □.

In order to separate ones and tens, the next step involves the use of the associative property where (20 + 3) + 2 becomes 20 + (3 + 2). The three and two are added and 20 + (3 + 2) becomes 20 + 5, then 25.

On the work pages the child will not write all of the steps in problems of this kind. He will only be asked to write that part of the problem which reveals his understanding of the concepts involved—the part requiring him to rewrite the number of ones and tens: 23 + 2 = 20 + 5.

Expanded notation used in the vertical form of a problem will be introduced. There will be several advantages to using expanded notation in this form. The child can see clearly his two addends broken down into ones and tens in such problems as 23 + 2 = □. Expanded notation helps the child to understand the addition algorism. The place-value meaning of the first addend is emphasized and quicker computation is possible. The child will have the opportunity to work in a simple setting with a form which he will use quite often to give meaning to the more complex concepts of addition and subtraction involving carrying and borrowing.

$$\begin{array}{r} 20+3 \\ +\quad 2 \\ \hline 20+5 \end{array}$$

The child will use his ever-increasing skills and abilities with numbers, as he compares numbers in the setting of addition. He should become aware that a problem such as 33 + 4 ○ 30 + 6 can be solved without actually adding. After he determines that there are the same number of tens on each side, he merely compares the number of ones.

See Key Topics in Mathematics for the Primary Teacher: Order—Relations—Operations; Addition of Whole Numbers.

Readiness for Understanding

An understanding of the concept of place value.

An understanding of the use of expanded notation.

An understanding of the associative property.

Mastery of number combinations of zero through nine.

Ability to work with placeholder equations and vertical notation.

Ability to compare numbers.

Developmental Experiences

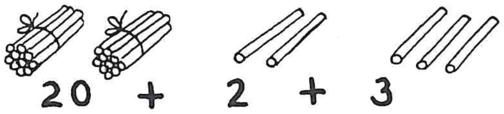
MATERIALS

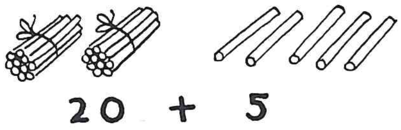
- sticks
- flannel board
- felt numerals
- operation and relation symbols
- 2 Countingmen
- pocket chart
- strips of colored paper
- rubber bands

▶ Review the addition of a one-digit addend to a two-digit addend. Place the equation 12 + 3 = □ on the flannel board. Have children set up on their desks the bundles of ten sticks and single sticks that are needed to picture the sets for this equation. Under the numeral 12 + 3 on the flannel board, build the expanded-notation form 10 + 2 + 3. Then ask the children to take the set of two from the set of ten and two and place it with the set of three. Have a child rewrite 10 + 2 + 3 so that it corresponds with the picture of the sets now on his desk and place this numeral (10 + 5) under the other two on the flannel board.

Guide the children in summarizing what happens when three is added to twelve. First, the ones are separated from the set of ten, then all of the ones are joined. As a last step, all of the ones are put back with the set of ten. Have the children complete and read the placeholder equation.

Place the equation 22 + 3 = □ on the flannel board. Then ask the children to tell how many bundles of ten and how many single sticks are needed to represent the first addend in this equation. Have children display the necessary bundles of ten sticks and single sticks on their desks to picture the sets for this equation. Under the numeral 22 + 3 on the flannel board have a child build the expanded-notation form 20 + 2 + 3. Ask the children to take a set of two from the set of twenty and two and place it with the set of three. Then have a child rewrite 20 + 2 + 3 so that it corresponds with the picture of the sets now on his desk. Have him place the numeral 20 + 5 with the other two on the flannel board.

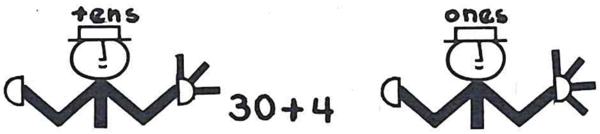




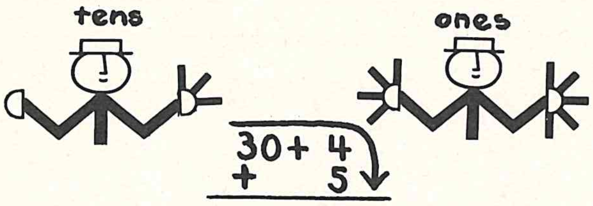
Help the children to summarize what happens when three is added to twenty-two. First, the ones are separated from the two sets of ten; then the ones are joined; and finally, all of the ones are put back with the two sets of ten to get the answer. Have the children complete and read the placeholder equation.

The children can use their sticks to work other problems of this type. Have them complete and read the related placeholder equation each time.

▶ Place the problem 34 + 5 = □ on the flannel board and display two Countingmen. Call on a child to operate the Tens-man and another one to operate the Ones-man. Have the children decide what number is represented first in the equation and show it on the Countingmen. Ask the class what the



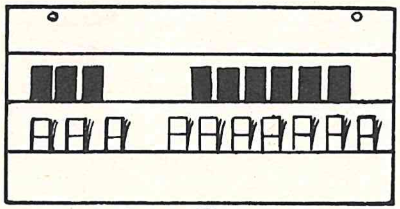
three fingers on the Tens-man mean; they will reply that the fingers stand for the three tens in 34. Then ask the class to tell what the four fingers on the Ones-man mean. They stand for the four ones in 34. Tell the class that you are going to write the numeral 34 in expanded-notation form, 30 + 4, on the chalkboard and that the operators must decide what to do next. The ones-operator must join a set of five ones to the set of four ones now on the Countingman. Write the numeral 5 under the 4 in the problem on the chalkboard. After the class tells how many ones are now on the Countingman, have them explain the final step in solving the problem. The children must think in terms of putting ones together with tens to get the answer. They will explain that there are three tens and nine ones, or 39, on the Countingmen. Now complete the problem on the chalkboard.



Tell the children that this is another way of writing problems involving ones and tens, and that they will be using it often from now on in their work. Help them to understand that they will find it easier to add ones in this form because the ones are lined up under each other. Then all they have to do is add ones to tens to get their answer.

Have the children think through other problems of this kind, using the Countingmen. Ask them to write the problems in vertical form, using expanded notation, and then read and discuss each one.

▶ On the chalkboard, write the numerals for two numbers to be compared: 34 + 2 ○ 30 + 7. Ask a child to place three bundles of ten paper strips along with four and two individual paper strips in the pocket chart. Have another child place directly under the first set of strips, the number of strips indicated by the numeral on the right. Ask the class to match the sets of strips so they can see which set represents the greater number. Help the children see that since the number of tens in each numeral is the same, a comparison of the ones will reveal which of the two numerals represents the greater number.

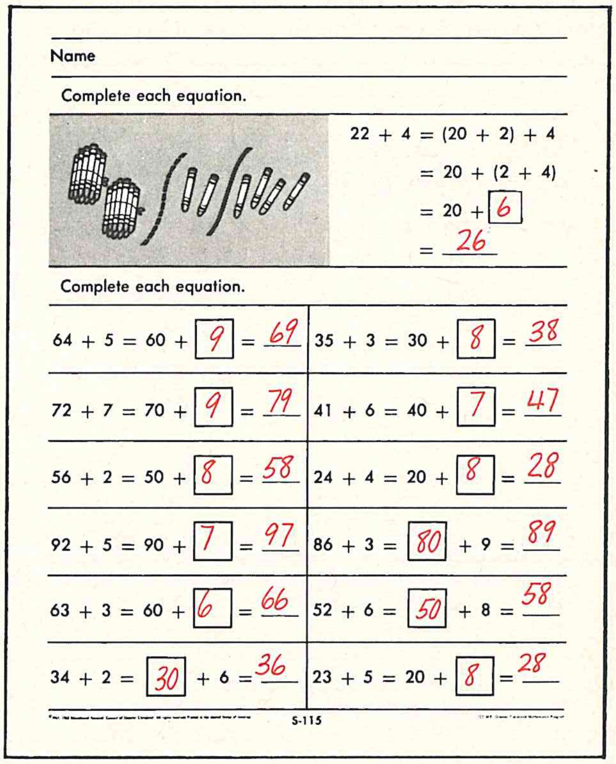


34+2 ○ 30+7

Set up other problems and have children determine quickly which number in the pair is greater. Ask them to explain their thinking in making their decision.

After several numbers have been compared with the aid of sets, have the children compare numbers without this visual aid. Help them to understand that when the number of tens is the same, they need concentrate only on the number of ones.

▶ Use work page S-115 to reinforce the children's



Name

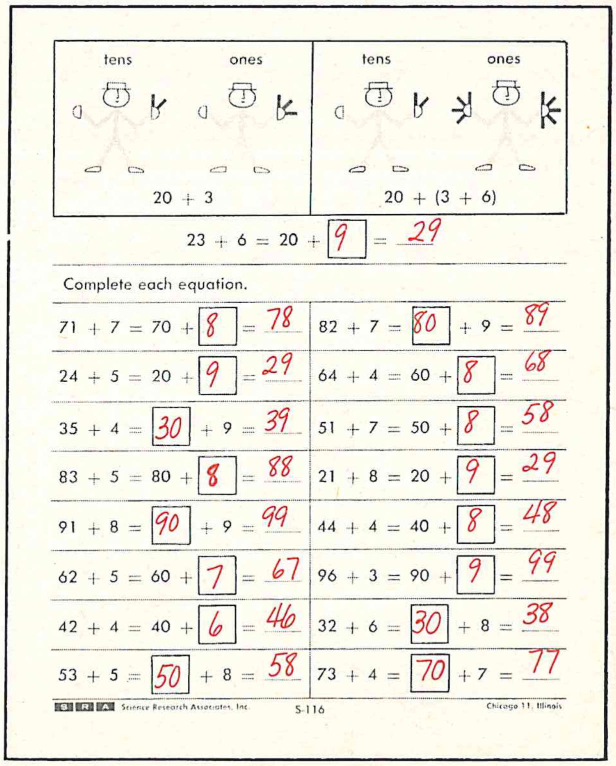
Complete each equation.

$22 + 4 = (20 + 2) + 4$
$= 20 + (2 + 4)$
$= 20 + \boxed{6}$
$= \underline{26}$

Complete each equation.

$64 + 5 = 60 + \boxed{9} = \underline{69}$	$35 + 3 = 30 + \boxed{8} = \underline{38}$
$72 + 7 = 70 + \boxed{9} = \underline{79}$	$41 + 6 = 40 + \boxed{7} = \underline{47}$
$56 + 2 = 50 + \boxed{8} = \underline{58}$	$24 + 4 = 20 + \boxed{8} = \underline{28}$
$92 + 5 = 90 + \boxed{7} = \underline{97}$	$86 + 3 = \boxed{80} + 9 = \underline{89}$
$63 + 3 = 60 + \boxed{6} = \underline{66}$	$52 + 6 = \boxed{50} + 8 = \underline{58}$
$34 + 2 = \boxed{30} + 6 = \underline{36}$	$23 + 5 = 20 + \boxed{8} = \underline{28}$

S-115


tens
ones
tens
ones

$20 + 3$

$20 + (3 + 6)$

$23 + 6 = 20 + \boxed{9} = \underline{29}$

Complete each equation.

$71 + 7 = 70 + \boxed{8} = \underline{78}$	$82 + 7 = \boxed{80} + 9 = \underline{89}$
$24 + 5 = 20 + \boxed{9} = \underline{29}$	$64 + 4 = 60 + \boxed{8} = \underline{68}$
$35 + 4 = \boxed{30} + 9 = \underline{39}$	$51 + 7 = 50 + \boxed{8} = \underline{58}$
$83 + 5 = 80 + \boxed{8} = \underline{88}$	$21 + 8 = 20 + \boxed{9} = \underline{29}$
$91 + 8 = \boxed{90} + 9 = \underline{99}$	$44 + 4 = 40 + \boxed{8} = \underline{48}$
$62 + 5 = 60 + \boxed{7} = \underline{67}$	$96 + 3 = 90 + \boxed{9} = \underline{99}$
$42 + 4 = 40 + \boxed{6} = \underline{46}$	$32 + 6 = \boxed{30} + 8 = \underline{38}$
$53 + 5 = \boxed{50} + 8 = \underline{58}$	$73 + 4 = \boxed{70} + 7 = \underline{77}$

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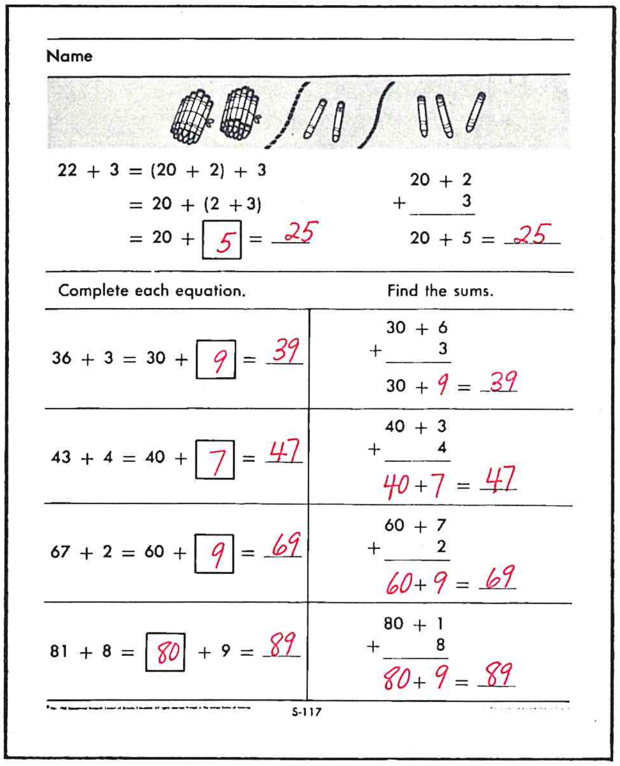
understanding of two-digit addition. Work through the steps in the first problem at the top of the page with the children. Then ask them to observe the second problem on the page, and think about how many ones and how many tens are represented in the equation. Have them fill in the placeholder and then write the sum on the line at the end of the equation. The children should be able to complete the remainder of the page on their own. Give help only if needed. Follow this procedure for work page S-116

▶ Use the problems on work page S-117 to give children practice in making the transition from the use of the equation form in working two-digit addition problems, to the use of expanded notation in vertical form. Work the first problem at the top of



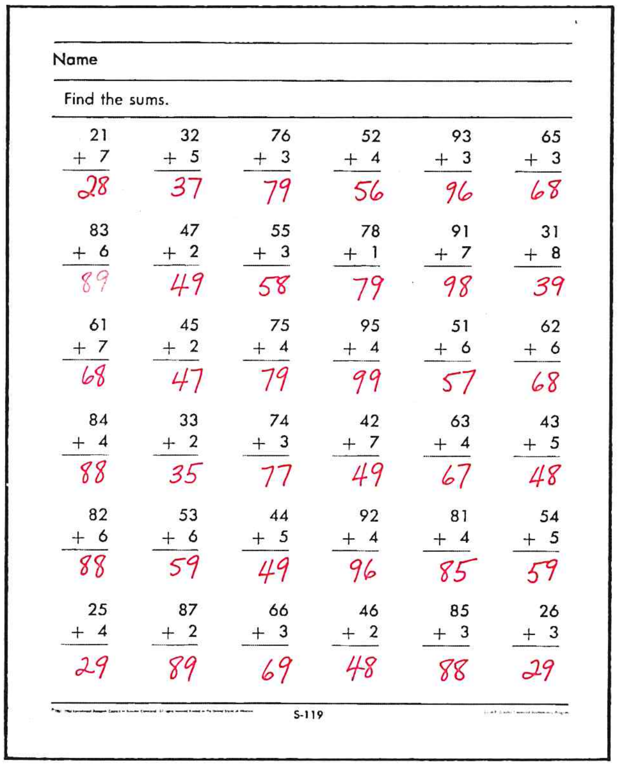
Name

22 + 3 = (20 + 2) + 3
= 20 + (2 + 3)
= 20 + $\boxed{5}$ = 25

20 + 2
+ 3
20 + 5 = 25

Complete each equation.	Find the sums.
36 + 3 = 30 + $\boxed{9}$ = 39	30 + 6 + 3 30 + 9 = 39
43 + 4 = 40 + $\boxed{7}$ = 47	40 + 3 + 4 40 + 7 = 47
67 + 2 = 60 + $\boxed{9}$ = 69	60 + 7 + 2 60 + 9 = 69
81 + 8 = $\boxed{80}$ + 9 = 89	80 + 1 + 8 80 + 9 = 89

S-117

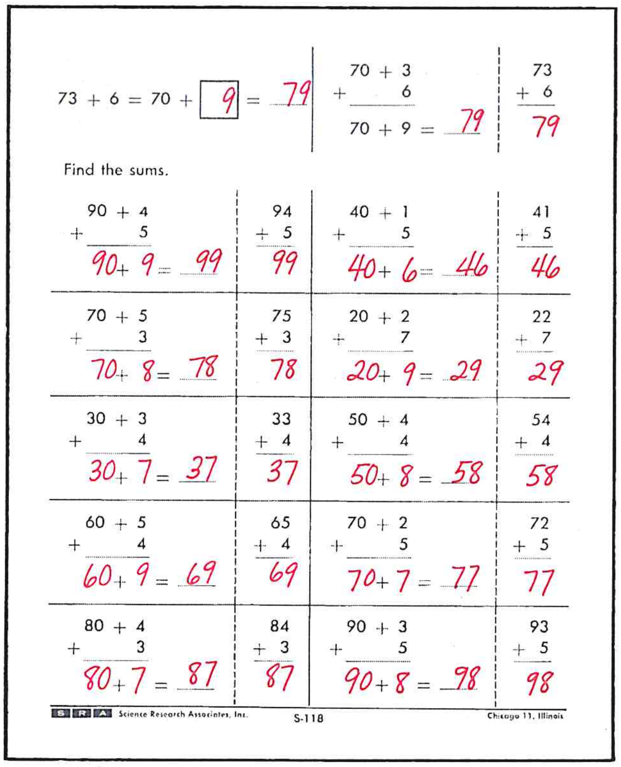


Name

Find the sums.

21 + 7 28	32 + 5 37	76 + 3 79	52 + 4 56	93 + 3 96	65 + 3 68
83 + 6 89	47 + 2 49	55 + 3 58	78 + 1 79	91 + 7 98	31 + 8 39
61 + 7 68	45 + 2 47	75 + 4 79	95 + 4 99	51 + 6 57	62 + 6 68
84 + 4 88	33 + 2 35	74 + 3 77	42 + 7 49	63 + 4 67	43 + 5 48
82 + 6 88	53 + 6 59	44 + 5 49	92 + 4 96	81 + 4 85	54 + 5 59
25 + 4 29	87 + 2 89	66 + 3 69	46 + 2 48	85 + 3 88	26 + 3 29

S-119



73 + 6 = 70 + $\boxed{9}$ = 79	70 + 3 + 6 70 + 9 = 79	73 + 6 79

Find the sums.

90 + 4 + 5 90 + 9 = 99	94 + 5 99	40 + 1 + 5 40 + 6 = 46	41 + 5 46
70 + 5 + 3 70 + 8 = 78	75 + 3 78	20 + 2 + 7 20 + 9 = 29	22 + 7 29
30 + 3 + 4 30 + 7 = 37	33 + 4 37	50 + 4 + 4 50 + 8 = 58	54 + 4 58
60 + 5 + 4 60 + 9 = 69	65 + 4 69	70 + 2 + 5 70 + 7 = 77	72 + 5 77
80 + 4 + 3 80 + 7 = 87	84 + 3 87	90 + 3 + 5 90 + 8 = 98	93 + 5 98

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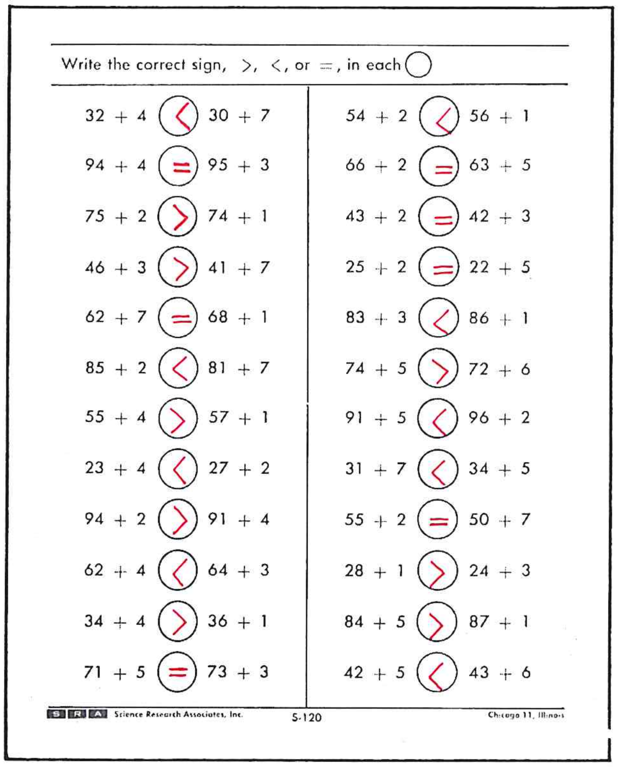

Write the correct sign, >, <, or =, in each ○

32 + 4 (<) 30 + 7
94 + 4 (=) 95 + 3
75 + 2 (>) 74 + 1
46 + 3 (>) 41 + 7
62 + 7 (=) 68 + 1
85 + 2 (<) 81 + 7
55 + 4 (>) 57 + 1
23 + 4 (<) 27 + 2
94 + 2 (>) 91 + 4
62 + 4 (<) 64 + 3
34 + 4 (>) 36 + 1
71 + 5 (=) 73 + 3

54 + 2 (<) 56 + 1
66 + 2 (=) 63 + 5
43 + 2 (=) 42 + 3
25 + 2 (=) 22 + 5
83 + 3 (<) 86 + 1
74 + 5 (>) 72 + 6
91 + 5 (<) 96 + 2
31 + 7 (<) 34 + 5
55 + 2 (=) 50 + 7
28 + 1 (>) 24 + 3
84 + 5 (>) 87 + 1
42 + 5 (<) 43 + 6

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the page with the children, emphasizing that it is quicker and easier to visualize ones and tens if they use expanded notation in vertical form when solving problems which have a two-digit numeral. Stress that the important idea to keep in mind is the number of ones and tens, and the correct placement of numerals representing ones and tens. After the first example has been completed, the children can complete the equations and find the sums for the rest of the problems on the page.

Use work page S-118 to help children make the transition from the expanded-notation form to the algorism for two-digit addition problems. Work the problem at the top of the page with the class, reminding them that the algorism is the quickest and easiest form for working problems that have two-digit numerals. Tell the children, however, that they do not have the advantage of seeing ones and tens written out. They must now think of what is behind each digit in the numeral; they must think about the number of ones or tens which each digit represents. Have them work the remainder of the page on their own, writing all the numerals needed to complete each problem and find a sum.

▶ Use work page S-119 to give the class practice in working two-digit addition problems in vertical-notation form. This form is presented only after a careful foundation has been laid for the meaning behind this algorism or computational rule. Have the children find the sums, using care in the correct placement of numerals representing the ones and tens. Work the first problem with the children, then have them complete the remaining problems on their own.

Work page S-120 offers children an opportunity to use their ability to reason. Work the first problem with the group, then have them do the remainder of the page by working independently. When the work on this page is completed, discuss each problem, asking the children to explain how they thought out their answers.

Supplemental Experiences

▶ Place a series of numerals on the chalkboard. Have the children take turns in writing the numerals in different ways.

$32 + 4 = 30 + 2 + 4$
$45 + 2 = 40 + 5 + 2$
$60 + 3 + 4 = 60 + 7$
$50 + 1 + 8 = 50 + 9$

Write related placeholder equations on the chalkboard.

$3 + \square = 8$ $\square + 2 = 6$
$13 + \square = 18$ $\square + 12 = 16$
$43 + \square = 48$ $\square + 42 = 46$

Have the children complete each equation. Help them to become aware of the relation among the three equations. Help them to realize that knowing how to work the equation involving only ones, will help them work equations involving tens and ones. Remind them to do the two-digit numeral problems by first adding the ones and then putting this sum with the tens to get the answer.

Two-Digit Subtraction—Addition

NO BORROWING

NO CARRYING

Work Pages S-121 Through S-126

Objectives

To extend the concept of two-digit subtraction without borrowing.

To re-emphasize the inverse relation between addition and subtraction.

To provide practice in the use of expanded notation in vertical form.

To provide practice in working two-digit subtraction problems in vertical-notation form.

Fundamentals

From addition problems involving a two-digit addend and a one-digit addend, the child will progress to problems in the subtraction of a number represented by a one-digit numeral from a number represented by a two-digit numeral. The child discovered that in such problems as $18 - 7 = \square$, he could first subtract the ones and then add the result to ten. The teacher will want to guide the child in his first attempt at finding out whether this will also work in problems such as $28 - 7 = \square$, $38 - 7 = \square$, or $48 - 7 = \square$. She will help him to first subtract seven from eight and then add this result to the tens to arrive at the answer. She will also help him check his answer by addition.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concept of place value.

An understanding of the concept of expanded notation.

Awareness of the inverse concept.

Mastery of number combinations with sums through nine.

Ability to work with placeholder equations and vertical-notation form.

Developmental Experiences

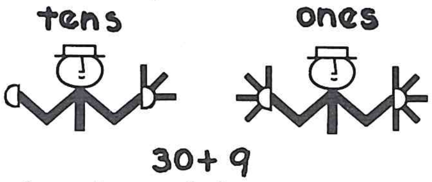
MATERIALS

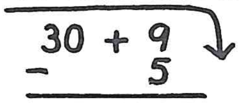
counting sticks
flannel board
felt numerals
operation and relation symbols
2 Countingmen

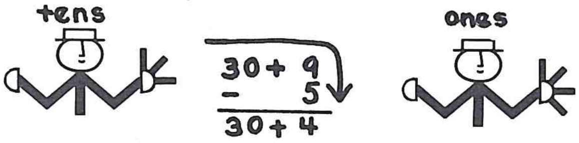
▶ Place the equation $15 - 3 = \square$ on the flannel board. Have children set up on their desks the bundles of ten sticks and single sticks that are needed to picture the sets for this equation. Ask how many sticks must be taken away from the one ten and five ones; have the sticks removed, and ask how many ones and how many tens remain. Ask the children to complete and read the placeholder equation.

Place the equation 25 − 3 = ☐ on the flannel board and follow the same procedure. Repeat the activity with similar equations.

▶ Place the problem 39 − 5 = ☐ on the flannel board, and display two Countingmen. Call on one child to operate the Tens-man and another to operate the Ones-man. Have the children decide what number is represented first in the equation and show this number on the Countingmen. Then write the expanded form for thirty-nine on the chalkboard, and tell the operators that they must decide what to do next. The ones-operator should remove a set of five ones from the set of nine ones. Write −5 under the 30 + 9 on the board. After the class tells how many ones are now represented on the Countingmen, have them explain the final step of the problem. The children must think in terms of putting ones together with tens to get the answer of three tens and four ones, or thirty-four.





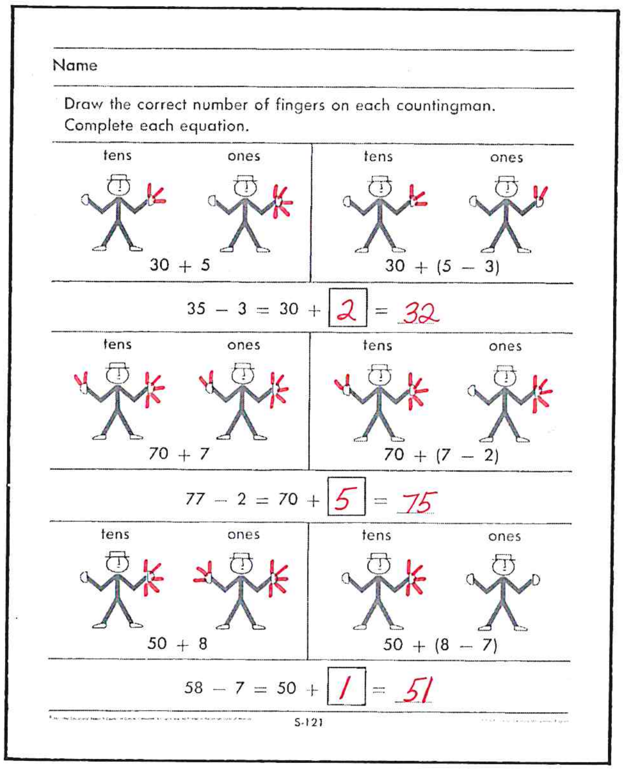


Complete the problem on the chalkboard and help the children summarize the steps involved in solving the problem.

Give the children other problems of this type to work on the Countingmen, and have them write the problems in vertical form using expanded notation. It is important for them to read and discuss each problem.

▶ Use work pages S-121 and S-122 to reinforce the children's understanding of two-digit subtraction. Work through the steps in the first problem on work page S-121 with the children. Remind them that they must think of how many ones and how many tens are represented by the first numeral in the equation. Have the children draw the correct number of fingers on the first set of Countingmen. Have them think of subtracting ones first. Have them draw the correct number of fingers on the second set of Countingmen to indicate the removal of three fingers from five fingers. Stress that the parentheses will help them to think of the numbers that are to be subtracted first. Then have them put the ones back with the tens, fill in the placeholder in the equation, and write the difference on the line at the end of the equation. The children should be able to complete the remainder of work page S-121 and all of work page S-122 on their own. Give help only if needed.

▶ Use work page S-123 to help the children make the transition from the equation form for working two-digit subtraction problems to the use of expanded notation in vertical form. Work the first problem at the top of the page with the children,



Name

Draw the correct number of fingers on each countingman.
Complete each equation.

tens
ones
tens
ones

30 + 5 | 30 + (5 − 3)

35 − 3 = 30 + $\boxed{2}$ = 32

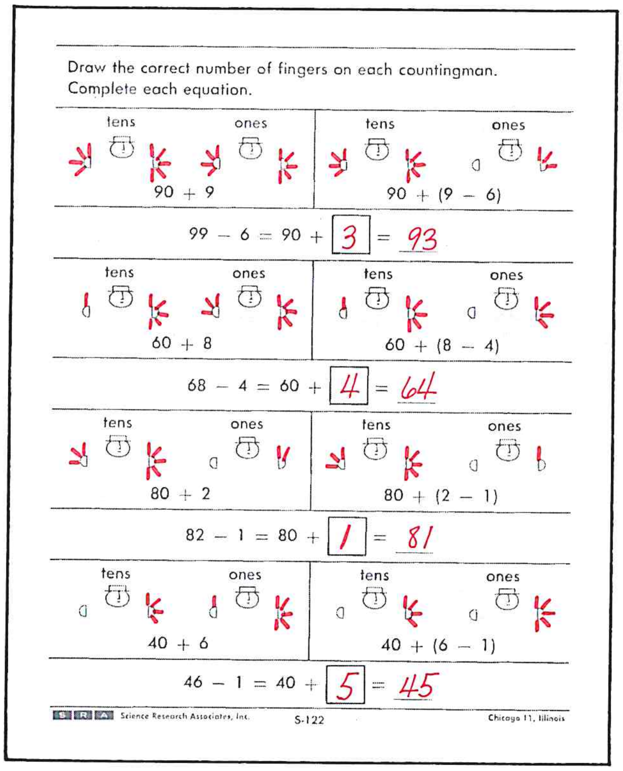
70 + 7 | 70 + (7 − 2)

77 − 2 = 70 + $\boxed{5}$ = 75

50 + 8 | 50 + (8 − 7)

58 − 7 = 50 + $\boxed{1}$ = 51

S-121



Draw the correct number of fingers on each countingman.
Complete each equation.

tens
ones
tens
ones

90 + 9 | 90 + (9 − 6)

99 − 6 = 90 + $\boxed{3}$ = 93

60 + 8 | 60 + (8 − 4)

68 − 4 = 60 + $\boxed{4}$ = 64

80 + 2 | 80 + (2 − 1)

82 − 1 = 80 + $\boxed{1}$ = 81

40 + 6 | 40 + (6 − 1)

46 − 1 = 40 + $\boxed{5}$ = 45

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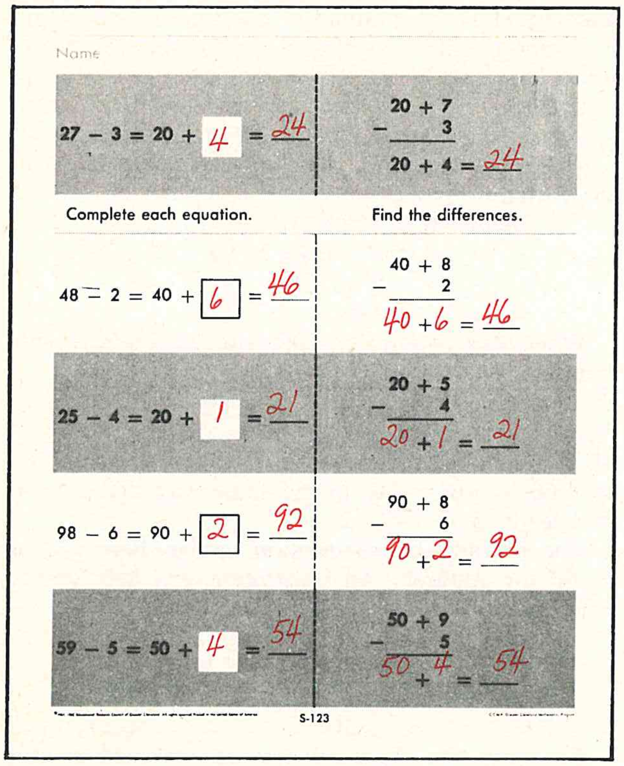
stressing that they will find it quicker and easier to visualize ones and tens when they use expanded notation in vertical form in problems having a two-digit numeral. Explain that the important idea to keep in mind is the number of ones and tens, and the correct placement of numerals representing ones and tens. After the example has been completed, have the children complete the equation and find the differences for the rest of the problems on the page.

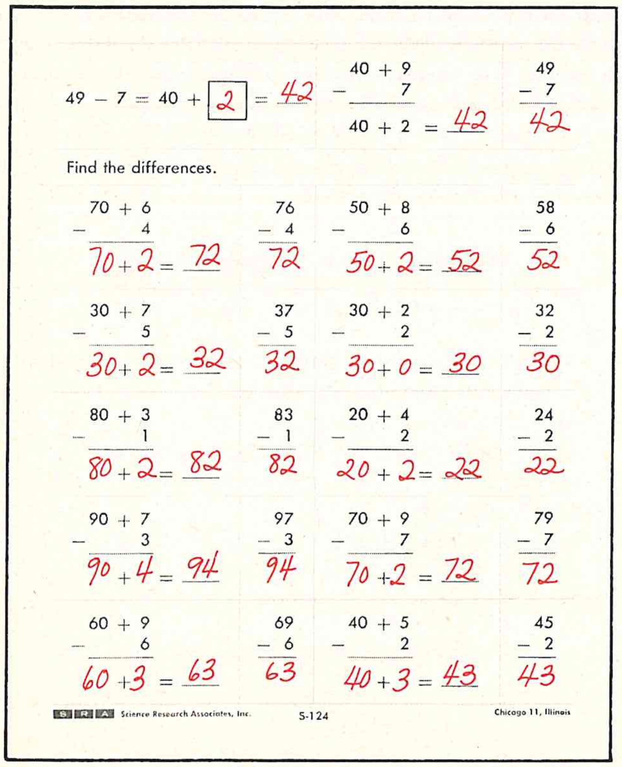
Use work page S-124 to help children make the transition from expanded-notation form to the algorism for two-digit subtraction problems. Work the first problem at the top of the page with the class. Remind them that the algorism is the quickest and easiest form for working problems that have two-



Name

27 − 3 = 20 + $\boxed{4}$ = 24	20 + 7 − 3 20 + 4 = 24
Complete each equation.	Find the differences.
48 − 2 = 40 + $\boxed{6}$ = 46	40 + 8 − 2 40 + 6 = 46
25 − 4 = 20 + $\boxed{1}$ = 21	20 + 5 − 4 20 + 1 = 21
98 − 6 = 90 + $\boxed{2}$ = 92	90 + 8 − 6 90 + 2 = 92
59 − 5 = 50 + $\boxed{4}$ = 54	50 + 9 − 5 50 + 4 = 54

S-123



49 − 7 = 40 + $\boxed{2}$ = 42		40 + 9 − 7 40 + 2 = 42	49 − 7 42

Find the differences.

70 + 6 − 4 70 + 2 = 72	76 − 4 72	50 + 8 − 6 50 + 2 = 52	58 − 6 52
30 + 7 − 5 30 + 2 = 32	37 − 5 32	30 + 2 − 2 30 + 0 = 30	32 − 2 30
80 + 3 − 1 80 + 2 = 82	83 − 1 82	20 + 4 − 2 20 + 2 = 22	24 − 2 22
90 + 7 − 3 90 + 4 = 94	97 − 3 94	70 + 9 − 7 70 + 2 = 72	79 − 7 72
60 + 9 − 6 60 + 3 = 63	69 − 6 63	40 + 5 − 2 40 + 3 = 43	45 − 2 43

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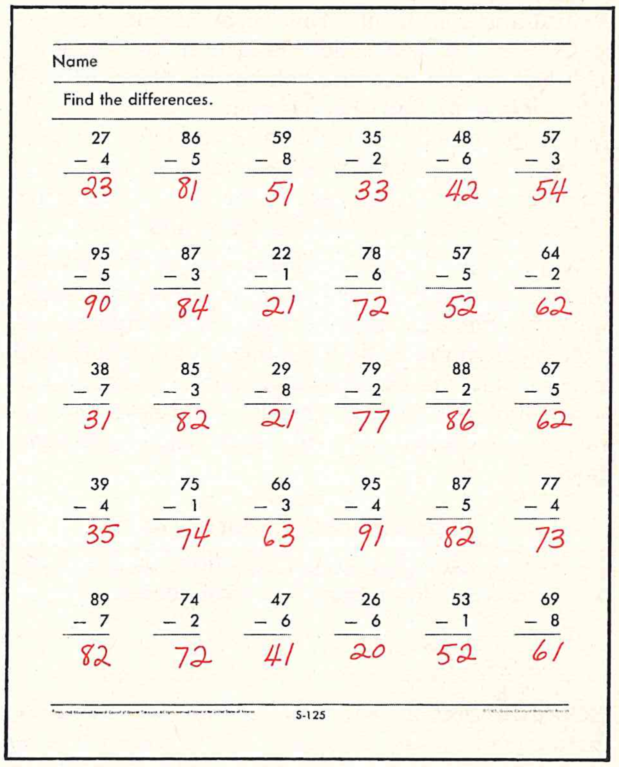
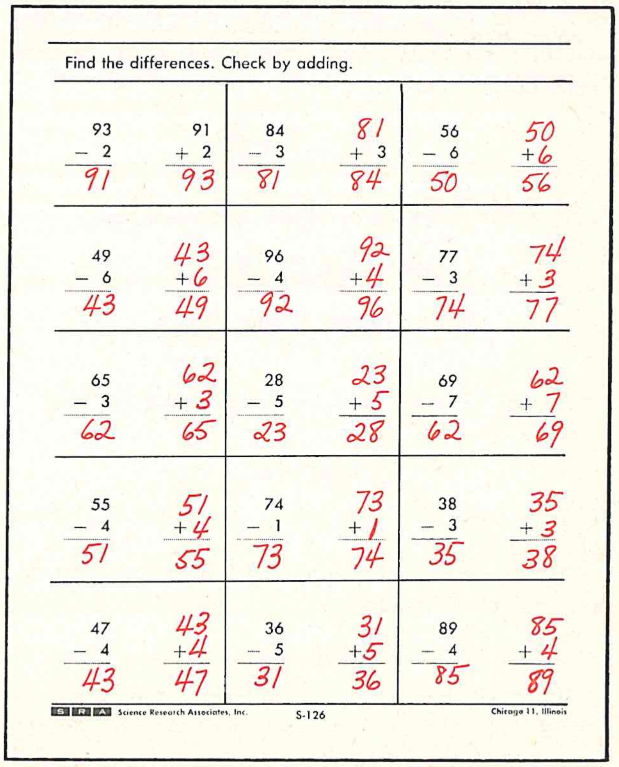


Name

Find the differences.

27 − 4 23	86 − 5 81	59 − 8 51	35 − 2 33	48 − 6 42	57 − 3 54
95 − 5 90	87 − 3 84	22 − 1 21	78 − 6 72	57 − 5 52	64 − 2 62
38 − 7 31	85 − 3 82	29 − 8 21	79 − 2 77	88 − 2 86	67 − 5 62
39 − 4 35	75 − 1 74	66 − 3 63	95 − 4 91	87 − 5 82	77 − 4 73
89 − 7 82	74 − 2 72	47 − 6 41	26 − 6 20	53 − 1 52	69 − 8 61

S-125



Find the differences. Check by adding.

93 − 2 91	91 + 2 93	84 − 3 81	81 + 3 84	56 − 6 50	50 + 6 56
49 − 6 43	43 + 6 49	96 − 4 92	92 + 4 96	77 − 3 74	74 + 3 77
65 − 3 62	62 + 3 65	28 − 5 23	23 + 5 28	69 − 7 62	62 + 7 69
55 − 4 51	51 + 4 55	74 − 1 73	73 + 1 74	38 − 3 35	35 + 3 38
47 − 4 43	43 + 4 47	36 − 5 31	31 + 5 36	89 − 4 85	85 + 4 89

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digit numerals. Point out to the children, however, that they do not have the advantage of seeing the ones and tens clearly written out. They must now think of what is behind each digit in the numeral; they must think about the number of ones or tens each digit represents. Have them write all the numerals needed to complete the remaining problems.

▶ Use work page S-125 to give the children practice in working two-digit subtraction problems in vertical-notation form. This form should be presented only after a solid foundation has been established for the meaning behind this algorism. Ask the children to find the differences, and work the first problem with the children. Then have the class complete the remainder of the page on their own, reminding them to use care in the correct placement of the numerals representing ones and tens.

Work page S-126 provides a review of two-digit addition and subtraction in vertical-notation form. Have the children find the sums or differences, reminding them to line up the numerals in the ones and the tens places as carefully as possible. Children who understand the inverse concept will have an advantage in being able to work these problems quickly.

Supplemental Experiences

▶ Write related placeholder equations on the chalkboard. Have children complete each equation.

$\square - 5 = 4$ $\quad 8 - \square = 2$
$\square - 5 = 14$ $\quad 18 - \square = 12$
$\square - 5 = 34$ $\quad 88 - \square = 82$

Help them become aware of the relation among the three equations and to realize that knowing how to work the equation involving only ones will help them work equations involving tens and ones. Remind them to do the two-digit problems by subtracting the ones first and then putting this result with the tens to get their answer.

Two-Digit Addition—Subtraction

NO CARRYING
NO BORROWING

Work Pages S-127 Through S-130

Objectives

To extend the development of two-digit addition without carrying.

To extend the development of two-digit subtraction without borrowing.

Fundamentals

The basic properties of addition alone will not provide the child with the tools necessary for finding the sum in a problem such as $20 + 40 = \square$. At this stage the child must use his intuition. Since he knows that $2 + 4 = 6$, it will not be too difficult for him to grasp the fact that $20 + 40 = 60$. The teacher can help the child see this by having him combine sets of ten and count the number of tens in the new group, and also by using the Countingman. The child will then learn to relate both the equation form and vertical notation to the union of sets.

The reason an intuitive approach is necessary for understanding the addition of multiples of ten is that it is not possible, at this time, to *mathematically* justify such addition. When multiplication and the distributive property are introduced in the third grade, the mathematical justification for adding tens will become apparent. The following example will show what is involved:

$$\begin{aligned}20 + 40 &= (2 \times 10) + (4 \times 10)\\ &= (2 + 4) \times 10\\ &= 6 \times 10\\ &= 60\end{aligned}$$

When developing two-digit subtraction involving multiples of ten, as in the problem $60 - 40 = \square$, the child should be guided to apply what he knows about adding multiples of ten. Knowing that $20 + 40 = 60$, the child should arrive at the conclusion that $60 - 40 = 20$, in the same way that $2 + 4 = 6$ leads to $6 - 4 = 2$.

The problem of justification applies here just as it did for addition. An illustration will help clarify what is involved:

$$\begin{aligned}60 - 40 &= (6 \times 10) - (4 \times 10)\\ &= (6 - 4) \times 10\\ &= 2 \times 10\\ &= 20\end{aligned}$$

For now, the child will not be bothered by this delay in mathematical justification. The addition and subtraction of tens will come so naturally that he will be able to acquire the skills he needs with little trouble. The justification of adding and subtracting tens will fall into place in the third grade.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Ability to combine sets of ten to make a new set.

Ability to remove a part of a set and determine the number of the set that remains.

Ability to add two numbers.

Ability to subtract two numbers.

Awareness of the inverse relation between addition and subtraction.

Understanding of the concept of place value.

Ability to work with placeholder equations and vertical notation.

Developmental Experiences

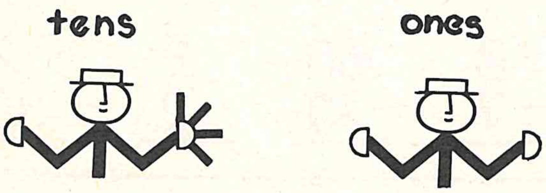
MATERIALS

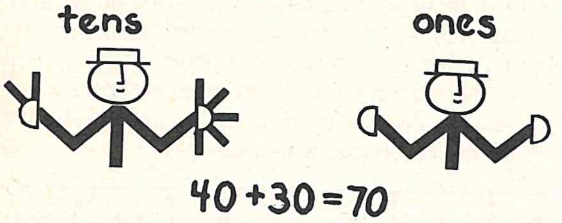
sticks
2 Countingmen

▶ Have the children work at their desks with bundles of ten sticks each. Have them join two bundles with six bundles and tell how many there are all together. Have a child tell the set story and write the related equation on the chalkboard: 20 + 60 = 80. Have the children reverse the steps: begin with eight bundles of ten, remove six bundles of ten, and tell how many remain. Have a child tell the set story and record the related subtraction equation: 80 − 60 = 20.

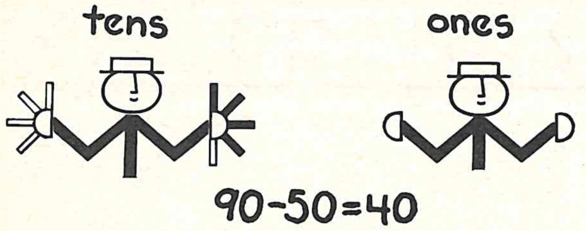
Continue with the joining and removal of sets. Have the children record the related addition and subtraction facts in equation form. Then ask other children to write, on the chalkboard, the vertical-notation form for the ideas involved.

▶ Display two Countingmen. Have a child show forty (or four tens) on the Countingmen. Have the child put thirty (or three tens) with the forty. Ask the class to tell how many he has now: "Seventy, or seven tens." On the chalkboard, have one child record an equation related to this activity; have another child write the vertical-notation form.



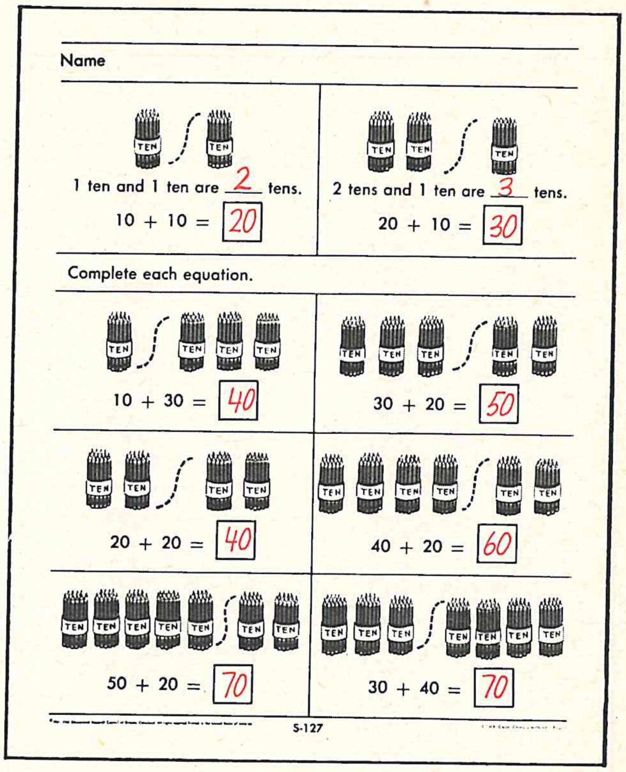


Have ninety shown in the Countingmen. Ask a child to remove fifty from ninety. Have one child record, on the chalkboard, an equation related to the activity; have a second child write the vertical-notation form for the ideas involved.



▶ Use work page S-127 as a follow-up to the manipulation of concrete objects. Work the first two problems at the top of the page with the children. Guide them to understand that they are to write the numeral that tells how many tens in the blank before the word "tens." As the next step, tell them that they are to work the addition problems using the place-value numerals for tens instead of numerals with words. Remind them that the pictures of the bundles of ten will help them. Direct them to complete the remainder of the page on their own.

Work page S-128 will give children the opportunity to work with the vertical-notation form when adding multiples of ten. Tell the children that they are to find the sums. Have them work the entire page independently.



Name

1 ten and 1 ten are 2 tens.
10 + 10 = 20

2 tens and 1 ten are 3 tens.
20 + 10 = 30

Complete each equation.

10 + 30 = 40

30 + 20 = 50

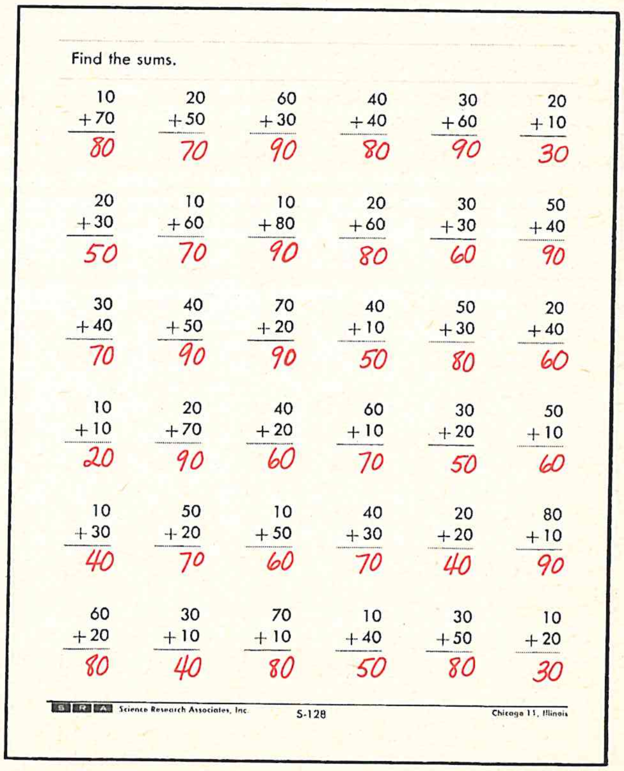
20 + 20 = 40

40 + 20 = 60

50 + 20 = 70

30 + 40 = 70

S-127



Find the sums.

10 +70 80	20 +50 70	60 +30 90	40 +40 80	30 +60 90	20 +10 30
20 +30 50	10 +60 70	10 +80 90	20 +60 80	30 +30 60	50 +40 90
30 +40 70	40 +50 90	70 +20 90	40 +10 50	50 +30 80	20 +40 60
10 +10 20	20 +70 90	40 +20 60	60 +10 70	30 +20 50	50 +10 60
10 +30 40	50 +20 70	10 +50 60	40 +30 70	20 +20 40	80 +10 90
60 +20 80	30 +10 40	70 +10 80	10 +40 50	30 +50 80	10 +20 30

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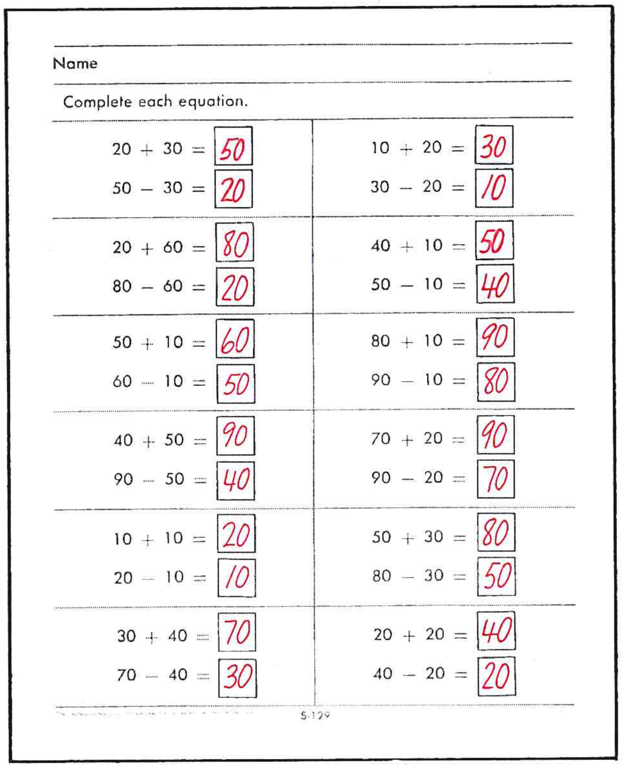
▶ Work page S-129 emphasizes the inverse relation between addition and subtraction. Discuss the inverse concept and work the first pair of equations with the children. Have the class work the remainder of the page on their own.

Work page S-130 will give children the opportunity to work with the vertical-notation form when subtracting tens. Tell the children that they are to find the differences. Have them work the entire page independently.

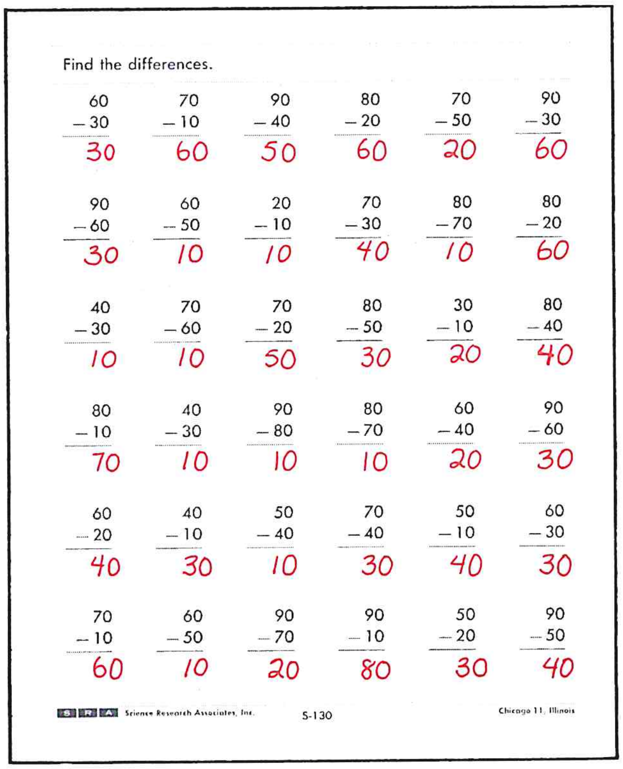


Name

Complete each equation.

20 + 30 = $\boxed{50}$ 50 − 30 = $\boxed{20}$	10 + 20 = $\boxed{30}$ 30 − 20 = $\boxed{10}$
20 + 60 = $\boxed{80}$ 80 − 60 = $\boxed{20}$	40 + 10 = $\boxed{50}$ 50 − 10 = $\boxed{40}$
50 + 10 = $\boxed{60}$ 60 − 10 = $\boxed{50}$	80 + 10 = $\boxed{90}$ 90 − 10 = $\boxed{80}$
40 + 50 = $\boxed{90}$ 90 − 50 = $\boxed{40}$	70 + 20 = $\boxed{90}$ 90 − 20 = $\boxed{70}$
10 + 10 = $\boxed{20}$ 20 − 10 = $\boxed{10}$	50 + 30 = $\boxed{80}$ 80 − 30 = $\boxed{50}$
30 + 40 = $\boxed{70}$ 70 − 40 = $\boxed{30}$	20 + 20 = $\boxed{40}$ 40 − 20 = $\boxed{20}$

S-129



Find the differences.

$$\begin{array}{rrrrrr} 60 & 70 & 90 & 80 & 70 & 90 \\ -30 & -10 & -40 & -20 & -50 & -30 \\ \hline 30 & 60 & 50 & 60 & 20 & 60 \end{array}$$

$$\begin{array}{rrrrrr} 90 & 60 & 20 & 70 & 80 & 80 \\ -60 & -50 & -10 & -30 & -70 & -20 \\ \hline 30 & 10 & 10 & 40 & 10 & 60 \end{array}$$

$$\begin{array}{rrrrrr} 40 & 70 & 70 & 80 & 30 & 80 \\ -30 & -60 & -20 & -50 & -10 & -40 \\ \hline 10 & 10 & 50 & 30 & 20 & 40 \end{array}$$

$$\begin{array}{rrrrrr} 80 & 40 & 90 & 80 & 60 & 90 \\ -10 & -30 & -80 & -70 & -40 & -60 \\ \hline 70 & 10 & 10 & 10 & 20 & 30 \end{array}$$

$$\begin{array}{rrrrrr} 60 & 40 & 50 & 70 & 50 & 60 \\ -20 & -10 & -40 & -40 & -10 & -30 \\ \hline 40 & 30 & 10 & 30 & 40 & 30 \end{array}$$

$$\begin{array}{rrrrrr} 70 & 60 & 90 & 90 & 50 & 90 \\ -10 & -50 & -70 & -10 & -20 & -50 \\ \hline 60 & 10 & 20 & 80 & 30 & 40 \end{array}$$

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Supplemental Experiences

▶ Place a series of vertical-notation problems on the chalkboard. Have children copy these problems on their writing paper and work them. Remind the children that they must carefully line up the ones and the tens.

$$\begin{array}{rrrr} 6 & 60 & 2 & 20 \\ +3 & +30 & +5 & +50 \\ \hline \end{array}$$

$$\begin{array}{rrrr} 8 & 80 & 9 & 90 \\ -7 & -70 & -2 & -20 \\ \hline \end{array}$$

▶ As a co-operative activity, have the children construct magic squares. Draw three rows of three boxes each on the chalkboard. Tell the children that they must place three given numerals in the boxes in such a way that the numbers they represent can be added in any direction—vertically, horizontally, and diagonally—with the same sum each time. For their first effort, let the children work with 1, 2, and 3. Later have them try to make a magic square using 2, 3, and 4. As a variation, have the children try using multiples of ten in the squares.

3	1	2
1	2	3
2	3	1

2	4	3
4	3	2
3	2	4

30	10	20
10	20	30
20	30	10

20	40	30
40	30	20
30	20	40

Two-Digit Addition

NO CARRYING

Work Pages S-131 Through S-136

Objectives

To extend the development of two-digit addition without carrying.

To prepare children for understanding the addition algorism.

Fundamentals

After the child develops the ability to add multiples of ten, it is possible to proceed with two-digit addition and to justify every step by the associative and commutative properties. The following example illustrates the procedure.

$$\begin{aligned} 27 + 30 &= (20 + 7) + 30 && \text{expanded notation} \\ &= 20 + (7 + 30) && \text{associative property} \\ &= 20 + (30 + 7) && \text{commutative property} \\ &= (20 + 30) + 7 && \text{associative property} \\ &= 50 + 7 && \text{addition of tens} \\ &= 57 && \text{place-value notation} \end{aligned}$$

The child can use the scrambling process as he works with addition problems of this kind. He can immediately rearrange the numbers in the addition problem without taking all of the steps in the above example. It is important, however, that the teacher understand how the commutative and associative properties of addition relate to this idea of scrambling.

The child will begin his work with sets and the Countingmen. He will learn to put his sets of tens together and then join them to the set of ones to get his answer. In working with equations, the child will learn to add his tens and then to add his ones to his tens to get his answer. In using expanded notation in vertical form, the child can clearly see his ones and tens lined up under each other for faster computation. In the algorism, the child will add ones, add tens, and then think in terms of adding the tens to the ones to explain his answer.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Ability to combine sets to make a new set.

Ability to add two numbers.

Understanding of the concept of place value.

Understanding of the concept of expanded notation.

Understanding of the associative and commutative properties.

Ability to work with placeholder equations and vertical-notation form.

Developmental Experiences

MATERIALS

sticks
2 Countingmen

▶ Place the equation $42 + 20 = \square$ on the chalkboard. Have children place four bundles of sticks and two loose sticks on the left side of their desks. Ask how many there are. Have them place two more bundles of sticks on the right side of their desks. Ask how many there are.

Have the children show how many sticks they have all together. Tell the children to put all their ones together, and then all the tens together. Have children tell how many ones and how many tens they now have. Have them join their ones to their tens and give the result. Place the related equation on the chalkboard under the placeholder equation: $40 + 20 + 2 = 62$.

Set up the expanded notation in vertical form for this problem. Guide the children to understand that

$$\begin{array}{r} 40 + 2 \\ +\ (20 + 0) \\ \hline 60 + 2 \end{array} = 62$$

the zero in the numeral $20 + 0$ will help them visualize that there are no ones in the number represented in the numeral. Explain that the parentheses around the expanded-notation form of the second addend will help separate the plus sign (which is the operational symbol in this problem) from the plus sign used in the numeral $20 + 0$.

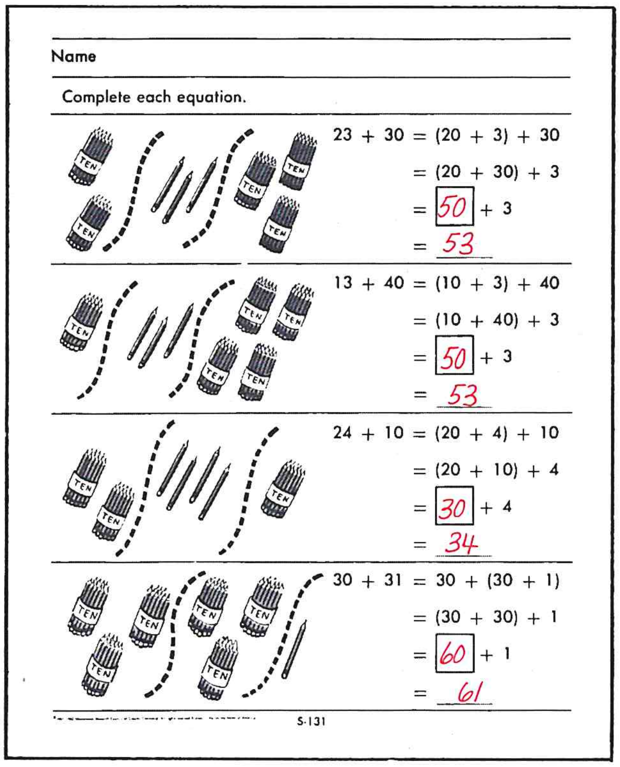
Continue in this way to work other problems of this kind. Have both forms of the problem, the equation form and the expanded notation in vertical form, written on the chalkboard and read as the thinking steps in the problems are summarized.

▶ Demonstrate the problem $65 + 30 = \square$ by using the Countingmen. Tell one child to show the number sixty-five on the Countingmen—six tens and five ones. Ask another child to show thirty more; he should place three more fingers on the Tens-man. Have someone explain what was done. Write the related vertical-notation form on the chalkboard. Have it read: "Five plus zero equals five; sixty plus thirty equals ninety; ninety plus five is ninety-five."

Continue to use the Countingmen to develop other problems of this type. Have the related vertical notation placed on the chalkboard and read for each problem.

▶ Use work page S-131 as follow-up to the manipulation of concrete objects. Work through the thinking steps with the children for each of the problems on the page. Refer to the pictures of bundles of sticks to guide the discussion of the associative property. Have the children complete each equation.

Use work page S-132 to help children think through two-digit addition problems in terms of ones and tens. Work the first problem on the top of the page with the children, then have them work the remainder of the page independently. Explain that they must fill in each placeholder with the numeral that indicates the number of tens in the problem, and then they must write the standard place-value numeral for the sum on the line at the end of the equation.



Name

Complete each equation.

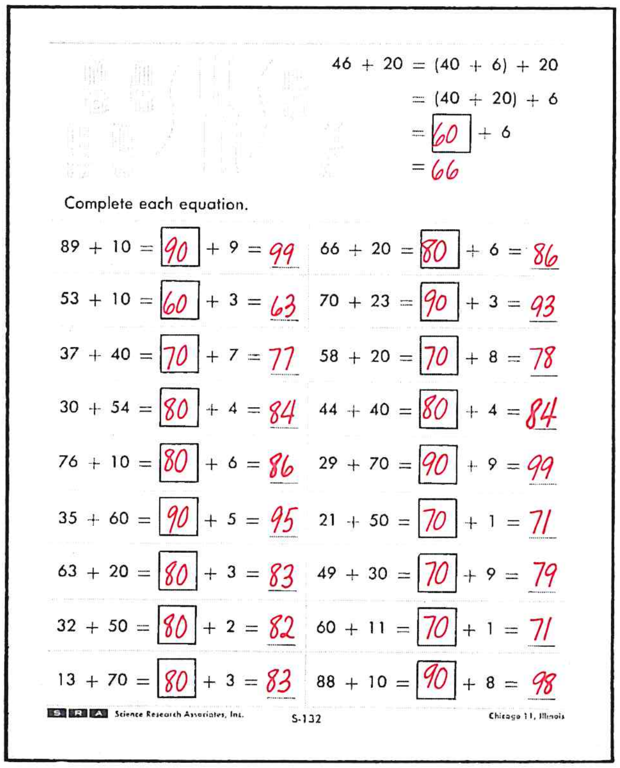
$23 + 30 = (20 + 3) + 30$
$= (20 + 30) + 3$
$= \boxed{50} + 3$
$= 53$

$13 + 40 = (10 + 3) + 40$
$= (10 + 40) + 3$
$= \boxed{50} + 3$
$= 53$

$24 + 10 = (20 + 4) + 10$
$= (20 + 10) + 4$
$= \boxed{30} + 4$
$= 34$

$30 + 31 = 30 + (30 + 1)$
$= (30 + 30) + 1$
$= \boxed{60} + 1$
$= 61$

S-131



$46 + 20 = (40 + 6) + 20$
$= (40 + 20) + 6$
$= \boxed{60} + 6$
$= 66$

Complete each equation.

$89 + 10 = \boxed{90} + 9 = 99$
$53 + 10 = \boxed{60} + 3 = 63$
$37 + 40 = \boxed{70} + 7 = 77$
$30 + 54 = \boxed{80} + 4 = 84$
$76 + 10 = \boxed{80} + 6 = 86$
$35 + 60 = \boxed{90} + 5 = 95$
$63 + 20 = \boxed{80} + 3 = 83$
$32 + 50 = \boxed{80} + 2 = 82$
$13 + 70 = \boxed{80} + 3 = 83$

$66 + 20 = \boxed{80} + 6 = 86$
$70 + 23 = \boxed{90} + 3 = 93$
$58 + 20 = \boxed{70} + 8 = 78$
$44 + 40 = \boxed{80} + 4 = 84$
$29 + 70 = \boxed{90} + 9 = 99$
$21 + 50 = \boxed{70} + 1 = 71$
$49 + 30 = \boxed{70} + 9 = 79$
$60 + 11 = \boxed{70} + 1 = 71$
$88 + 10 = \boxed{90} + 8 = 98$

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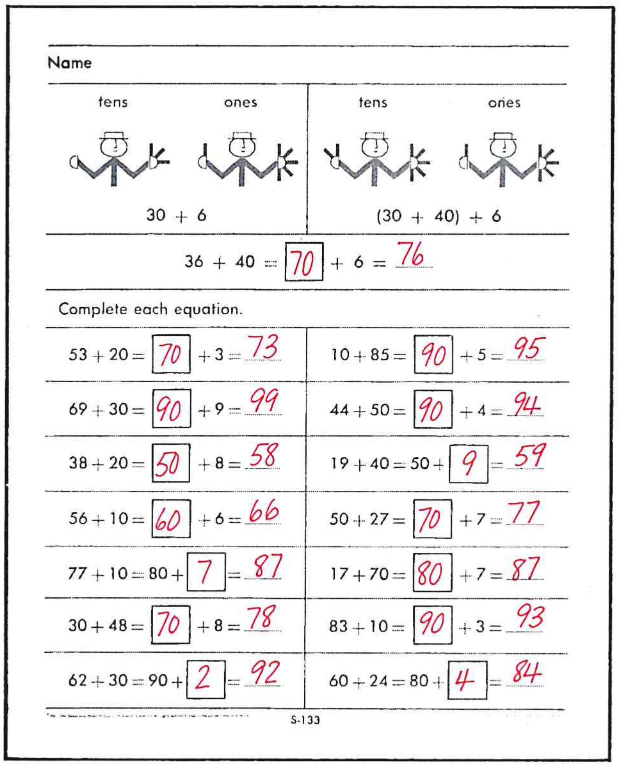


Name

$30 + 6$

$(30 + 40) + 6$

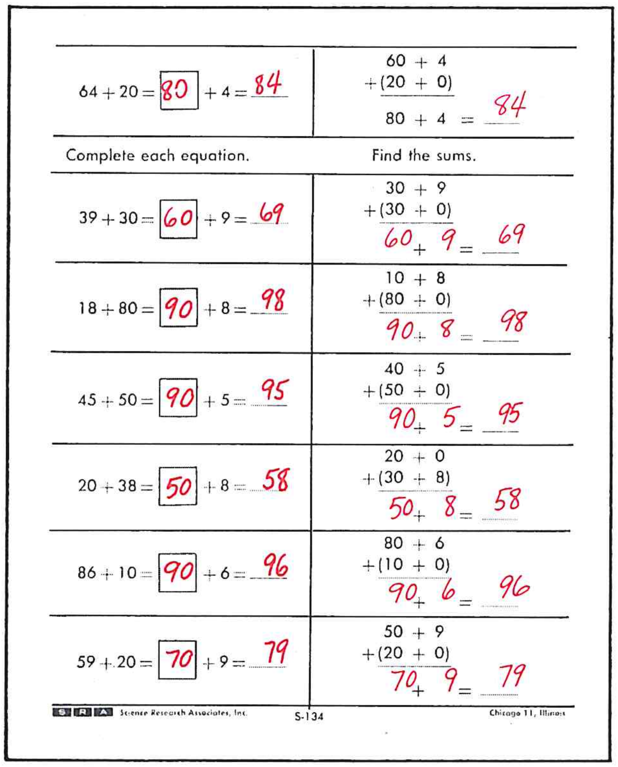
$36 + 40 = \boxed{70} + 6 = 76$

Complete each equation.

$53 + 20 = \boxed{70} + 3 = 73$
$69 + 30 = \boxed{90} + 9 = 99$
$38 + 20 = \boxed{50} + 8 = 58$
$56 + 10 = \boxed{60} + 6 = 66$
$77 + 10 = 80 + \boxed{7} = 87$
$30 + 48 = \boxed{70} + 8 = 78$
$62 + 30 = 90 + \boxed{2} = 92$

$10 + 85 = \boxed{90} + 5 = 95$
$44 + 50 = \boxed{90} + 4 = 94$
$19 + 40 = 50 + \boxed{9} = 59$
$50 + 27 = \boxed{70} + 7 = 77$
$17 + 70 = \boxed{80} + 7 = 87$
$83 + 10 = \boxed{90} + 3 = 93$
$60 + 24 = 80 + \boxed{4} = 84$

S-133



$64 + 20 = \boxed{80} + 4 = 84$

60 + 4
+(20 + 0)
80 + 4 = 84

Complete each equation.

$39 + 30 = \boxed{60} + 9 = 69$
$18 + 80 = \boxed{90} + 8 = 98$
$45 + 50 = \boxed{90} + 5 = 95$
$20 + 38 = \boxed{50} + 8 = 58$
$86 + 10 = \boxed{90} + 6 = 96$
$59 + 20 = \boxed{70} + 9 = 79$

Find the sums.

30 + 9
+(30 + 0)
60 + 9 = 69

10 + 8
+(80 + 0)
90 + 8 = 98

40 + 5
+(50 + 0)
90 + 5 = 95

20 + 0
+(30 + 8)
50 + 8 = 58

80 + 6
+(10 + 0)
90 + 6 = 96

50 + 9
+(20 + 0)
70 + 9 = 79

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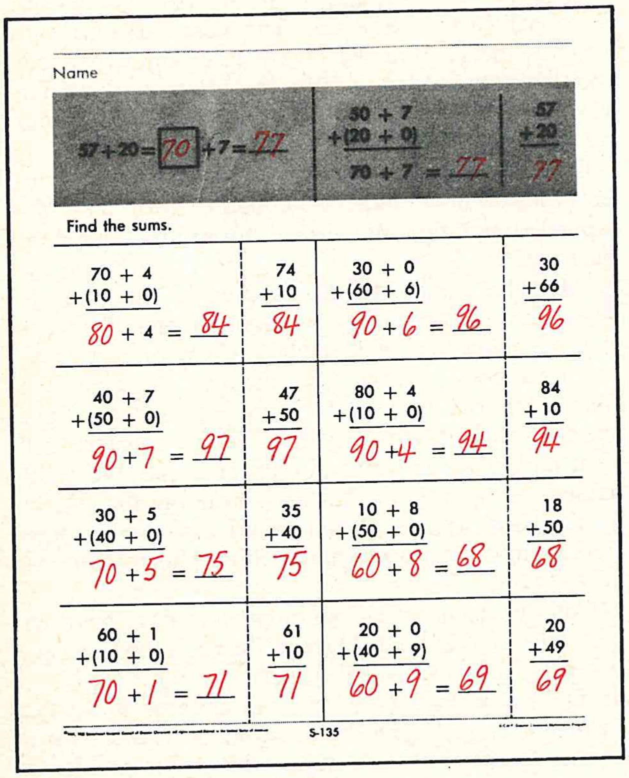
▶ Work page S-133 can also be used to help children think through two-digit addition problems in terms of ones and tens. Follow the procedure suggested for the work on page S-132.

Use work page S-134 to help children make the transition from the equation form to expanded notation in vertical form for working these problems. Work the problem at the top of the page with the children. Tell them that they will find it quicker and easier to visualize ones and tens by using expanded notation in vertical form for problems which have two-digit numerals. Tell them to keep in mind the number of ones and tens and the correct placement of numerals representing ones and tens. After the example has been completed, direct children to complete each equation and find the sums for the rest of the problems on the page.



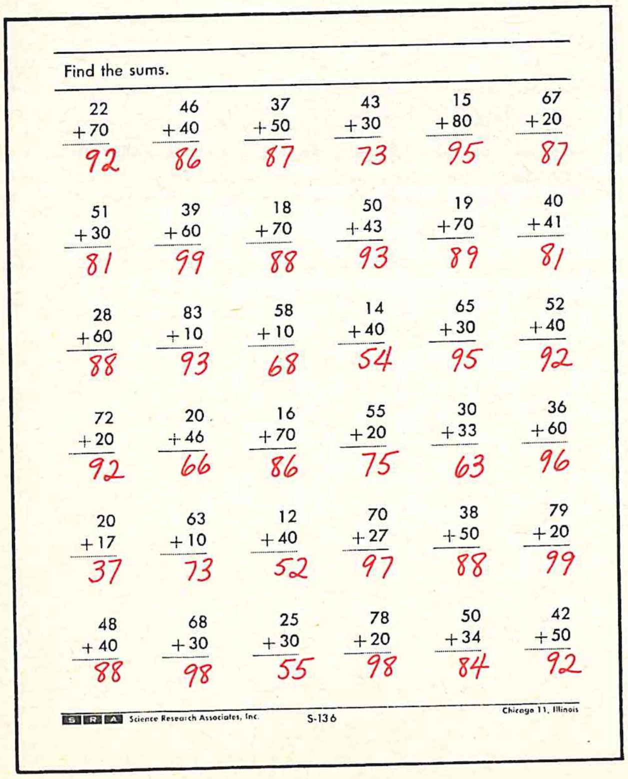
Name

57 + 20 = 70 + 7 = 77	50 + 7 +(20 + 0) 70 + 7 = 77	57 +20 77

Find the sums.

70 + 4 +(10 + 0) 80 + 4 = 84	74 +10 84	30 + 0 +(60 + 6) 90 + 6 = 96	30 +66 96
40 + 7 +(50 + 0) 90 + 7 = 97	47 +50 97	80 + 4 +(10 + 0) 90 + 4 = 94	84 +10 94
30 + 5 +(40 + 0) 70 + 5 = 75	35 +40 75	10 + 8 +(50 + 0) 60 + 8 = 68	18 +50 68
60 + 1 +(10 + 0) 70 + 1 = 71	61 +10 71	20 + 0 +(40 + 9) 60 + 9 = 69	20 +49 69

S-135



Find the sums.

22 +70 92	46 +40 86	37 +50 87	43 +30 73	15 +80 95	67 +20 87
51 +30 81	39 +60 99	18 +70 88	50 +43 93	19 +70 89	40 +41 81
28 +60 88	83 +10 93	58 +10 68	14 +40 54	65 +30 95	52 +40 92
72 +20 92	20 +46 66	16 +70 86	55 +20 75	30 +33 63	36 +60 96
20 +17 37	63 +10 73	12 +40 52	70 +27 97	38 +50 88	79 +20 99
48 +40 88	68 +30 98	25 +30 55	78 +20 98	50 +34 84	42 +50 92

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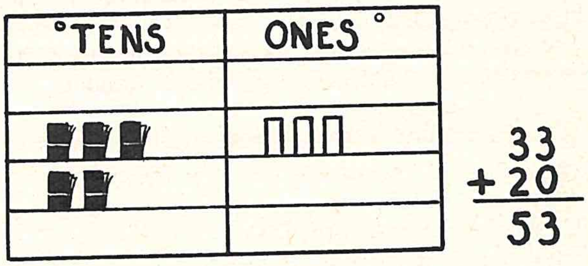
▶ Use work page S-135 to help children make the transition from the expanded-notation form to the algorism for two-digit addition problems. Remind the children that the algorism is the quickest and easiest form for working problems which have two-digit numerals. Tell them, however, that they do not have the advantage of seeing ones and tens clearly written out. They must now think what is behind each digit in the numeral—they must think about the number of ones or tens each digit represents. Have them work the entire page on their own. They must write all of the needed numerals to complete each problem and find each sum.

Use work page S-136 to give practice in working two-digit addition problems in vertical-notation form. This form should be presented only after a careful foundation for the meaning behind this algorism, or computational method, has been established. Work the first problem with the children and then have them complete the remainder of the page on their own. Tell the children to find the sums; remind them to use care in the correct placement of numerals representing ones and tens.

Supplemental Experiences

▶ Use the pocket chart as a place-value device. Put a piece of yarn down the center; label the left side "tens" and the right side "ones."

Place three bundles of ten red strips in the tens place and three separate blue strips in the ones place. In another pocket, put two bundles of red strips. Call upon a child to write the related vertical-notation form for this activity on the chalkboard. Have the problem read in terms of ones and tens.



Continue in this way to build set stories related to two-digit addition with no carrying. Have children write the corresponding algorism for the activity they viewed.

▶ Present oral chain problems. Say: "Twenty plus thirty"—pause—"plus six." Call upon someone to give the answer. Give another problem: "Ten plus seven"—pause—"plus fifty." Call upon someone to give the answer.

Continue in this way with other oral problems. At first, some children may need to work with bundles of sticks and loose sticks to get the answers. Encourage them to work as much as possible, however, without this aid. The children will enjoy working faster and faster as they become more familiar with two-digit addition.

Two-Digit Subtraction

NO BORROWING

Work Pages S-137 and S-138

Objectives

To extend the development of two-digit subtraction without borrowing.

Fundamentals

To justify subtracting in problems such as 36 − 20 = □, the teacher will again refer to work done in addition. The child discovered that he could find the sum of thirty-six and twenty by adding ones, adding tens, and then adding the two sums. He learned to think of 36 + 20 as (30 + 20) + 6, as 50 + 6, and finally as 56. This procedure could be justified by the use of the commutative and associative properties.

These two properties do not apply to subtraction. The teacher will have to guide the child to try to subtract ones, subtract tens, and then add ones and tens to arrive at his answer. She will have to help him think of 36 − 20 as (30 − 20) + 6, as 10 + 6, and finally as 16. The child will work first with sets and equations, and then with expanded notation in vertical form. Finally, he will be introduced to the subtraction algorism.

See Key Topics in Mathematics for the Primary Teacher: Subtraction of Whole Numbers.

Readiness for Understanding

Awareness of the inverse relation between addition and subtraction.

Understanding of subtraction.

Understanding of expanded notation.

Ability to work with vertical notation.

Developmental Experiences

MATERIALS

sticks
2 Countingmen

▶ Place the equation 42 − 20 = □ on the chalkboard. Have the children place forty-two sticks (four bundles of ten and two singles) on their desks. Have them remove twenty sticks (two bundles of ten). Ask how many ones and how many tens they have left, and have them complete the placeholder equation.

Write the problem in vertical form using expanded notation. Explain that the parentheses around the

$$\begin{array}{r} 40 + 2 \\ -\ (20 + 0) \\ \hline 20 + 2 \end{array} = 22$$

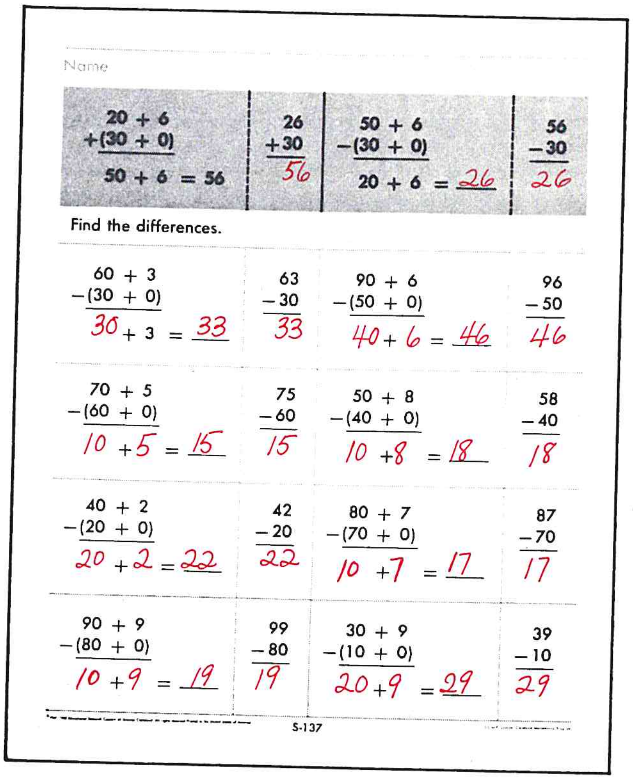
numeral 20 + 0 will help them think of the number to be subtracted. Remind them that the plus sign between the 20 and the 0 is part of the expanded-notation numeral for twenty. The minus sign in front of (20 + 0) means that both the ones and the tens are to be subtracted.

Continue in this way to work other similar problems. Show how both the equation form and the expanded notation in vertical form are related to each activity.

▶ Help the child find the solution to the problem 84 − 50 = □ by using the Countingmen. Tell one child to show the number eighty-four, or eight tens and four ones, on the Countingmen. Ask another child to remove fifty, or five tens. He should remove five fingers from the Tens-man. Have someone tell what took place in terms of ones and tens. Write the related subtraction in vertical form on the chalkboard. Have it read: "Four minus zero equals four; eighty minus fifty equals thirty; thirty plus four equals thirty-four."

Continue to use the Countingmen to develop other problems of this type. Have children record the action each time in vertical form.

▶ Use work page S-137 to help children make the transition from expanded notation in vertical form to the algorism for two-digit subtraction problems.



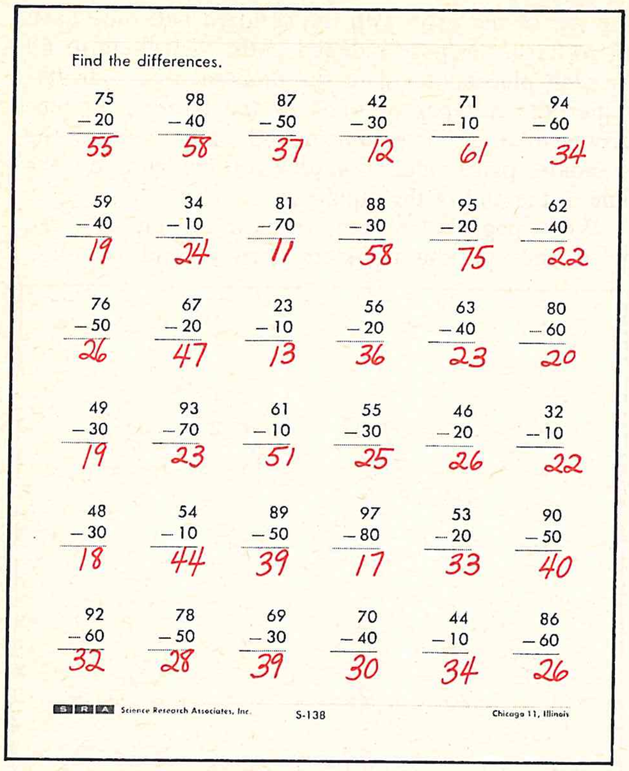
Name

20 + 6 +(30 + 0) 50 + 6 = 56	26 +30 56	50 + 6 −(30 + 0) 20 + 6 = 26	56 −30 26

Find the differences.

60 + 3 −(30 + 0) 30 + 3 = 33	63 −30 33	90 + 6 −(50 + 0) 40 + 6 = 46	96 −50 46
70 + 5 −(60 + 0) 10 + 5 = 15	75 −60 15	50 + 8 −(40 + 0) 10 + 8 = 18	58 −40 18
40 + 2 −(20 + 0) 20 + 2 = 22	42 −20 22	80 + 7 −(70 + 0) 10 + 7 = 17	87 −70 17
90 + 9 −(80 + 0) 10 + 9 = 19	99 −80 19	30 + 9 −(10 + 0) 20 + 9 = 29	39 −10 29

S-137



Find the differences.

75 − 20 55	98 − 40 58	87 − 50 37	42 − 30 12	71 − 10 61	94 − 60 34
59 − 40 19	34 − 10 24	81 − 70 11	88 − 30 58	95 − 20 75	62 − 40 22
76 − 50 26	67 − 20 47	23 − 10 13	56 − 20 36	63 − 40 23	80 − 60 20
49 − 30 19	93 − 70 23	61 − 10 51	55 − 30 25	46 − 20 26	32 − 10 22
48 − 30 18	54 − 10 44	89 − 50 39	97 − 80 17	53 − 20 33	90 − 50 40
92 − 60 32	78 − 50 28	69 − 30 39	70 − 40 30	44 − 10 34	86 − 60 26

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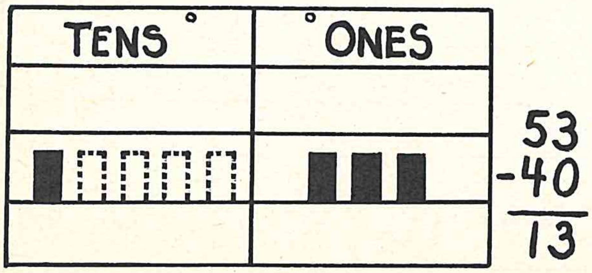
Work the two problems at the top of the page with the children. Remind them that the algorism is the quickest and easiest form for working problems which have two-digit numerals. Tell them to think about the number of ones or tens each digit represents. Have them work the remainder of the problems independently.

Use work page S-138 to give practice in working two-digit subtraction problems in vertical-notation form. Work the first problem with the group and then have them complete the remainder of the page on their own. Tell the children to find the differences; remind them to use care in the placement of ones and tens.

Supplemental Experiences

▶ Use the pocket chart as a place-value device. Divide the chart into two parts and label one part "Tens" and the other part "Ones." Place five bundles of ten paper strips under the part labeled "Tens" and three separate strips under the part labeled "Ones." Remove four bundles of ten. Call upon a child to write the related vertical-notation form for this activity on the chalkboard and have the problem read.



Continue in this way to build set stories related to two-digit subtraction problems in which no borrowing is involved. Have children write the corresponding algorism for each activity.

▶ Present oral chain problems. Say, "Fifty minus thirty"—pause—"plus six." Call upon someone to give the answer. Give another problem: "Forty plus seven"—pause—"minus ten." Call upon someone to give the answer. Present a third problem: "Ninety-eight minus thirty"—pause—"plus twenty."

Continue in this way with other oral problems. The children will enjoy working faster and faster as they become more familiar with two-digit subtraction.

Two-Digit Addition

NO CARRYING

Work Pages S-139 Through S-144

Objectives

To extend the development of two-digit addition without carrying.

To provide practice with addition involving three and four two-digit addends.

To give children experience in searching for missing addends.

Fundamentals

Problems such as $23 + 45 = \square$ constitute the last step at the second grade level in the development of two-digit addition without carrying. The justification for adding first the ones and then the tens will be outlined.

$$\begin{aligned} 23 + 45 &= (20 + 3) + (40 + 5) \\ &\qquad \text{expanded notation} \\ &= [(20 + 3) + 40] + 5 \\ &\qquad \text{associative property} \\ &= [20 + (3 + 40)] + 5 \\ &\qquad \text{associative property} \\ &= [20 + (40 + 3)] + 5 \\ &\qquad \text{commutative property} \\ &= [(20 + 40) + 3] + 5 \\ &\qquad \text{associative property} \\ &= (20 + 40) + (3 + 5) \\ &\qquad \text{associative property} \end{aligned}$$

The child is not expected to follow all of the detailed steps listed above. The teacher will give the child a simplified version of this problem.

$$\begin{aligned} 23 + 45 &= (20 + 3) + (40 + 5) \\ &= (20 + 40) + (3 + 5) \end{aligned}$$

The careful groundwork which has been laid in previous lessons will help the child understand this transition. The teacher will want to review the commutative and associative properties and emphasize that they make it possible to change the order and grouping of addends in any way that is convenient.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Ability to add two numbers.

Understanding of the concept of place value.

Understanding of expanded notation.

Understanding of the associative and commutative properties of addition.

Understanding of the process of scrambling.

Ability to work with placeholder equations and vertical notation.

Ability to find a missing addend.

Developmental Experiences

MATERIALS	VOCABULARY
sticks	reconstruction
2 Countingmen	problem

▶ Write the equation 42 + 26 = ☐ on the chalkboard. Have children place forty-two sticks (four bundles of ten and two singles) on the left side of their desks. Have them place twenty-six sticks (two bundles of ten and six singles) on the right side of their desks.

Help the children show how many sticks they have altogether. Tell them to first put the ones together, then put the tens together, and finally put the ones and tens together. Write the related equation on the chalkboard under the placeholder equation: $40 + 20 + 2 + 6 = 60 + 8 = 68$.

Set up the expanded notation in vertical form for

$$\begin{array}{r} 40 + 2 \\ +\ \underline{(20 + 6)} \\ 60 + 8 \end{array} = 68$$

this problem. Remind the children that the plus sign in front of the parentheses tells them to add and the parentheses help them think of the number to be added.

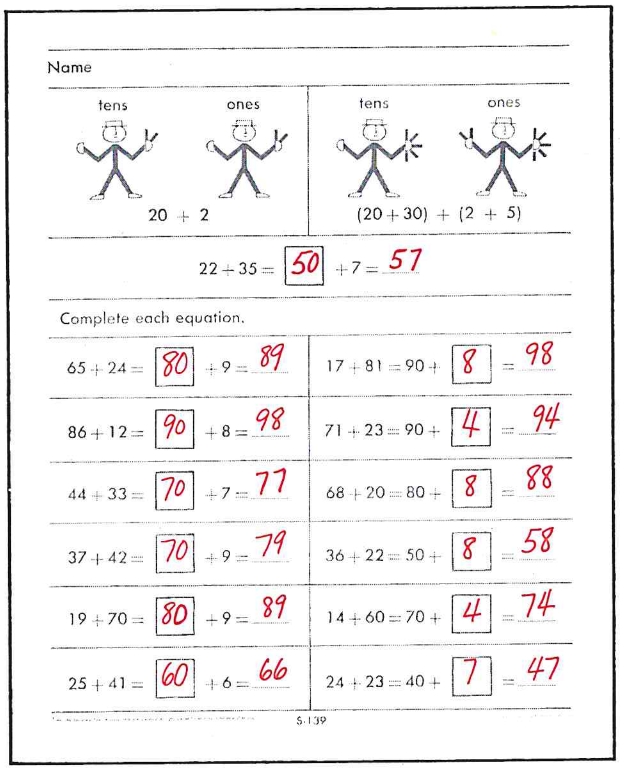
Work other problems of this kind in the same way. Have both the equation form and expanded notation in vertical form written for each activity. Have the thinking steps in each problem summarized.

▶ Help the children find the solution to the problem 35 + 24 = ☐ by using the Countingmen. Tell one child to show the number thirty-five on the Countingmen—three tens and five ones. Ask another child to show twenty-four more. He should place four more fingers on the Ones-man and two more fingers on the Tens-man. Have someone tell how many ones and tens are now shown. Write the related vertical-notation form on the chalkboard, and have it read: "Five plus four equals nine; thirty plus twenty equals fifty; fifty plus nine equals fifty-nine."

Use the Countingmen to develop other addition problems of this type. Have the related vertical notation read each time.

▶ Use work page S-139 to help children with two-digit addition problems. Work the first problem on the top of the page with the children and then have them finish the page independently. Tell them to fill in each placeholder with the numeral that indicates either the number of ones or the number of tens represented in the problem and then to write the standard place-value numeral for the sum on the line at the end of the equation.

Work page S-140 can be used to help children work through the thinking steps behind two-digit



Name

tens
ones
tens
ones

20 + 2

(20 + 30) + (2 + 5)

22 + 35 = $\boxed{50}$ + 7 = 57

Complete each equation.

65 + 24 = $\boxed{80}$ + 9 = 89

86 + 12 = $\boxed{90}$ + 8 = 98

44 + 33 = $\boxed{70}$ + 7 = 77

37 + 42 = $\boxed{70}$ + 9 = 79

19 + 70 = $\boxed{80}$ + 9 = 89

25 + 41 = $\boxed{60}$ + 6 = 66

17 + 81 = 90 + $\boxed{8}$ = 98

71 + 23 = 90 + $\boxed{4}$ = 94

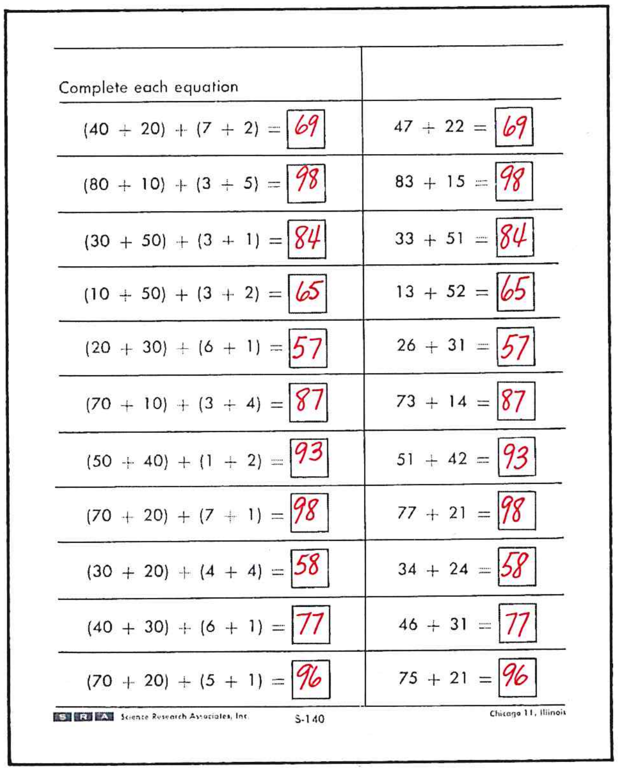
68 + 20 = 80 + $\boxed{8}$ = 88

36 + 22 = 50 + $\boxed{8}$ = 58

14 + 60 = 70 + $\boxed{4}$ = 74

24 + 23 = 40 + $\boxed{7}$ = 47

S-139



Complete each equation

(40 + 20) + (7 + 2) = $\boxed{69}$	47 + 22 = $\boxed{69}$
(80 + 10) + (3 + 5) = $\boxed{98}$	83 + 15 = $\boxed{98}$
(30 + 50) + (3 + 1) = $\boxed{84}$	33 + 51 = $\boxed{84}$
(10 + 50) + (3 + 2) = $\boxed{65}$	13 + 52 = $\boxed{65}$
(20 + 30) + (6 + 1) = $\boxed{57}$	26 + 31 = $\boxed{57}$
(70 + 10) + (3 + 4) = $\boxed{87}$	73 + 14 = $\boxed{87}$
(50 + 40) + (1 + 2) = $\boxed{93}$	51 + 42 = $\boxed{93}$
(70 + 20) + (7 + 1) = $\boxed{98}$	77 + 21 = $\boxed{98}$
(30 + 20) + (4 + 4) = $\boxed{58}$	34 + 24 = $\boxed{58}$
(40 + 30) + (6 + 1) = $\boxed{77}$	46 + 31 = $\boxed{77}$
(70 + 20) + (5 + 1) = $\boxed{96}$	75 + 21 = $\boxed{96}$

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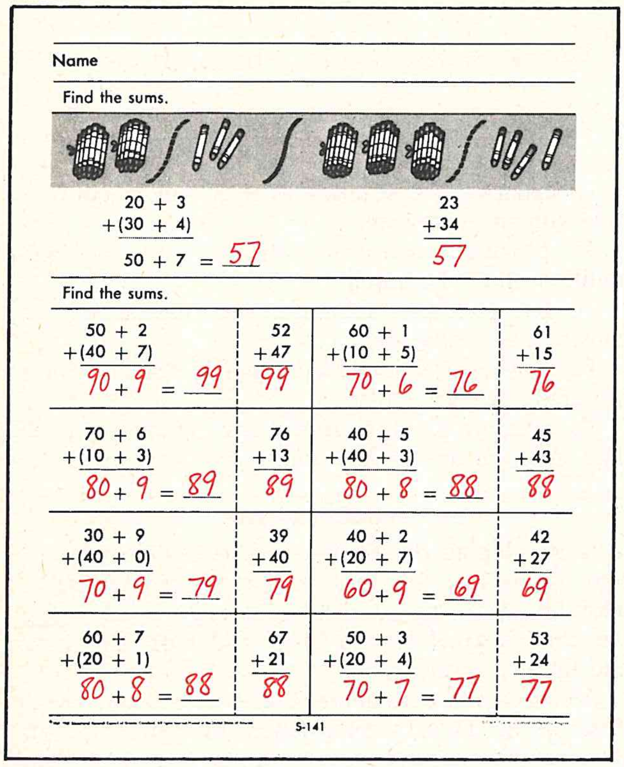
addition problems in order to give more meaning to the algorism. Work the first pair of equations across the top of the page with the children. Point out how easy it is to see the tens and ones in the equations on the left side of the page. Tell the children that they must think about the tens and ones in the equations on the right side of the page even though the expanded-notation form is not there to help them. Have the children work the remainder of the problems on their own.

▶ Use work page S-141 to help children make the transition from the expanded-notation form to the algorism for two-digit addition problems. Discuss the picture and work the problem at the top of the page with the children. Have them complete the remainder of the page on their own. Tell them to



Name

Find the sums.

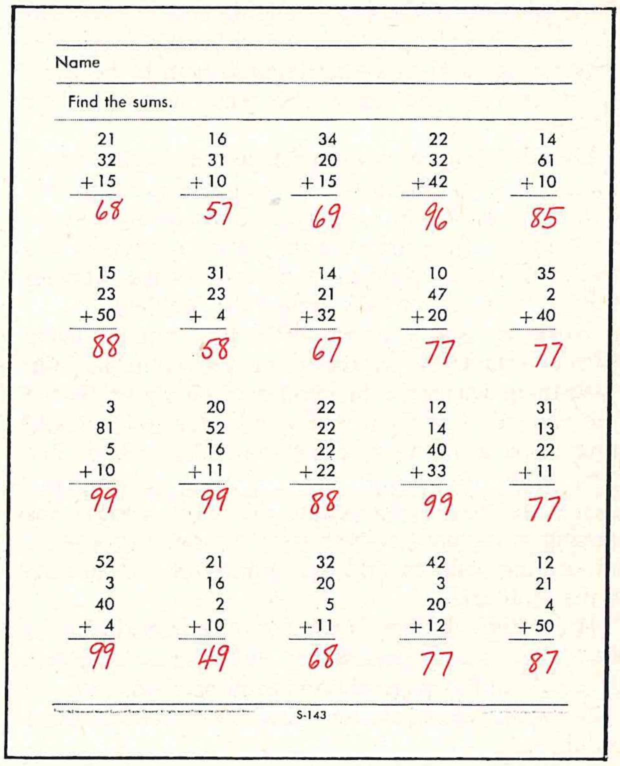
20 + 3
+(30 + 4)
50 + 7 = 57

23
+34
57

Find the sums.

50 + 2 +(40 + 7) 90 + 9 = 99	52 +47 99	60 + 1 +(10 + 5) 70 + 6 = 76	61 +15 76
70 + 6 +(10 + 3) 80 + 9 = 89	76 +13 89	40 + 5 +(40 + 3) 80 + 8 = 88	45 +43 88
30 + 9 +(40 + 0) 70 + 9 = 79	39 +40 79	40 + 2 +(20 + 7) 60 + 9 = 69	42 +27 69
60 + 7 +(20 + 1) 80 + 8 = 88	67 +21 88	50 + 3 +(20 + 4) 70 + 7 = 77	53 +24 77

S-141

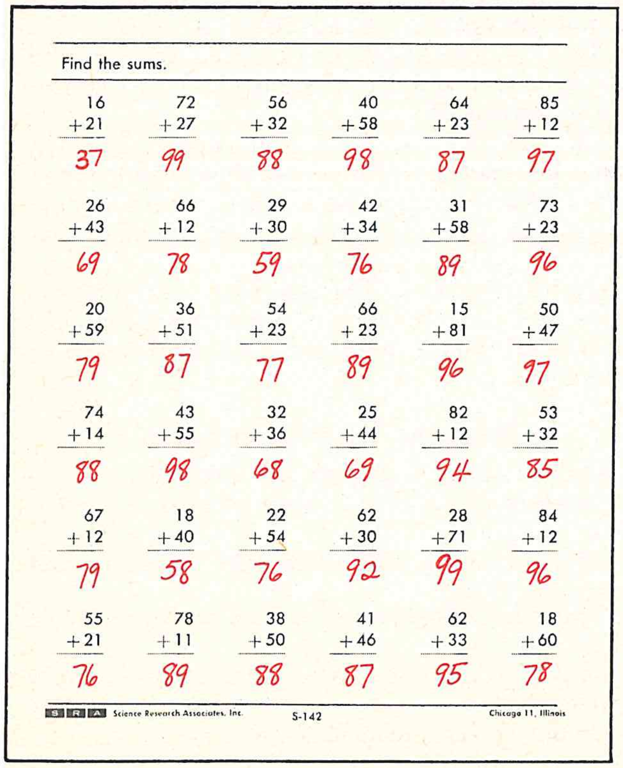


Name

Find the sums.

21 32 +15 68	16 31 +10 57	34 20 +15 69	22 32 +42 96	14 61 +10 85
15 23 +50 88	31 23 + 4 58	14 21 +32 67	10 47 +20 77	35 2 +40 77
3 81 5 +10 99	20 52 16 +11 99	22 22 22 +22 88	12 14 40 +33 99	31 13 22 +11 77
52 3 40 + 4 99	21 16 2 +10 49	32 20 5 +11 68	42 3 20 +12 77	12 21 4 +50 87

S-143



Find the sums.

16 +21 37	72 +27 99	56 +32 88	40 +58 98	64 +23 87	85 +12 97
26 +43 69	66 +12 78	29 +30 59	42 +34 76	31 +58 89	73 +23 96
20 +59 79	36 +51 87	54 +23 77	66 +23 89	15 +81 96	50 +47 97
74 +14 88	43 +55 98	32 +36 68	25 +44 69	82 +12 94	53 +32 85
67 +12 79	18 +40 58	22 +54 76	62 +30 92	28 +71 99	84 +12 96
55 +21 76	78 +11 89	38 +50 88	41 +46 87	62 +33 95	18 +60 78

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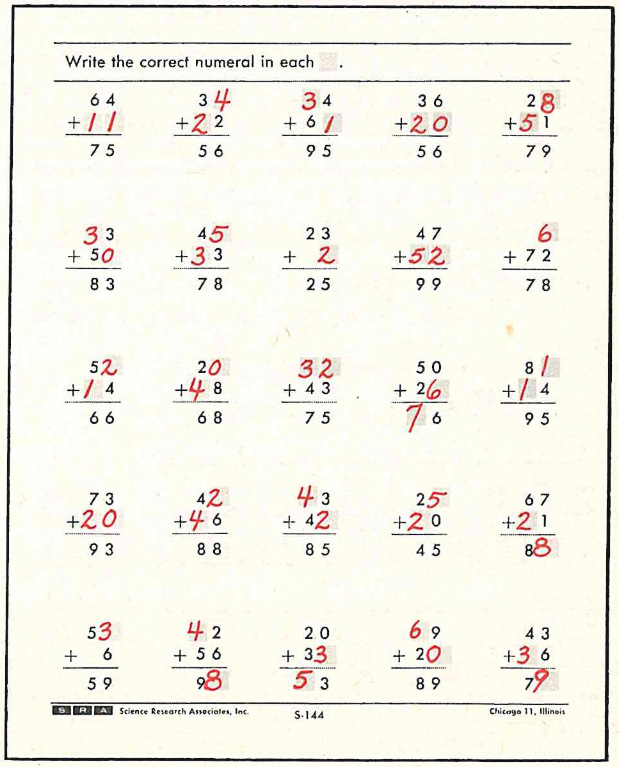

Write the correct numeral in each ▢.

6 4 + 1 1 7 5	3 4 +2 2 5 6	3 4 + 6 1 9 5	3 6 +2 0 5 6	2 8 +5 1 7 9
3 3 + 5 0 8 3	4 5 +3 3 7 8	2 3 + 2 2 5	4 7 +5 2 9 9	6 + 7 2 7 8
5 2 + 1 4 6 6	2 0 +4 8 6 8	3 2 + 4 3 7 5	5 0 + 2 6 7 6	8 1 + 1 4 9 5
7 3 +2 0 9 3	4 2 +4 6 8 8	4 3 + 4 2 8 5	2 5 +2 0 4 5	6 7 +2 1 8 8
5 3 + 6 5 9	4 2 + 5 6 9 8	2 0 + 3 3 5 3	6 9 + 2 0 8 9	4 3 +3 6 7 9

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write all the numerals to complete each problem.

Use work page S-142 to give practice in working two-digit addition problems in vertical form. Direct the children to find the sums; remind them to think carefully about the ones and tens in these problems.

▶ Work page S-143 will give children practice in column addition with two-digit addends. Work the first problem with the class. Tell the children to add from top to bottom and then have them check their answers by adding from bottom to top. Review the application of the scrambling process in problems of this type. Have children complete the remainder of the page on their own. Remind them to be careful in placing the ones and tens digits in their answers.

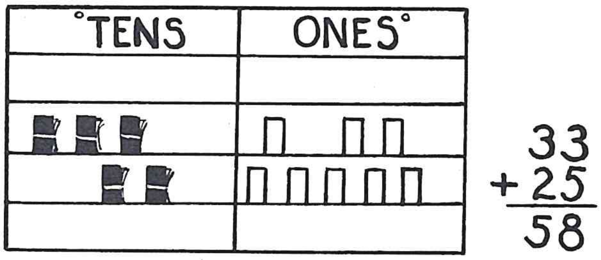
Use work page S-144 to provide practice in finding the missing number in the vertical-notation form for addition problems involving two-digit addends. Problems of this type are referred to as *reconstruction problems,* for the child must reconstruct the problem in his mind in order to complete the problem.

Work the first problem with the children. Have them determine the number of ones given in the sum. Have them determine the number of ones given in the first addend. Ask them how many more ones would have to be added to four ones in order to have five ones. When the answer one is offered, tell the children that they have found the digit which was missing in the ones column. Use the same procedure for helping children find the number of tens needed in the problem.

Have the children complete the remainder of the work page independently. When they complete the work on this page, discuss each problem with the class. Have them explain how they worked these problems.

Supplemental Experiences

▶ Use the pocket chart as a place-value device. Divide the chart into two halves and label one half "Tens" and the other half "Ones." Place three bundles of ten paper strips on the tens side and three separate strips on the ones side. To these sets join two bundles of ten strips and five single strips. Call upon a child to write the related vertical-notation form for this activity on the chalkboard, and have the problem read: "Three plus five equals eight; thirty plus twenty equals fifty; fifty plus eight is fifty-eight."



Continue in this way to build other set stories involving two-digit addition with no carrying. Have children write the corresponding algorism for each set story.

Two-Digit

Addition and Subtraction

COMPARISON

Work Pages S-145 Through S-150

Objectives

To extend the development of two-digit subtraction without borrowing.

To provide practice in working with two-digit addition and subtraction.

To give children experience with subtraction reconstruction problems.

To provide practice in determining the operation required in a given equation.

To compare numbers in the environment of two-digit addition and subtraction.

Fundamentals

In developing the final step of two-digit subtraction, the teacher will again refer to the work done in addition. In solving problems such as $36 + 22 = \square$, the child learned to add ones, add tens, and then add the two results.

Although the commutative and associative properties do not hold for subtraction, the child will find that he can subtract ones, subtract tens, and then add ones and tens to arrive at his answer. He will learn to think of $36 - 22$ as $(30 - 20) + (6 - 2)$, as $10 + 4$, and finally as 14. After working with sets, equations, and expanded notation, the algorism will be introduced.

The child will be given many opportunities to apply his understanding of two-digit addition and subtraction as he compares numbers, determines the operational sign needed in a given equation, and works reconstruction problems.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers; Order—Relations—Operations.

Readiness for Understanding

Understanding of addition and subtraction.

Understanding of the concept of expanded notation.

Ability to work with placeholder equations and vertical notation.

Awareness of the inverse relation between addition and subtraction.

Ability to determine the operation required in a given equation.

Ability to compare numbers.

Developmental Experiences

MATERIALS

- sticks
- 2 Countingmen
- pocket chart
- strips of colored paper
- rubber bands
- individual sets of plastic numerals
- triangles
- operation and relation symbols

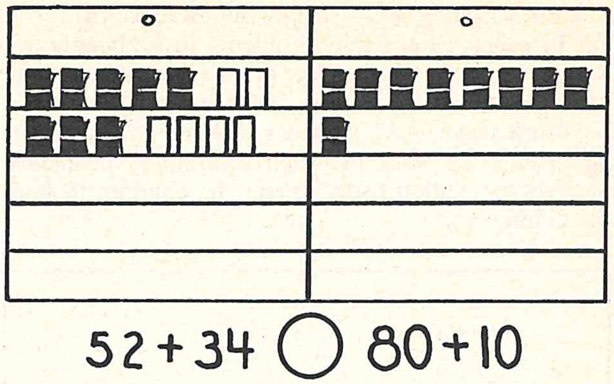
▶ Write the equation 68 − 42 = ☐ on the chalkboard. Have children place sixty-eight sticks (six bundles of ten and eight singles) on their desks. Have them remove forty-two sticks (four bundles of ten and two singles). Have the children tell how many ones and how many tens remain. Complete the placeholder equation and work the same problem using expanded notation in vertical form. Remind the children that the parentheses around the 40 + 2 will help them think of the ones and tens to be subtracted.

$$\begin{array}{r} 60 + 8 \\ -\ (40 + 2) \\ \hline 20 + 6 \end{array} = 26$$

Continue in this way to work similar problems. Have both the equation form and the expanded notation in vertical form written on the chalkboard and read.

▶ Demonstrate the problem 88 − 37 = ☐ by using the Countingmen. Tell a child to show the number eighty-eight on the Countingmen. Ask another child to remove thirty-seven: he should remove seven fingers from the eight ones and three fingers from the eight tens. Have one child tell the set story in terms of ones and tens and another child complete the equation. Write the related algorism form on the chalkboard. Have it read: "Eight minus seven equals one; eighty minus thirty equals fifty; fifty plus one equals fifty-one." Continue to use the Countingmen to develop other problems of this type. Have children record each problem in algorism form.

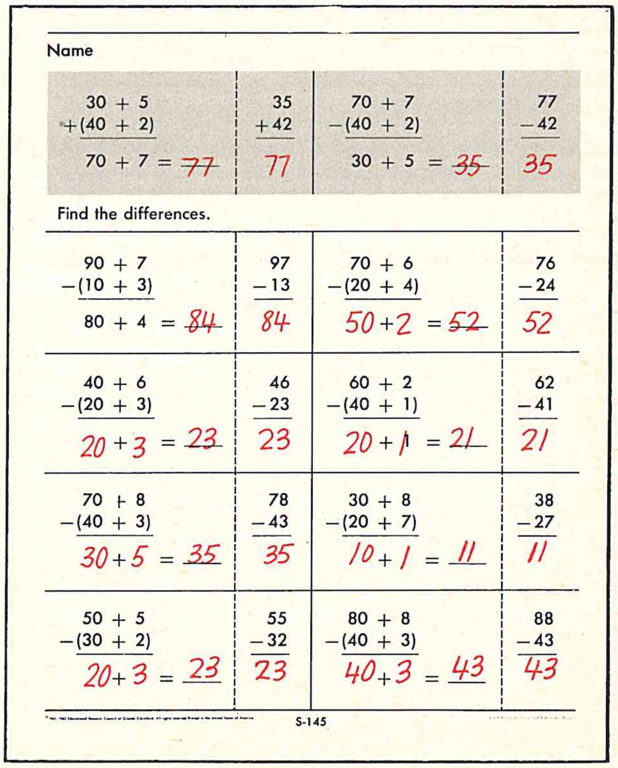
▶ Write an inequality such as 52 + 34 ○ 80 + 10 on the chalkboard. Have a child place corresponding sets of paper strips (bundles of ten and single strips) in the pocket chart as illustrated. Have the class compare the number of tens first. They will discover that there are more tens in the number represented on the right side of the chart than in the number represented on the left side. Ask if there are enough ones on the left to make another set of ten. Since there are not, comparison of the tens alone will determine which number is greater. Have the children check on the accuracy of their decision by adding the numbers represented on either side of the circle. Have the < symbol written in the circle.



Set up other problems of this type and have children quickly determine which number in the pair is greater. Have them explain their thinking as they make their decision. After several numbers have been compared with the aid of sets, have children compare numbers without this visual clue.

▶ Have the children use their plastic numerals, plus signs, minus signs, equal signs, and triangle to build equations requiring operational signs. Give directions to build such equations as 5 + 31 = 30 △ 6, or 77 − 7 = 60 △ 10. After the children have replaced the triangle with the correct operational sign, have the completed equation read and discussed.

▶ Use work page S-145 to help children make the transition from the expanded-notation form to the algorism for two-digit subtraction problems. Work the two problems at the top of the page with the children and have them work the remainder of the page on their own.



Name

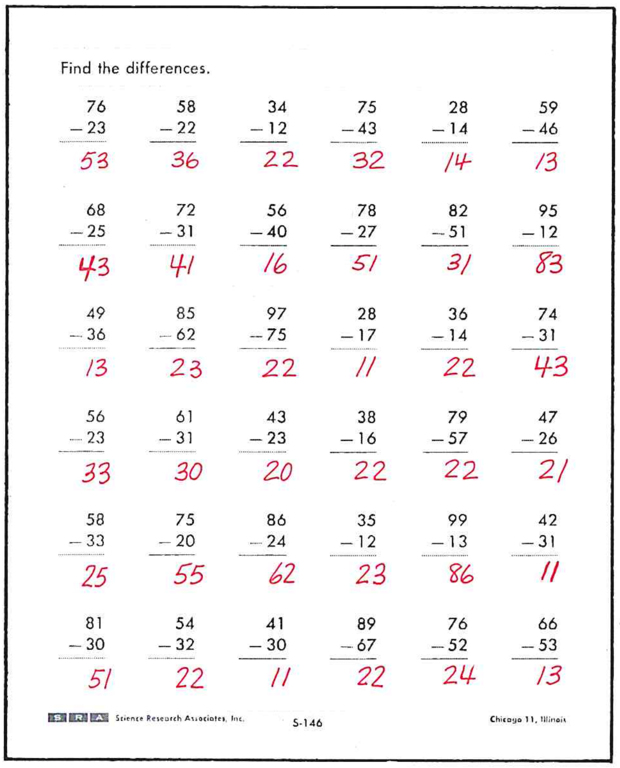
30 + 5 +(40 + 2) 70 + 7 = 77	35 +42 77	70 + 7 −(40 + 2) 30 + 5 = 35	77 −42 35

Find the differences.

90 + 7 −(10 + 3) 80 + 4 = 84	97 −13 84	70 + 6 −(20 + 4) 50 + 2 = 52	76 −24 52
40 + 6 −(20 + 3) 20 + 3 = 23	46 −23 23	60 + 2 −(40 + 1) 20 + 1 = 21	62 −41 21
70 + 8 −(40 + 3) 30 + 5 = 35	78 −43 35	30 + 8 −(20 + 7) 10 + 1 = 11	38 −27 11
50 + 5 −(30 + 2) 20 + 3 = 23	55 −32 23	80 + 8 −(40 + 3) 40 + 3 = 43	88 −43 43

S-145

Use work page S-146 to provide practice in working two-digit subtraction problems in vertical-notation form. Work the first few problems together, then have the children complete the page independently. ▶ Work page S-147 provides children with another opportunity to work two-digit subtraction problems in vertical-notation form. Direct the children to find the differences.



Find the differences.

76	58	34	75	28	59
−23	−22	−12	−43	−14	−46
53	36	22	32	14	13
68	72	56	78	82	95
−25	−31	−40	−27	−51	−12
43	41	16	51	31	83
49	85	97	28	36	74
−36	−62	−75	−17	−14	−31
13	23	22	11	22	43
56	61	43	38	79	47
−23	−31	−23	−16	−57	−26
33	30	20	22	22	21
58	75	86	35	99	42
−33	−20	−24	−12	−13	−31
25	55	62	23	86	11
81	54	41	89	76	66
−30	−32	−30	−67	−52	−53
51	22	11	22	24	13

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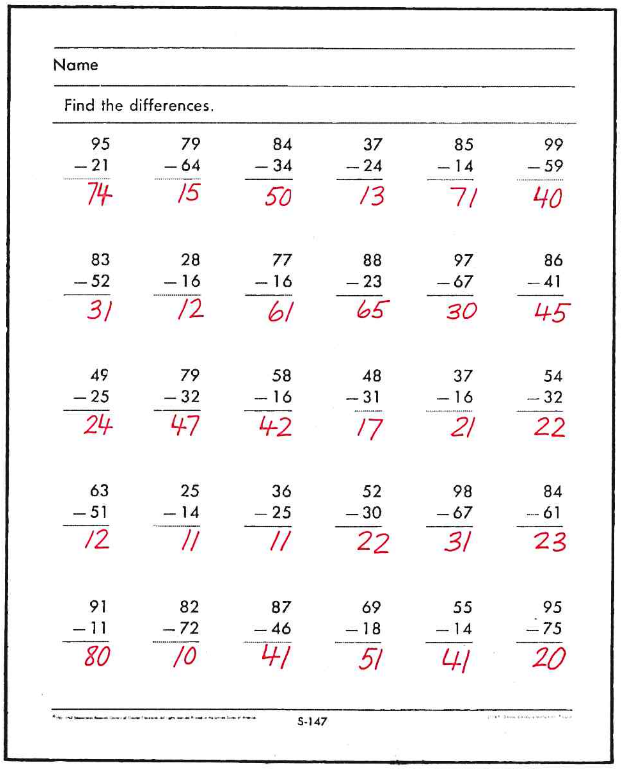


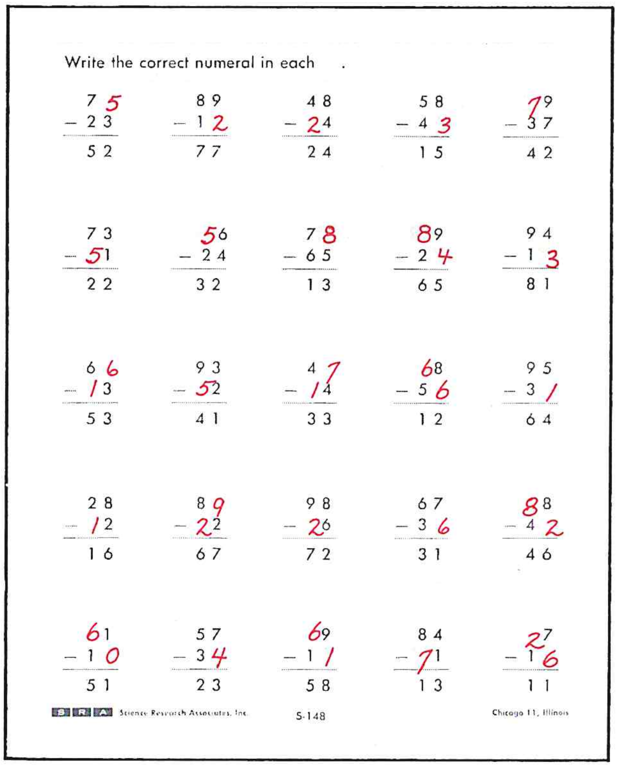
Name

Find the differences.

95	79	84	37	85	99
−21	−64	−34	−24	−14	−59
74	15	50	13	71	40
83	28	77	88	97	86
−52	−16	−16	−23	−67	−41
31	12	61	65	30	45
49	79	58	48	37	54
−25	−32	−16	−31	−16	−32
24	47	42	17	21	22
63	25	36	52	98	84
−51	−14	−25	−30	−67	−61
12	11	11	22	31	23
91	82	87	69	55	95
−11	−72	−46	−18	−14	−75
80	10	41	51	41	20

S-147

Use work page S-148 to provide practice in working reconstruction problems involving two-digit subtraction. Work the first problem on the page with the children. First have them determine the number of ones in the difference. Then have them determine the number of ones in the number which was subtracted. Ask, "From how many ones were the three ones subtracted to give two ones?" When the answer,



Write the correct numeral in each .

75	89	48	58	79
−23	−12	−24	−43	−37
52	77	24	15	42
73	56	78	89	94
−51	−24	−65	−24	−13
22	32	13	65	81
66	93	47	68	95
−13	−52	−14	−56	−31
53	41	33	12	64
28	89	98	67	88
−12	−22	−26	−36	−42
16	67	72	31	46
61	57	69	84	27
−10	−34	−11	−71	−16
51	23	58	13	11

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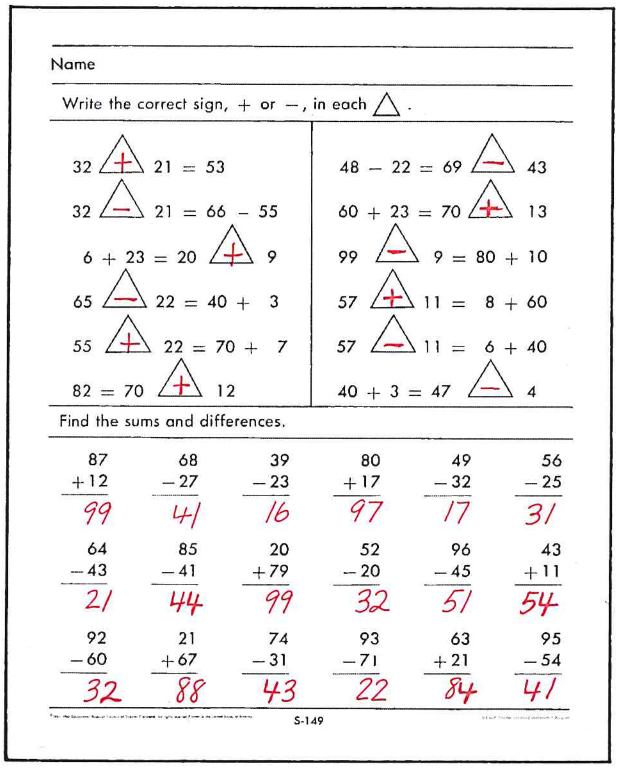


Name

Write the correct sign, + or −, in each △.

32 △+ 21 = 53
32 △− 21 = 66 − 55
6 + 23 = 20 △+ 9
65 △− 22 = 40 + 3
55 △+ 22 = 70 + 7
82 = 70 △+ 12

48 − 22 = 69 △− 43
60 + 23 = 70 △+ 13
99 △− 9 = 80 + 10
57 △+ 11 = 8 + 60
57 △− 11 = 6 + 40
40 + 3 = 47 △− 4

Find the sums and differences.

87	68	39	80	49	56
+12	−27	−23	+17	−32	−25
99	41	16	97	17	31
64	85	20	52	96	43
−43	−41	+79	−20	−45	+11
21	44	99	32	51	54
92	21	74	93	63	95
−60	+67	−31	−71	+21	−54
32	88	43	22	84	41

S-149

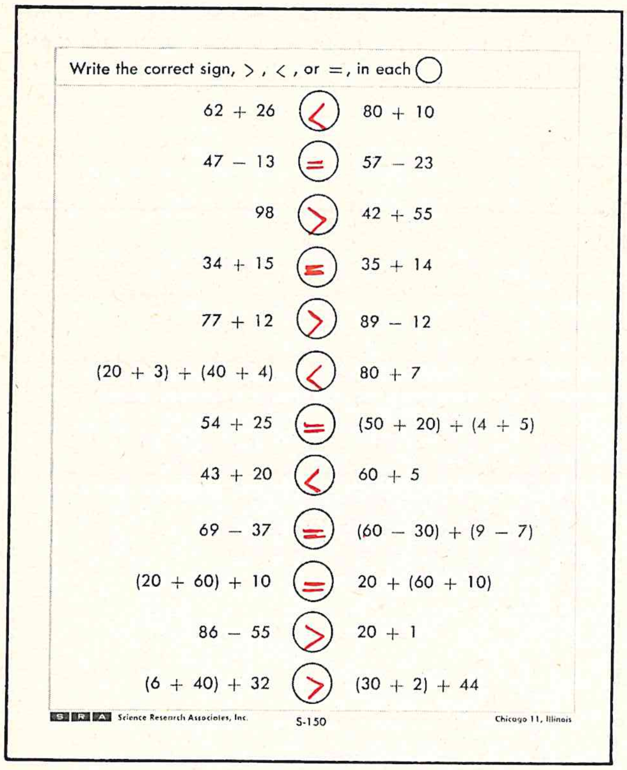
"Five," is given, have the children write the digit 5 in the color square. Work the first problem in the second row with the children; help them find the needed number of tens. If children wish to add to find the missing numbers in these subtraction problems, encourage them to do so.

Have the children complete the remainder of the page independently; then discuss each problem. Have them explain their thinking as they worked these problems.

▶ Use the top part of work page S-149 to give children an opportunity to select the operational sign to complete the equations correctly. Work the first equation with the group. Have them complete the remainder of these equations independently.

The lower section of work page S-149 provides a quick review of two-digit addition and subtraction. Tell the children to find the sums or differences. After directions have been given, have children complete this section of the work page on their own.

Work page S-150 offers children an opportunity to use their ability to reason. Encourage them to find answers, whenever possible, without computing; then to check their work by adding or subtracting. When work on this page is complete, discuss each problem with the class; have different children explain how they thought out their answer.



Write the correct sign, >, <, or =, in each ○

62 + 26 < 80 + 10

47 − 13 = 57 − 23

98 > 42 + 55

34 + 15 = 35 + 14

77 + 12 > 89 − 12

(20 + 3) + (40 + 4) < 80 + 7

54 + 25 = (50 + 20) + (4 + 5)

43 + 20 < 60 + 5

69 − 37 = (60 − 30) + (9 − 7)

(20 + 60) + 10 = 20 + (60 + 10)

86 − 55 > 20 + 1

(6 + 40) + 32 > (30 + 2) + 44

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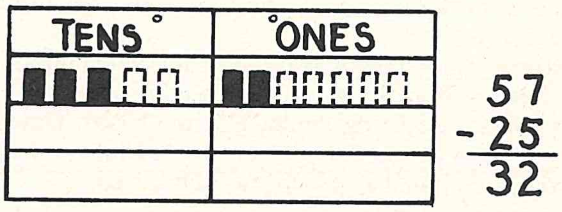
Supplemental Experiences

▶ Place a list of numbers in a vertical line on the chalkboard. Above and to either side of the numerals write +12 and −12. Call upon children to come to the chalkboard and write the sum and difference involved with each given number.

−12		+12
31	43	55
42	54	66
	65	
	76	
	87	

Repeat the exercise with different numerals in the center strip; add and subtract thirteen from each number represented.

▶ Place five bundles of ten paper strips and seven separate strips in a pocket chart which has been divided into two sections as illustrated. From these sets remove five single strips and two bundles of ten strips. Call upon a child to write the related number story in vertical form on the chalkboard and read the problem in terms of ones and tens.



Continue in this way to build set stories related to subtraction with two-digit numerals not involving borrowing. Have children write the corresponding algorism for each activity.

Story Problems

Work Pages S-151 and S-152

Objectives

To increase skill in solving story problems.

To build a foundation for solving more difficult addition and subtraction problems.

To develop the "think-through" procedure.

Fundamentals

In order to develop an adequate foundation for problem solving, regular review is needed. Much emphasis should be placed on the important concepts that addition is the operation that relates to joining sets or putting things together and subtraction relates to separating sets or taking things away. The inverse relation between addition and subtraction should be used for checking problems. Again, stress should be placed on adding or subtracting first in the ones place and then in the tens place.

Readiness for Understanding

Ability to interpret simple story problems.

An understanding of the inverse relation between addition and subtraction.

Ability to add and subtract numbers represented by two-digit numerals.

Developmental Experiences

MATERIALS

2 Countingmen

▶ Write several addition problems with two-digit addends, with no carrying, on the chalkboard. Have one child work out the first problem using two Countingmen, while a second child works the same problem on the board. Point out that the usual way

$$\begin{array}{r}43\\+\,24\\\hline\end{array}\qquad\begin{array}{r}65\\+\,34\\\hline\end{array}\qquad\begin{array}{r}56\\+\,32\\\hline\end{array}$$

to add is in the ones place first and then the tens place.

Have a child show a two-digit subtraction problem with no borrowing on the Countingmen while another child works the problem at the board. Have a third child check the problem by adding to the difference the number which was subtracted.

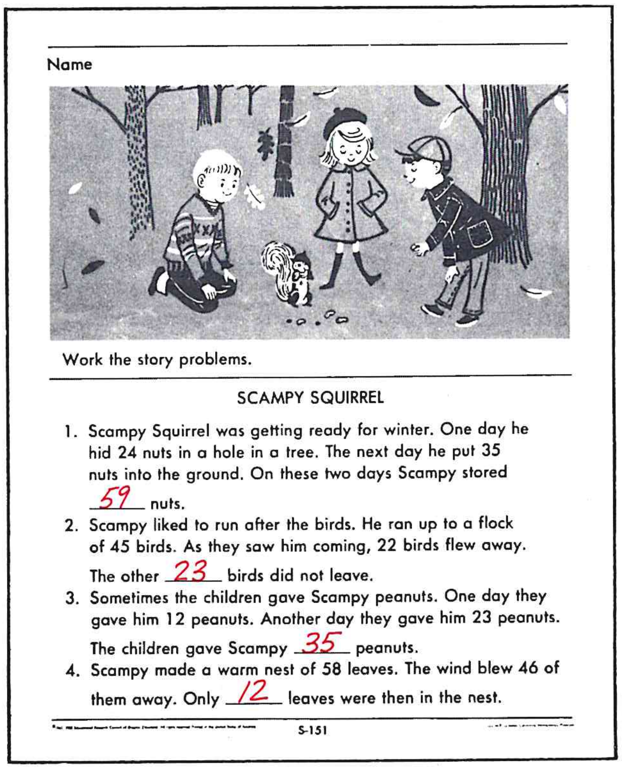
$$\begin{array}{r}76\\-\,23\\\hline 53\end{array}\qquad\begin{array}{r}53\\+\,23\\\hline 76\end{array}$$

▶ Use work page S-151 to give practice in two-digit addition and subtraction. Discuss the picture and the story title. Ask one child to read the first problem to the class. Have another child tell how he plans to solve the problem and explain why he chose the selected method. Ask the other children if they think this method will give the correct answer. Have them try it to see if they get a reasonable answer. Work through the second problem in the same way, having the children check this answer by adding. Have the children work the third and fourth problems independently. Then discuss these problems and have the children tell how they determined their answers.

Have the children work the problems on work page S-152 by themselves. Give individual help where needed. Some children may profit from using the Countingmen to find the correct answers.

Supplemental Experiences

▶ Play the game "I saw." The teacher begins by saying, "On the way to school this morning, I saw nine brown ______. (She may wave her arms to imitate birds' wings.) I saw two red ______, and three blue ______." Call on one child to guess what she saw and how many she saw all together. Then the child is leader and goes through the same procedure using different gestures to fit different things seen on the way to school—cars, bicycles, children, squirrels. If there is difficulty in remembering the numbers, a child can be the recorder and write each numeral on the board.



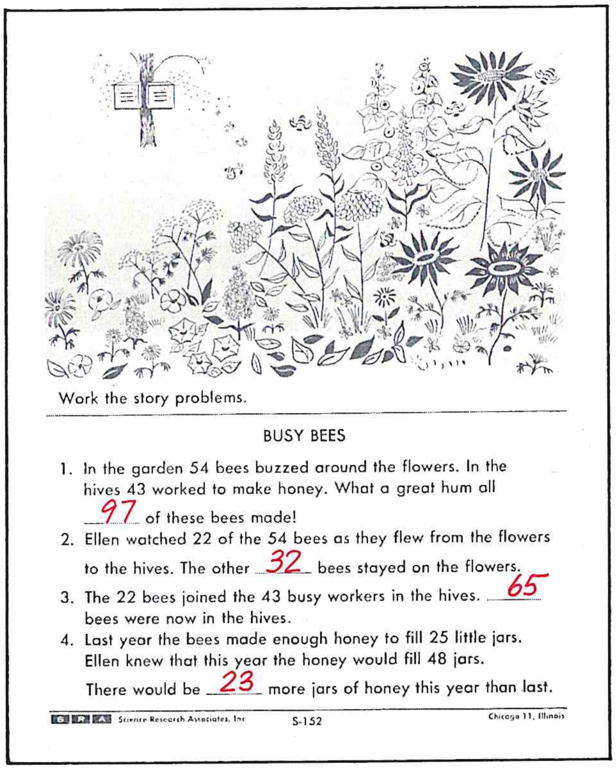
Name ______

Work the story problems.

SCAMPY SQUIRREL

1. Scampy Squirrel was getting ready for winter. One day he hid 24 nuts in a hole in a tree. The next day he put 35 nuts into the ground. On these two days Scampy stored 59 nuts.
2. Scampy liked to run after the birds. He ran up to a flock of 45 birds. As they saw him coming, 22 birds flew away. The other 23 birds did not leave.
3. Sometimes the children gave Scampy peanuts. One day they gave him 12 peanuts. Another day they gave him 23 peanuts. The children gave Scampy 35 peanuts.
4. Scampy made a warm nest of 58 leaves. The wind blew 46 of them away. Only 12 leaves were then in the nest.

S-151



Work the story problems.

BUSY BEES

1. In the garden 54 bees buzzed around the flowers. In the hives 43 worked to make honey. What a great hum all 97 of these bees made!
2. Ellen watched 22 of the 54 bees as they flew from the flowers to the hives. The other 32 bees stayed on the flowers.
3. The 22 bees joined the 43 busy workers in the hives. 65 bees were now in the hives.
4. Last year the bees made enough honey to fill 25 little jars. Ellen knew that this year the honey would fill 48 jars. There would be 23 more jars of honey this year than last.

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Time Calendar

Work Pages S-153 Through S-158

Objectives

To give practice in reading the calendar.

To reinforce the children's understanding of day, week, and month.

To introduce the year as a unit of time measurement.

Fundamentals

The child was introduced to the calendar in the first grade. He developed some understanding of the day, the week, and the month as units of time.

The second-grade teacher will help the child recall that there are seven days in each week, and that seven days measure the same amount of time as one week. She will review the names of the days and their order in the week. The child should learn that one month does not always measure the same amount of time as another month, and that a month does not always begin with the same day of the week. He should learn that the year consists of twelve months and that these months occur in a particular order.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

Familiarity with the concept of time.

Ability to use ordinal numerals, first through seventh.

Developmental Experiences

MATERIALS

- large demonstration clock
- large calendar

VOCABULARY

- day
- week
- month
- Sunday, Monday . . .
- today
- tomorrow
- yesterday
- date

▶ Have one of the children read the time on the classroom clock at the beginning of a school day. Have him go to the chalkboard and draw a clock face to show the time. Leave this clock on the board throughout the day. The next morning at the same time, call attention to this clock. Explain that the time that elapsed between the time school began *yesterday* morning and the time school began this morning is called a *day*.

Ask if anyone knows the name of *today*. Record the name of the day on a large piece of paper. Record the name of each successive day until the end of the school *week*. Ask the children to be prepared on *Monday* to tell how many days they did not go to school and to be able to name these days. On Monday morning record *Saturday* and *Sunday* on the chart. Continue in this manner until seven days have passed. Remind the children of the day they started the chart and explain that a week has passed since that time. Help the children count the names of the days and conclude that one week measures the same amount of time as seven days.

Now explain to the children that although any seven days can also be called a week, many people think of Sunday as the first day in the week. Write the names of the days in order on the chalkboard. Label each with a numeral, 1 through 7. Say, "If Sunday is the first day, which is the second day? . . . The fourth day? . . . The seventh day?" Ask them which day of the week today is, and then which day *tomorrow* will be. Have them give both the name and ordinal numeral. Help the children see that after Saturday, the next day is called Sunday and a new week begins.

▶ Call on the children's knowledge of the order of the days and the number of days in a week to develop an understanding of the *month* as a period of time. Write the names of the days in order on the chalkboard. Beneath each, write a numeral 1 through 7.

Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
1	2	3	4	5	6	7

Ask the children what day comes after Saturday. Write "Sunday" under the numeral 1 on the chalkboard and say, "Now we have begun a new week." Write the other names of the days of the week under the appropriate numerals. Say, "Since we have a new week, we'll have to give each day a new number, won't we? What number shall we give Sunday?" Encourage them to give the correct answer by counting through the last week again. Write the numeral 8 under the second Sunday, and proceed through the rest of the week.

Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
1	2	3	4	5	6	7
Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
8	9	10	11	12	13	14

Ask, "What is the name of the first day? . . . The fifth day? . . . The ninth day? . . . The twelfth day? What are the number names of the two Mondays? . . . The two Thursdays? . . . The two Saturdays?" Ask them what the name of the fifteenth day should be. Write, "Sunday" under the numeral 8. Write the rest of the names of the weekdays under the second row of numerals. Have one of the children fill in the correct numerals for this new week. Give them more practice in reading the numerals and the names of the days of the week.

Show them that the days in the column at the left are all Sundays, that the days in the next column are all Mondays, etc. Erase the "Monday" written above the numeral 9. Ask one of the children to tell you what the name of the ninth day is. Erase all of the names of the days except for the top row, and give the children practice in supplying the missing name for a given day.

Show the children a month on a real calendar, preferably one that begins on a Sunday, such as July, 1962, or September or December, 1963. Show them that it is the same as the one on the board for the first three weeks, and give them some practice in reading it.

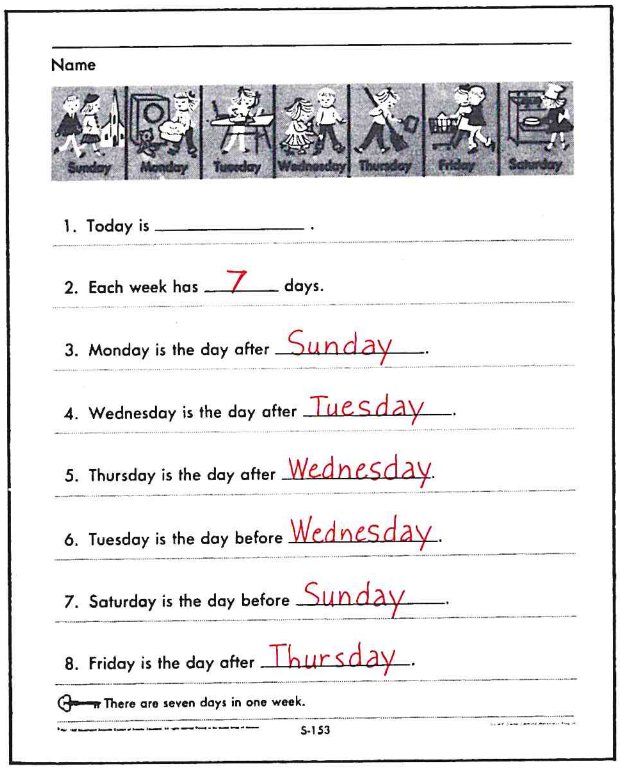
Explain that all the days on this calendar, taken together, are called a month. Explain that each numeral on the calendar is called a *"date."* Give them some practice in the use of this word. Ask them which day is the last day of this month and what day of the week it is. Ask them which day of the week it would be if a new month began after this month ended. Turn to the next calendar month and show them that this new month begins on the day after the last month ended. Have the children compare the number of days in the two months. Show other months and discuss the number of days in each.

Turn to the current month on the calendar and say, "This is the month we are in now." Show them which day today is. Ask one child what day of the week today is and what the date is. Ask another to point to the day on the calendar that was yesterday, and tell what day of the week that was and what the date was. Do the same with "tomorrow" and "the day after tomorrow."

▶ Help the children develop a feeling for how long a month is. Construct a large calendar, and record the weather every day for a month. At the end of each day, paste a picture characterizing the weather for that day—a yellow sun, a snowflake, a fluffy cloud, or a few rain drops. At the end of the month, discuss the month's weather. Ask the children how many sunny days there were, how many snowy days there were, and so forth.

On the first day of the next month, begin an "events" calendar. At the end of each day, have the children decide upon one event of the day to record in writing or pictures on the calendar. If one of the children is having a birthday, record this with a picture of a birthday cake and candles. Record holidays with an appropriate picture—a Christmas tree for Christmas, a heart for Valentine's day. At the end of the month, have one child recall for the class what the picture on the first day of the month stands for. Go through the rest of the month in order, having a different child describe the pictured event of each day.

▶ Use work page S-153 to reinforce the concepts of day and week. Discuss each picture and the name of the day which is written below the picture. Have

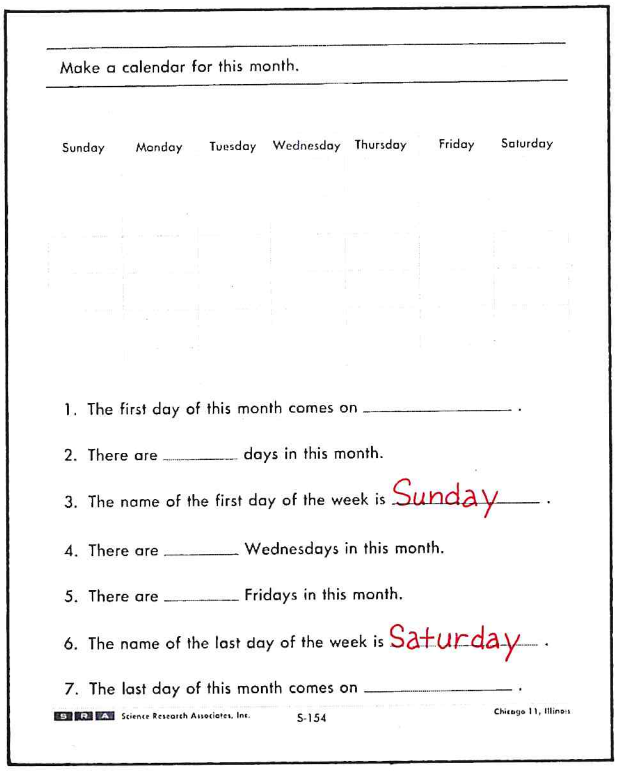

Sunday
Monday
Tuesday
Wednesday
Thursday
Friday
Saturday

Name

1. Today is ______________ .
2. Each week has 7 days.
3. Monday is the day after Sunday.
4. Wednesday is the day after Tuesday.
5. Thursday is the day after Wednesday.
6. Tuesday is the day before Wednesday.
7. Saturday is the day before Sunday.
8. Friday is the day after Thursday.

There are seven days in one week.

S-153



Make a calendar for this month.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday

1. The first day of this month comes on ______________ .
2. There are ________ days in this month.
3. The name of the first day of the week is Sunday .
4. There are ________ Wednesdays in this month.
5. There are ________ Fridays in this month.
6. The name of the last day of the week is Saturday .
7. The last day of this month comes on ______________ .

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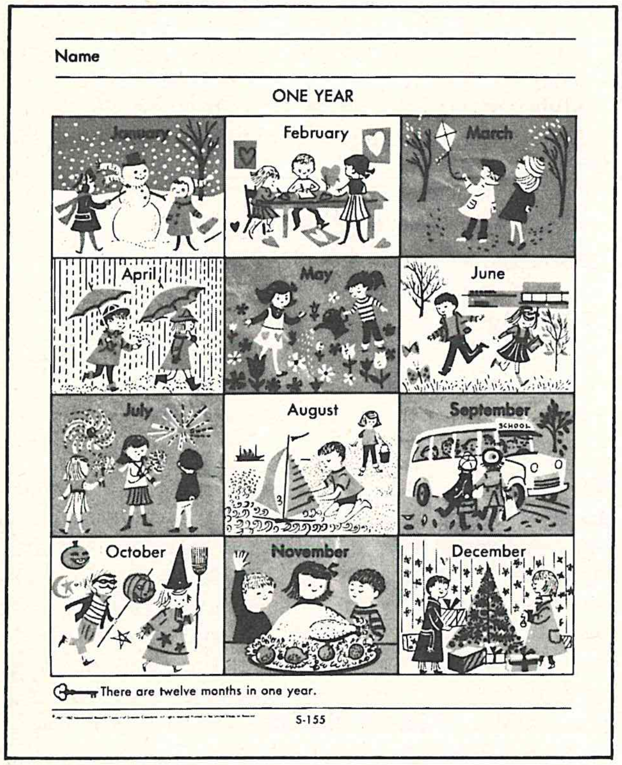
the children read the names of the days of the week in order. Then direct them to read each sentence and to fill in the blanks.

Use work page S-154 to reinforce the understanding of month and the order of the days in the month. Work with the children to help them make a calendar for the month and then fill in the blanks.

▶ Work page S-155 shows the twelve months in the year with appropriate activity pictures. Discuss the pictures with the children, and have them read the names of the months in order. There is no writing to be done on this page.

Work page S-156 shows the order of the months in the year and the days in the week. Read the page



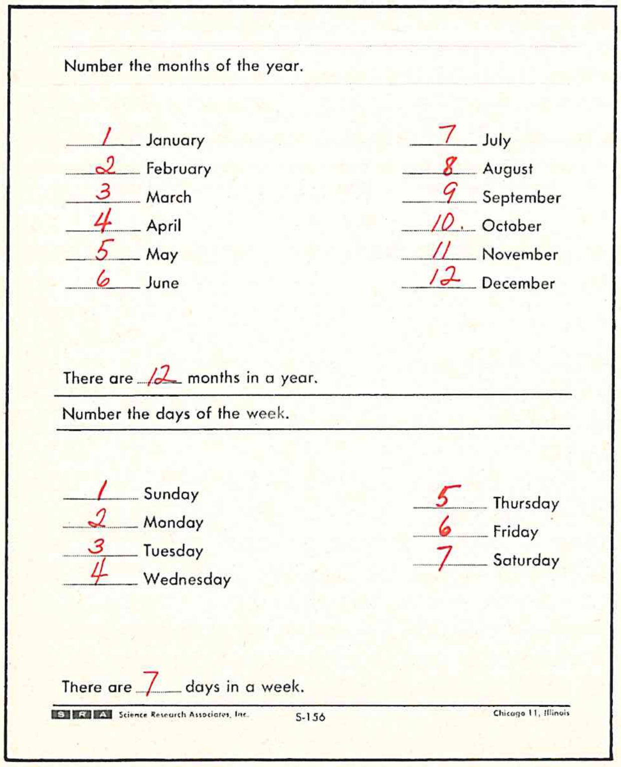
Name

ONE YEAR

January
February
March
April
May
June
July
August
September
October
November
December

There are twelve months in one year.

S-155



Number the months of the year.

1 January
2 February
3 March
4 April
5 May
6 June
7 July
8 August
9 September
10 October
11 November
12 December

There are 12 months in a year.

Number the days of the week.

1 Sunday
2 Monday
3 Tuesday
4 Wednesday
5 Thursday
6 Friday
7 Saturday

There are 7 days in a week.

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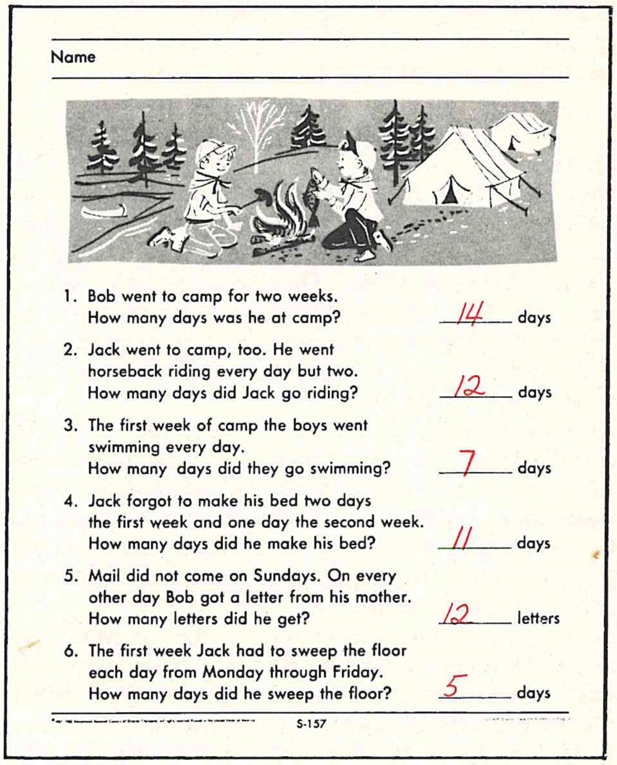
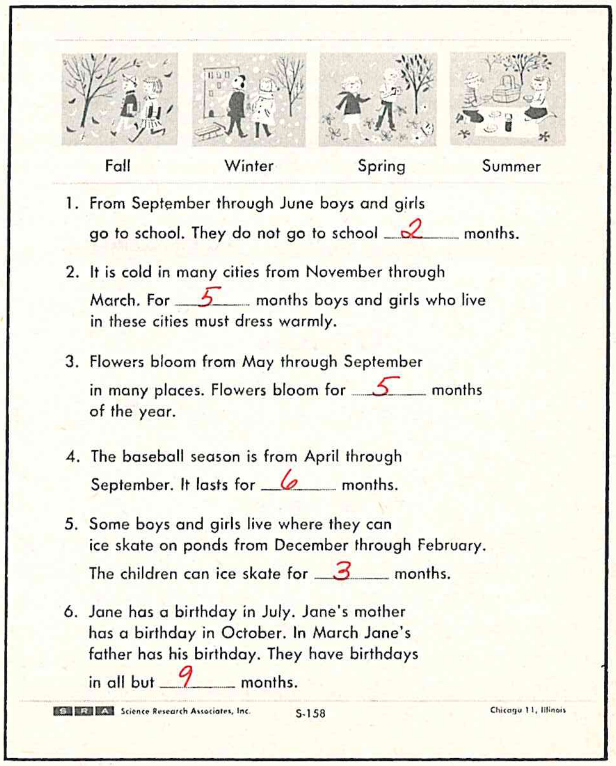


Name

1. Bob went to camp for two weeks. How many days was he at camp? 14 days
2. Jack went to camp, too. He went horseback riding every day but two. How many days did Jack go riding? 12 days
3. The first week of camp the boys went swimming every day. How many days did they go swimming? 7 days
4. Jack forgot to make his bed two days the first week and one day the second week. How many days did he make his bed? 11 days
5. Mail did not come on Sundays. On every other day Bob got a letter from his mother. How many letters did he get? 12 letters
6. The first week Jack had to sweep the floor each day from Monday through Friday. How many days did he sweep the floor? 5 days

S-157



Fall Winter Spring Summer

1. From September through June boys and girls go to school. They do not go to school 2 months.
2. It is cold in many cities from November through March. For 5 months boys and girls who live in these cities must dress warmly.
3. Flowers bloom from May through September in many places. Flowers bloom for 5 months of the year.
4. The baseball season is from April through September. It lasts for 6 months.
5. Some boys and girls live where they can ice skate on ponds from December through February. The children can ice skate for 3 months.
6. Jane has a birthday in July. Jane's mother has a birthday in October. In March Jane's father has his birthday. They have birthdays in all but 9 months.

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with the children. Direct them to write numerals to show the order of the days and months, and to tell how many days are in a week and how many months are in a year.

▶ Work page S-157 contains story problems relating to the number of days in two weeks. Read and discuss them with the children.

Work page S-158 shows the four seasons at the top of the page. The story problems relate to the number of months in a given period and call for a clear knowledge of the order of the months. Discuss each problem with the children.

Supplemental Experiences

▶ Construct a set of cards bearing the names of the days of the week. Distribute the cards to the children. Ask the child who has the card bearing the name of the first day of the week to come to the front of the room. Then ask the child whose card bears the name of the second day of the week to stand beside the first child. In this manner have the children arrange themselves to show the order of the days of the week.

▶ Construct a set of cards bearing the names of the months of the year. Distribute them to the children. Ask a question such as, "Which month comes next after April?" The child whose card bears the word "May" must rise and display his card. He then gives his card to another child and has a turn to ask the next question.

▶ Display a large calendar of the month. Ask questions such as: "What was the date of last Tuesday?" "How many Fridays are in this month?" "What will be the date of a week from yesterday?"

The child who answers correctly may then ask the next question and call upon another child to give the answer.

Time

Work Pages S-159 and S-160

Objectives

To review measuring time by the hour and the half-hour.

Fundamentals

Although the work pages that deal with telling time are arranged as a unit, many teachers will find it advantageous to split the unit and spread the work over the remainder of the year.

Before progressing to more difficult concepts, the child in the second grade should be given the opportunity to review time-telling to the hour and the half-hour. In the first grade, he learned the appearance of the clock with the hands on the hour.

He should also be aware of how the clock looks when the hands are on the half-hour; that the long hand has moved halfway round the face of the clock and that the short hand points halfway between two numerals.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

An understanding of the numbers one through twelve.

Ability to read the numerals 1 through 12.

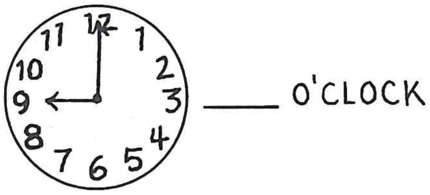
Ability to read whole hours and half-hours on the clock.

Developmental Experiences

MATERIALS	VOCABULARY
large demonstration clock	time clock o'clock hour half-hour half past

▶ Hold up a large demonstration *clock* that has movable hands. Discuss the clock, asking such questions as, "What do you see on the clock?" "Are the hands the same length?" "Do both hands move at the same *time?*" "Which hand moves faster?" Set the long hand of the clock on 12 and the short hand on 3, and review the fact that at the *hour* the long hand points to 12 and the short hand indicates the hour. Have a child tell the time shown by the clock as three *o'clock.*

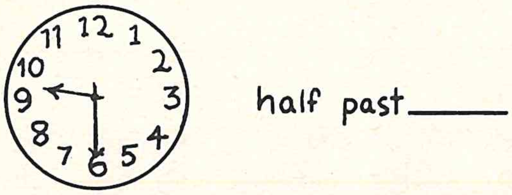
▶ Draw a large clock face showing nine o'clock on the chalkboard. Beside the clock write "____o'clock."



Have a child write the correct numeral on the line. Continue this activity by setting different hours on the clock and ask the children to tell the time as shown.

▶ When the class has had sufficient practice in the recognition of hours, review the *half-hour.* Set the hands of the demonstration clock to nine o'clock, and have a child read this time. Now move the long hand slowly round the clock to the numeral 6. Ask if anyone can tell the time now. Remind them that the short hand moved slowly as the long hand moved. The clock now reads *half past* the hour. The children should see that the long hand has gone half-way round the clock, and the short hand has moved slowly-as one half hour has passed to a point halfway to the next numeral.

On the chalkboard draw a large clock face showing half past nine. Beside the clock write "half past ___." Have a child read the time and write the correct numeral in the blank. Repeat with other half-hours.



The teacher can assure herself that the children understand this concept by varying the activity: Write a time such as three o'clock or half past seven on the chalkboard and have the child draw hands on the clock to indicate the time.

▶ Use work page S-159 to give the children practice in working with hours and half-hours. Discuss the pictures at the top of the page, and have the children read the time on the first clock and on the second clock. Ask them to compare the hands on these clocks to see how they differ. Now have the children read the time on each clock on the page and write the correct numeral on the line. The key at the bottom of the page will help them choose the correct time.

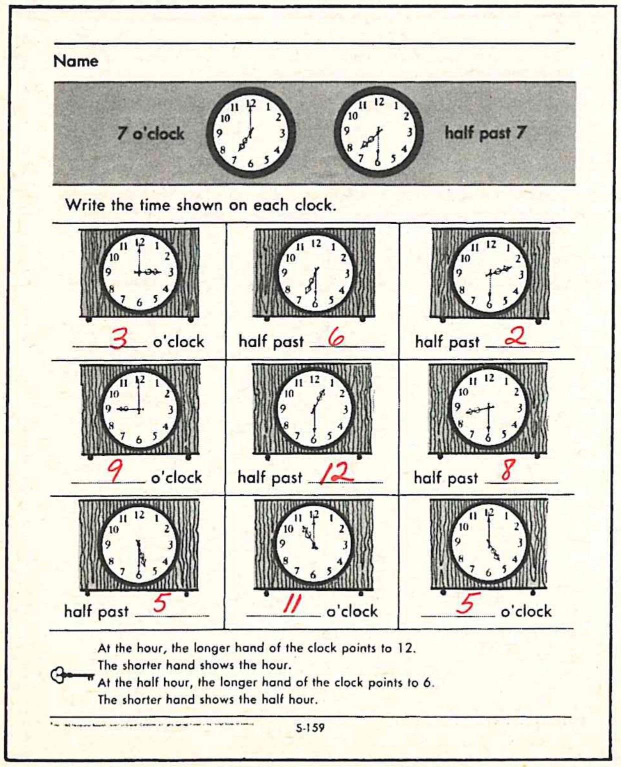
Work page S-160 gives the child an opportunity to show the correct time on the clock. Practice at the chalkboard in drawing hands on clocks should precede independent work on this page.

Supplemental Experiences

▶ During art period have each child make a model clock from a paper plate or cardboard circle. The hands of the clock can be cut from heavy cardboard and attached with a paper fastener or brad. Have the children write the numerals in the correct places and, if they wish, the face of the clock can be decorated.

▶ Construct a set of cards showing clock faces with the hands set to the hour and half-hour. Combine this set with a set of cards showing the form 1 o'clock through 12 o'clock and half past 1 through half past 12. Distribute the cards to the class; then ask a child who has a card with a clock face to stand and show his card. Ask the child whose card has the written form of this time to stand next to the child holding the clock card. The children then tell what time their cards indicate. To vary the activity begin with the written form.

▶ To play "Going to the Moon," divide the class into two teams. Have each team line up, one child behind the other, facing the front of the room. Put two clock models with movable hands on the chalk tray. At blast-off time, hold up a card labeled "half past 7." The first child in each row goes to the chalk tray and sets the hands of the clock to the indicated time. If they set the clock hands to the correct time, then they are "on the moon" and may step aside into an area designated as the moon. If a child does not set the time correctly, he must go back to earth, which is the end of his line, to await another turn. Continue the game until all the children from one team have reached the moon.



Name

7 o'clock

half past 7

Write the time shown on each clock.

3 o'clock

half past 6

half past 2

9 o'clock

half past 12

half past 8

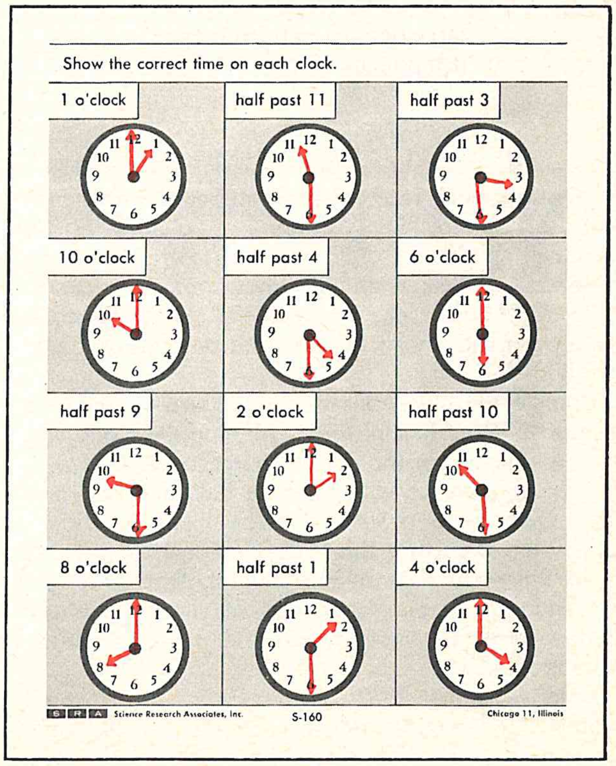
half past 5

11 o'clock

5 o'clock

At the hour, the longer hand of the clock points to 12.
The shorter hand shows the hour.
At the half hour, the longer hand of the clock points to 6.
The shorter hand shows the half hour.

S-159



Show the correct time on each clock.

1 o'clock

half past 11

half past 3

10 o'clock

half past 4

6 o'clock

half past 9

2 o'clock

half past 10

8 o'clock

half past 1

4 o'clock

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Time

Work Pages S-161 and S-162

Objectives

To review the use of numerals to denote the hour and the half-hour.

Fundamentals

In the first grade the child learned that there are two ways to denote the hour; for example, 3 o'clock and 3:00 are both symbols for three o'clock. The half-hour can also be indicated in two ways—half past 3 and 3:30 are both symbols for three-thirty. The numeral notation indicates the number of minutes past the hour and perhaps its greatest value is when there is need to denote such intervals of time as 5:05, 5:25, 7:35.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

Ability to read the numerals 1 through 12.

Ability to read hours and half-hours.

Ability to denote the time using the words "o'clock" and "half past the hour."

Ability to denote the time using numerals.

Developmental Experiences

MATERIALS

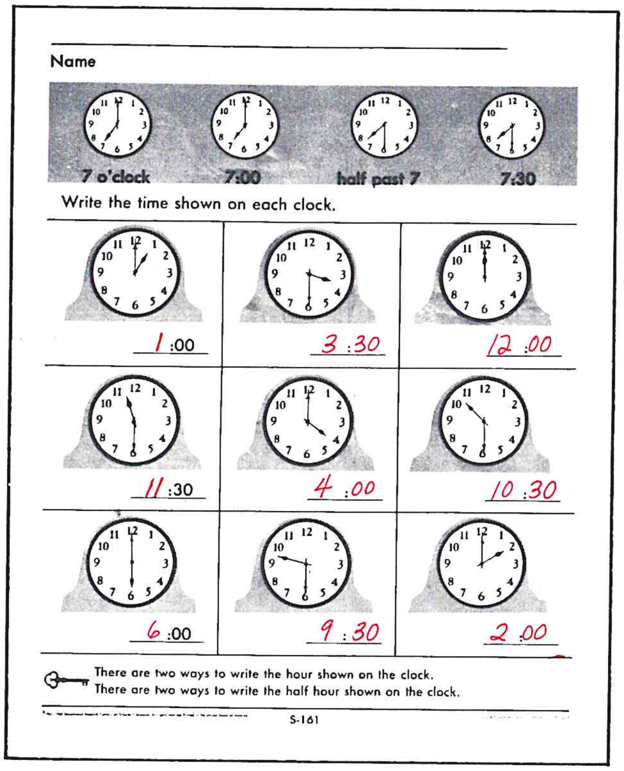
large demonstration clock
fish pole made of:
  yardstick
  string
  spring-type clothespin

▶ Draw a clock face on the chalkboard with hands indicating six o'clock. Write ______ o'clock beside the clock and have a child write the correct numeral in the blank. Ask if anyone knows another way to write this. Under the "6 o'clock," have someone write 6:00 and review the fact that both are symbols for six o'clock.

Change the clock on the chalkboard to indicate six-thirty. Write half past _____ beside the clock and have a child fill in the correct numeral. Ask if anyone knows another way to write half past six, and have them write 6:30 under the "half past six." Review the fact that this numerical symbol is read as six-thirty. Explain to the children that there are sixty minutes in one hour, and see if you can get them to discover that in a half-hour there are thirty minutes.

Emphasize that thirty minutes past the hour is expressed by the numerals ____:30 and that the minute hand has moved halfway round the clock.

▶ Write a time such as 6:00 on the chalkboard. Ask a child to set the hands on the demonstration clock to indicate this hour. Repeat with other hours and half-hours: 3 o'clock, 7:30, and half past 8. Have the children at their seats use their individual clocks to denote the time.



Name

7 o'clock   7:00   half past 7   7:30

Write the time shown on each clock.

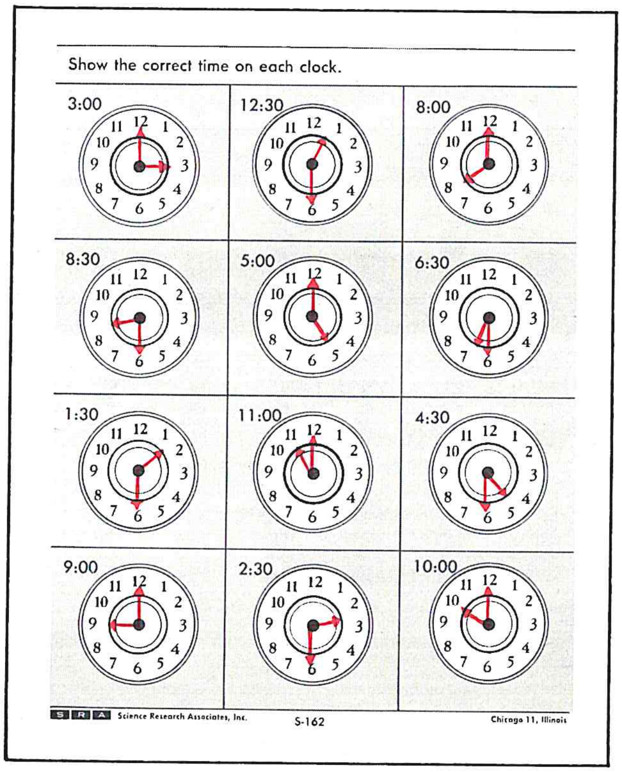
1 :00   3 :30   12 :00

11 :30   4 :00   10 :30

6 :00   9 :30   2 :00

There are two ways to write the hour shown on the clock.
There are two ways to write the half hour shown on the clock.

S-161



Show the correct time on each clock.

3:00   12:30   8:00

8:30   5:00   6:30

1:30   11:00   4:30

9:00   2:30   10:00

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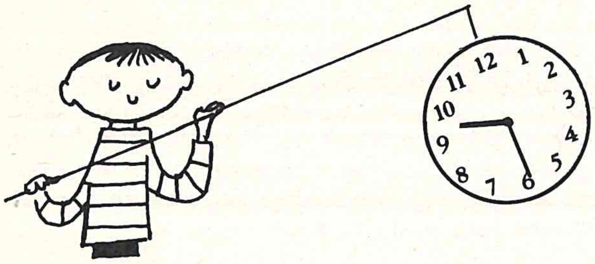
▶ Work page S-161 provides practice for the class in indicating the hour and half-hour in numeral notation. Discuss the two ways of denoting the hour and the half-hour as indicated at the top of the page. Have the children work independently to complete the page.

The child has an opportunity on work page S-162 to show the indicated time on the clock face. Remind the class that the long hand of the clock points to 12 at the hour; the short hand indicates the hour. At the half-hour the long hand of the clock points to 6; the short hand indicates the half-hour.

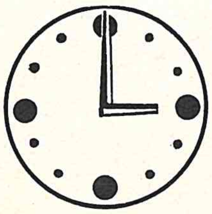
Supplemental Experiences

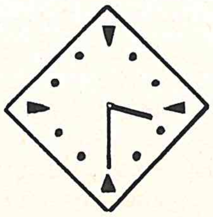
▶ Construct a set of cards indicating the hours 1:00 through 12:00 and the half-hours 1:30 through 12:30. Combine this set with cards that carry the "o'clock" and "half past" notation and distribute them to the class. Then display one clock-face card at a time. The two children who have cards that denote the same time as the clock must quickly rise and call out the hour or half-hour. The first child who answers correctly may display the next clock-face card.

▶ Use a fish pond game to provide practice for the class in writing and recognizing time notation. This fish pond is operated by one child, and the other children take turns being a fisherman. The fisherman writes an hour or half-hour in numeral notation (1:00, 9:30) on the chalkboard. When the fisherman lowers his pole into the fish pond, the operator of the pond attaches the clock-face card that matches the hour denoted on the board. The operator keeps his job as long as he selects the correct card; but the fisherman becomes the operator if an incorrect card is attached to his pole. It is possible to have several fishermen and fish ponds in action at the same time.



▶ The children may find it challenging to read the hours or half-hours from modern clock faces on which the numerals are omitted.





Time

Work Pages S-163 and S-164

Objectives

To develop an understanding of the passing of time.

To review the concept of an earlier and later hour.

To extend the concept of the interval between two hours of the day.

Fundamentals

The child has become aware of how the hour and half-hour are indicated by the clock, and has some idea of the amount of time involved in an hour and a half-hour. He has been introduced to the concept of an earlier and a later hour in the first grade, and has had experience in comparing two hours of the day whose difference was a whole number of hours. In the second grade the child will be asked to compare two hours of the day which differ by half-hours as well as by hours.

See Key Topics in Mathematics for the Primary Teacher: Measurement.

Readiness for Understanding

Ability to read the time on a clock to the hour and half-hour.

An understanding of the concept of an earlier and later hours.

Developmental Experiences

MATERIALS	VOCABULARY
large demonstration clock	earlier later
individual clock models	
set of clock-face cards	

▶ Draw two clock faces on the chalkboard with hands on the first clock face indicating eight o'clock, and then tell a story similar to this:

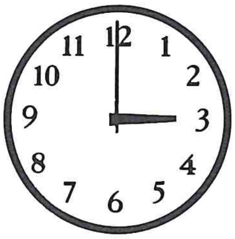
> John left home for school this morning at 8:00. His little sister, Judy, left for nursery school one hour later. What time did Judy leave for nursery school?

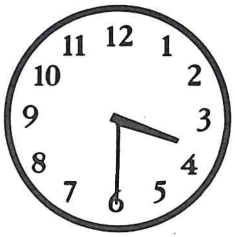
Ask a child to draw the hands on the second clock indicating nine o'clock. Now have the class compare the two clocks as you point out that nine o'clock is one hour *later* than eight o'clock. Also point out that eight o'clock is one hour *earlier* than nine o'clock.

Use other story situations to continue this activity, and have the children at their desks set their individual clocks one, two, or three hours earlier or later as indicated in each story.

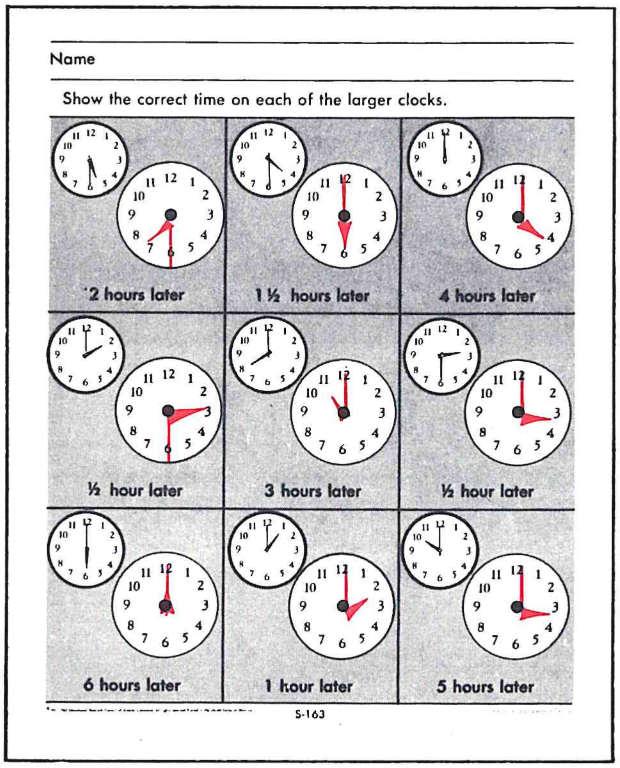
▶ Set the demonstration clock to three o'clock, and tell a story similar to the following story:

> Bill went to the school library at 3:00. His friend, Jim, went to the library a half-hour later. What time did Jim go to the library?





Have a child move the long hand of the demonstration clock to the correct position. The clock should show 3:30. Continue the activity, specifying a time in each story that is a half-hour earlier or later than the time first indicated by the demonstration clock. Include directions, such as: "Show the time which is one half-hour earlier than three o'clock," or "Show the time which is one half-hour later than 9:30."



Name

Show the correct time on each of the larger clocks.

2 hours later

1 ½ hours later

4 hours later

½ hour later

3 hours later

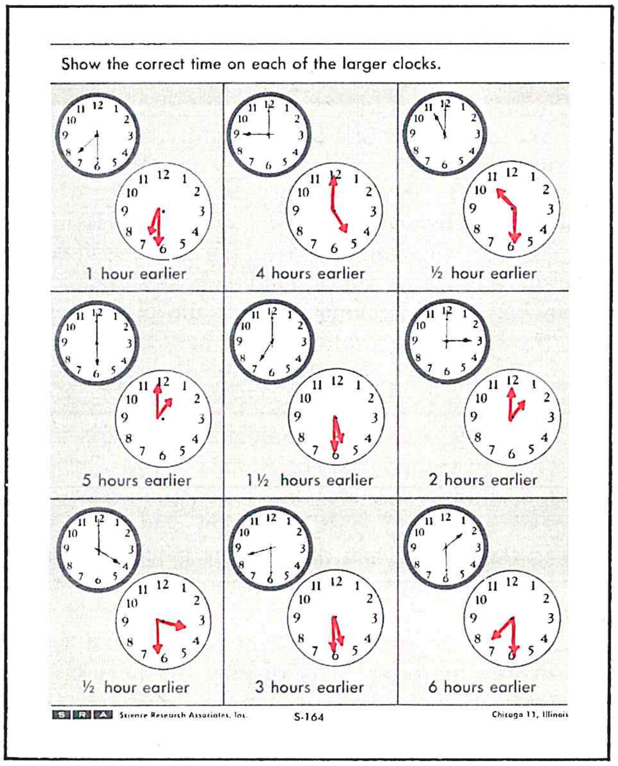
½ hour later

6 hours later

1 hour later

5 hours later

S-163



Show the correct time on each of the larger clocks.

1 hour earlier

4 hours earlier

½ hour earlier

5 hours earlier

1 ½ hours earlier

2 hours earlier

½ hour earlier

3 hours earlier

6 hours earlier

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▶ Use work page S-163 to provide practice for the children in indicating time which is a whole hour or half-hour later than the hour indicated by the clock. Have the children read the time indicated by the smaller clock and then draw hands on the larger clock to show the later hour that is indicated. Work the first three examples with the children, then encourage them to complete the page by working independently.

Use work page S-164 to reinforce the concept of time which is earlier than a given time. Direct the children to look at the smaller clock, then draw hands on the larger clock to indicate the hour which is earlier by the amount of time indicated. Work the first example with the children and have them complete the page on their own.

Supplemental Experiences

▶ Distribute a set of cards bearing clock faces. Ask a child to display his card, then ask the child who has a clock-card indicating the time that is three hours later to stand and tell what time his card shows. Have another child whose clock-card indicates the hour which is three hours earlier than the first clock, stand and tell what time his clock shows. Give the class time to make comparisons between the earlier and later hours shown on the clock-cards. Repeat the activity; call for time intervals that are half an hour earlier or later, as well as for time intervals that are hours earlier or later.

Time

Work Pages S-165 and S-166

Objectives

To teach children how to tell time to the quarter-hour.

Fundamentals

In teaching a child how to tell time to the quarter-hour, the teacher will call on the child's knowledge of fractional numbers. He is already aware that at half past an hour the long hand has moved halfway around the clock, while the short hand has moved halfway from one numeral to the next. In his first experiences with quarter past the hour, he will discover that the long hand moves a quarter of the way past the hour; and the short hand moves one-quarter, or one-fourth, of the way between two numerals. The child will also learn that when the clock indicates the interval of time as quarter to the hour, then the long hand has moved three-quarters of the way past the hour. He will understand that it is easier to express this time as a quarter to the next hour rather than as three-quarters past the last hour.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

An understanding of the fractions one-quarter, two-quarters or one-half, and three-quarters.

Ability to read the clock to the hour and half-hour.

Developmental Experiences

MATERIALS	VOCABULARY
large demonstration clock	quarter past
individual clock models	quarter to

▶ Use the demonstration clock to show an hour, perhaps six o'clock. Then move the long hand slowly around the clock to show the passing of one hour. As the long hand points to six-thirty, pause and discuss the half-hour (6:30), stressing that it indicates that thirty minutes have passed. Point out that this is halfway between six o'clock and seven o'clock. Move the hands ahead to seven o'clock and stress that there is a half-hour elapsing between six-thirty and seven o'clock.

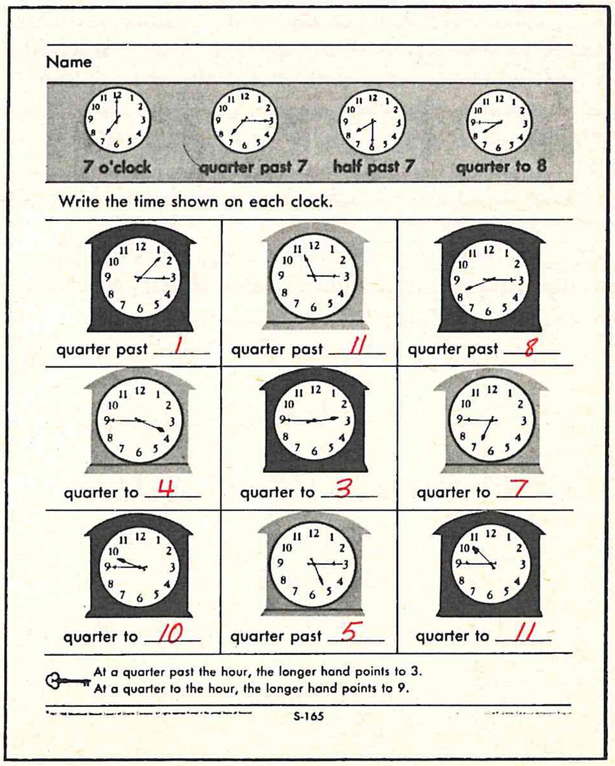
Now indicate six o'clock again. This time, as the long hand proceeds around the clock, stop the long hand on the three. Have the children read and discuss the time that is indicated. Help them understand that the long hand has moved one-fourth, or one-quarter, of the way around the clock, and that the short hand has moved one-quarter of the way between the 6 and 7. Explain that at a *quarter past* the hour, the long hand points to the numeral 3.

Continue the progress of the hour hand to the half past six mark. Point out that the long hand has moved halfway, or two-quarters of the way, around the clock; that the short hand has moved halfway, or two quarters of the way, between the 6 and 7.

Continue the progress of the hour hand to the *quarter to* seven mark. Talk about the fact that while the long hand has moved three-quarters of the way around the clock, it still has to move one-quarter of the way in order to show the next hour. Have the class read the time as a quarter to seven. Tell the children that it is usually spoken of as "a quarter to" the next hour rather than as "three quarters past" the last hour. Lead the discussion in such a way as to have them generalize that at a quarter to the hour, the long hand is at the 9.

▶ Have the children show designated times on their individual clocks. Include time on the hour, half-hour, quarter past the hour, quarter to the hour. Then call on children to tell what time is indicated by their clocks.

▶ Use work page S-165 to give the children practice in the recognition of quarter past the hour and quarter to the hour. Discuss the shading in the first four clocks on the page. The shaded area in each clock indicates the part of the hour that has passed when the time is quarter past 7, half past 7, or quarter to 8. Have the children write the time shown by each clock.



Name

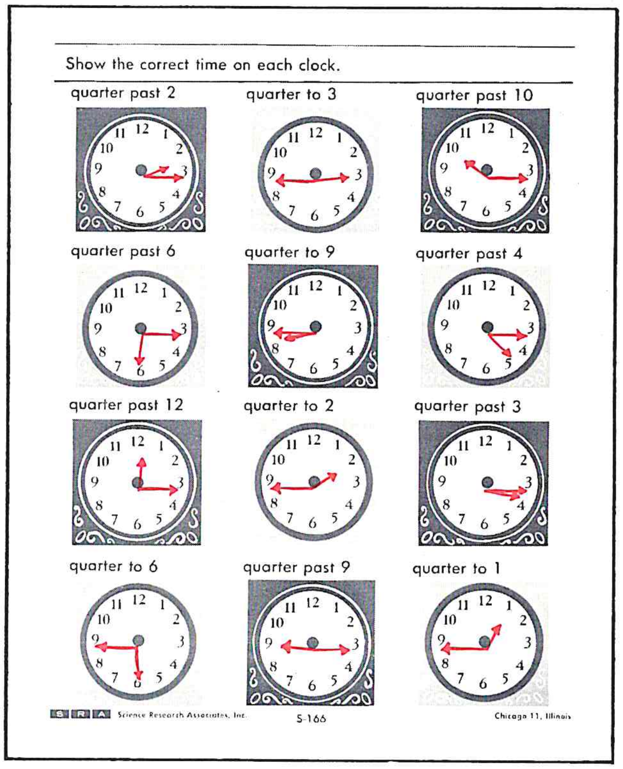
Write the time shown on each clock.

quarter past 1
quarter past 11
quarter past 8
quarter to 4
quarter to 3
quarter to 7
quarter to 10
quarter past 5
quarter to 11

At a quarter past the hour, the longer hand points to 3.
At a quarter to the hour, the longer hand points to 9.

S-165

On work page S-166 have the children show the correct positions of the hands. Using the demonstration clock, review the positions of the long hand and the short hand which indicate quarter past and quarter to the hour.



Show the correct time on each clock.

quarter past 2	quarter to 3	quarter past 10
quarter past 6	quarter to 9	quarter past 4
quarter past 12	quarter to 2	quarter past 3
quarter to 6	quarter past 9	quarter to 1

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Supplemental Experiences

▶ Select two teams, and have each child hold his model clock. The teacher calls out a time, such as quarter to four or quarter past six. The first child on each team sets his clock at the designated time. The first child who sets his clock correctly earns a point for his team. Continue to designate the time until the rest of the children on the teams have had a chance to set their clocks, or until a given interval of time has gone by.

You can vary this activity by using cards designating time (quarter past eight, quarter to ten) to direct the team members in setting their clocks.

Time

Work Pages S-167 and S-168

Objectives

To teach the children that there are sixty minutes in one hour.

To teach the children that five minutes have passed when the long hand of the clock moves from one numeral to the next.

Fundamentals

The child is introduced now to the fact that there are sixty minutes in one hour. He is also introduced to the fact that when the hour hand moves from one numeral to the next numeral, then five minutes have passed. The space between any two numerals on the clock indicates the passing of five minutes.

Readiness for Understanding

Ability to count by fives to sixty.

Ability to tell time by the hour, half-hour, and quarter-hour.

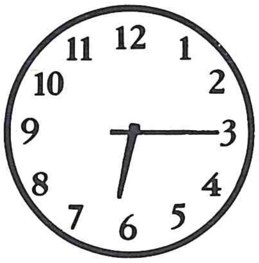
Developmental Experiences

MATERIALS

large demonstration clock
numeral cards 1 through 12

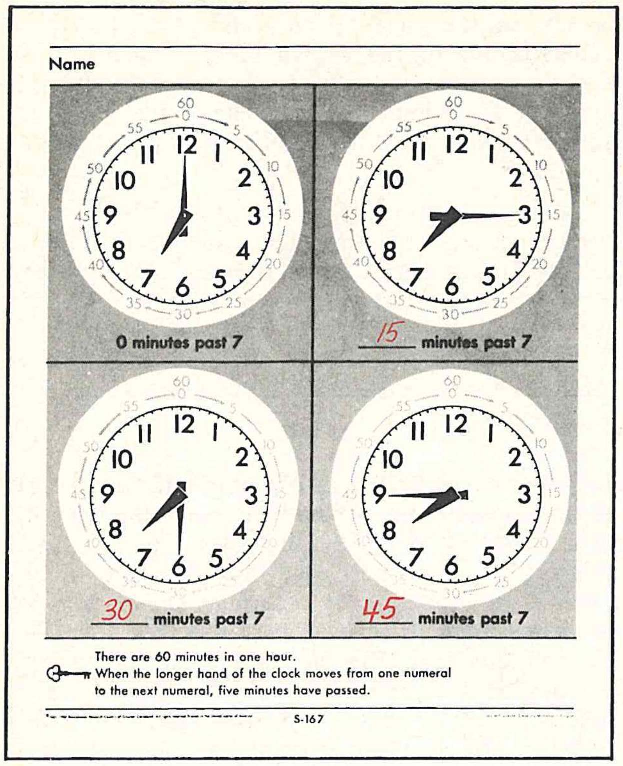
▶ Use the demonstration clock to point out the minute marks to the children. Have a child count the marks to discover that there are sixty. Emphasize that this shows that each hour has sixty minutes. Now have another child count the minute spaces between the numerals 12 and 1. He will find five minute spaces. Now have others count the minute spaces between any two consecutive numerals on the face of the clock. The children discover in doing so, that counting the minute marks one by one takes longer than counting the minutes by fives. They will observe that the same number of minutes is counted by either method.

Set the demonstration clock to indicate six o'clock. Have a child read the time shown by the clock. Help the class to see that another way to say this would be zero minutes past six. Now set the clock to show 6:15; have someone read the time on the clock. Then help the children to see that this is fifteen minutes past six. Write "____ minutes past 6" beside the clock. Continue the activity with other indications of the hour, half-hour, and quarter-hour.



____minutes past 6

▶ Use work pages S-167 and S-168 to provide the children with more experience in counting the five-minute intervals past the hour. Have them write the number of minutes past the hour indicated by each clock.


Name

0 minutes past 7

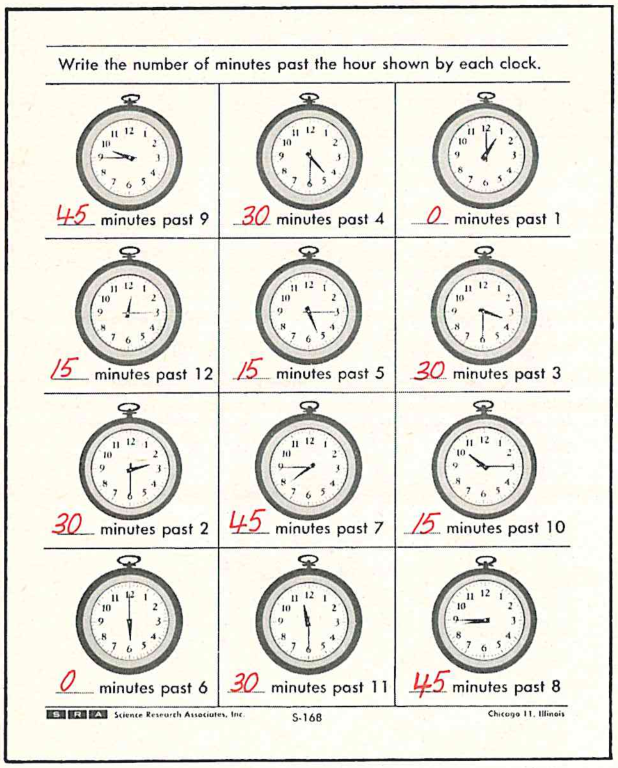
15 minutes past 7

30 minutes past 7

45 minutes past 7

There are 60 minutes in one hour.
When the longer hand of the clock moves from one numeral to the next numeral, five minutes have passed.

S-167


Write the number of minutes past the hour shown by each clock.

45 minutes past 9
30 minutes past 4
0 minutes past 1
15 minutes past 12
15 minutes past 5
30 minutes past 3
30 minutes past 2
45 minutes past 7
15 minutes past 10
0 minutes past 6
30 minutes past 11
45 minutes past 8

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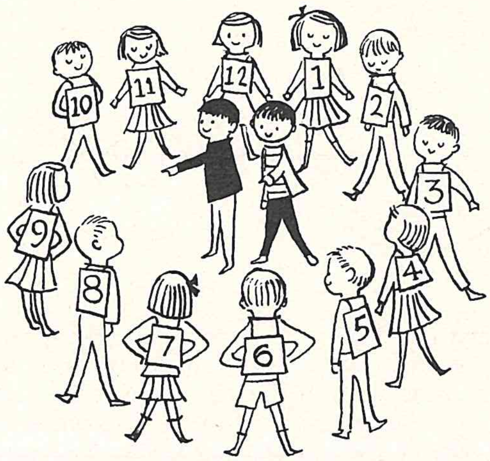
Supplemental Experiences

▶ Use a "living" clock to portray the movement of clock hands. Twelve children, in ring formation, represent the numerals on the clock, with each child wearing a numeral card. Two children stand in the center of the ring; one is the hour-man and the other is the minute-man. When a child holding a numeral card names a time, the hour- and minute-men must point to the correct numerals. If the caller says "forty-five minutes past six," the hour-man extends his arm between the 6 numeral and the 7 numeral, the minute-man counts by fives from the numeral 12 and points to the child wearing the numeral 9. The children take turns acting as the hour- and minute-man.



Time

Work Pages S-169 Through S-172

Objectives

To teach the children that there are two ways of writing quarter past the hour.

To teach the children that there are two ways of writing a quarter to the hour.

Fundamentals

The child is familiar with the written form of quarter to the hour and quarter past the hour. And since he has been introduced to the concept of an hour having sixty minutes, it will not be too difficult for the child to see that quarter past six can also be written as 6:15, and quarter to seven as 6:45. Counting by fives he can see that a quarter past the hour is fifteen minutes past the hour, and that a quarter to the hour is forty-five minutes past the hour.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

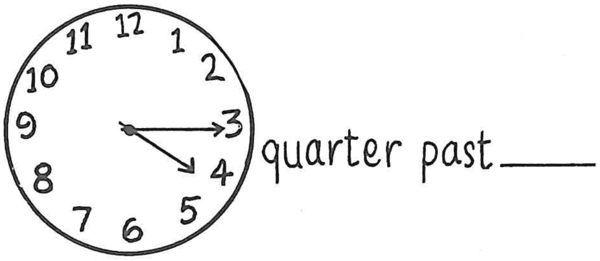
Ability to count by fives.
Ability to tell time on the quarter hour.

Developmental Experiences

MATERIALS

large demonstration clock
individual clock models
set of clock-face cards
cards showing numerical notation of time

▶ Draw a clock face on the chalkboard. Set the hands to show quarter past four. Write "quarter past_____" beside the clock. Have a child read the time and fill in the correct numeral. Ask if anyone knows another way to write this time. After a discussion write 4:15 below the other form. Let the children move the hands of the demonstration clock and count by fives to see that fifteen minutes have passed since four o'clock.



Continue this activity to show the class that quarter to five is written as 4:45. Have the minutes past the hour counted by fives so the children can see that forty-five minutes have passed since four o'clock.

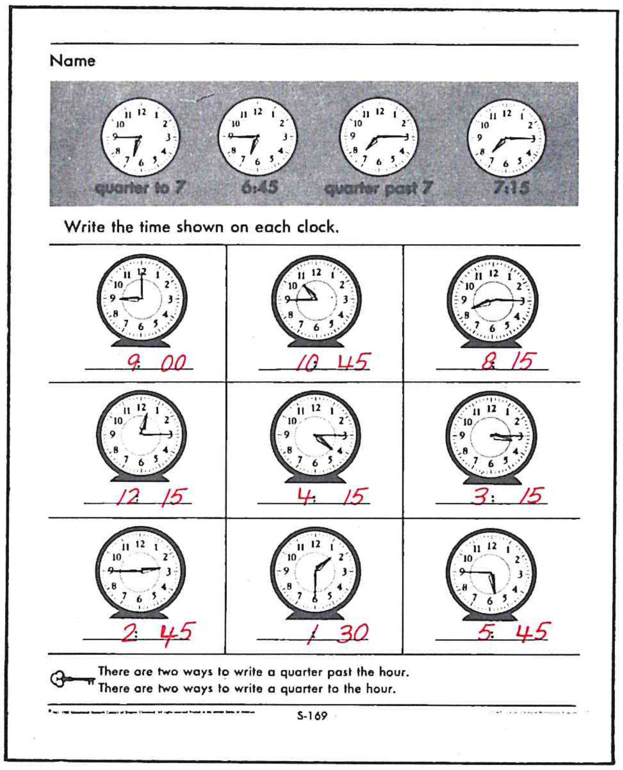
▶ Use work page S-169 to reinforce the children's understanding of the written form of quarter past the hour and quarter to the hour. Direct the children to count the minutes past the hour and write the numeral notation for the time shown by each clock.

Before using work page S-170, the children will need experience in showing the positions of the hands at the quarter-hours. Cards showing the numerical form to represent the quarter-hours can be used with the demonstration clock or the individual clocks. When using the work page, direct the children to draw the hands on the clock to show the correct time as indicated.

▶ Work pages S-171 and S-172 provide experience in working problems which will strengthen the children's concept of earlier and later times. In each problem the child must write the time that is indicated by the clocks, and determine the number of minutes or hours that have passed between the earlier and later time. This necessitates the counting of minutes between the first and second notations of time on the two clocks. A demonstration clock will prove helpful in showing how many minutes have passed between the time indicated first and then second.

Supplemental Experiences

▶ Provide additional cards showing the numerical notation form (9:15, 9:45) of the quarter-hour. Have the children match clock-face cards with the numerical notation cards.



Name

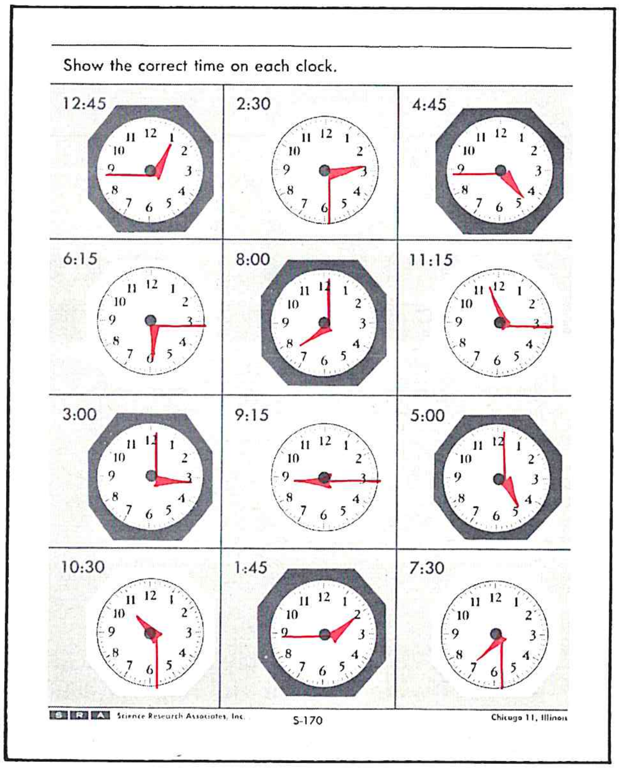
quarter to 7 | 6:45 | quarter past 7 | 7:15

Write the time shown on each clock.

9: 00	10: 45	8: 15
12: 15	4: 15	3: 15
2: 45	1: 30	5: 45

There are two ways to write a quarter past the hour.
There are two ways to write a quarter to the hour.

S-169



Show the correct time on each clock.

12:45	2:30	4:45
6:15	8:00	11:15
3:00	9:15	5:00
10:30	1:45	7:30

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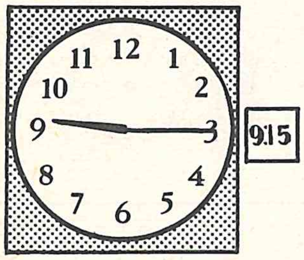
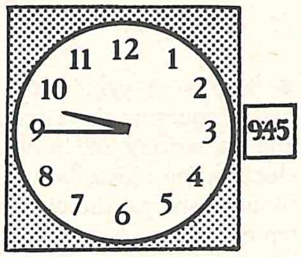
▶ Let the children make up their own story problems for the class to solve. Call on a child to tell his story problem, using his individual clock. The child may tell the complete story problem, or he may set the clock hands at the starting time involved in the problem and then ask another child to complete the problem by changing the hands of the clock.

▶ Use the clock faces on the set of cards to represent two different times. Have the children determine how many minutes have passed between the two.

▶ Narrate the schedule of the school-day, instructing the children to follow the narration by setting their individual clocks as the time schedule is given in detail. "The first school bell rings at quarter to nine, school begins at nine o'clock, morning exercises conclude at quarter past nine," and so on through each activity of the day.





Time

Work Pages S-173 and S-174

Objective

To teach the children how to recognize the number of minutes past the hour and before the hour.

Fundamentals

The child has been concerned with telling time on the hour, half-hour, and quarter-hour. He is familiar with counting by fives around the clock to find out how many minutes have elapsed since the hour. He will now be introduced to the minutes past the hour between the half- and quarter-hours. He will learn to read such times as 7:10, 6:50, 10:35, and 8:20. The teacher will want to remind the child that the long hand of the clock indicates the number of minutes past the hour.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

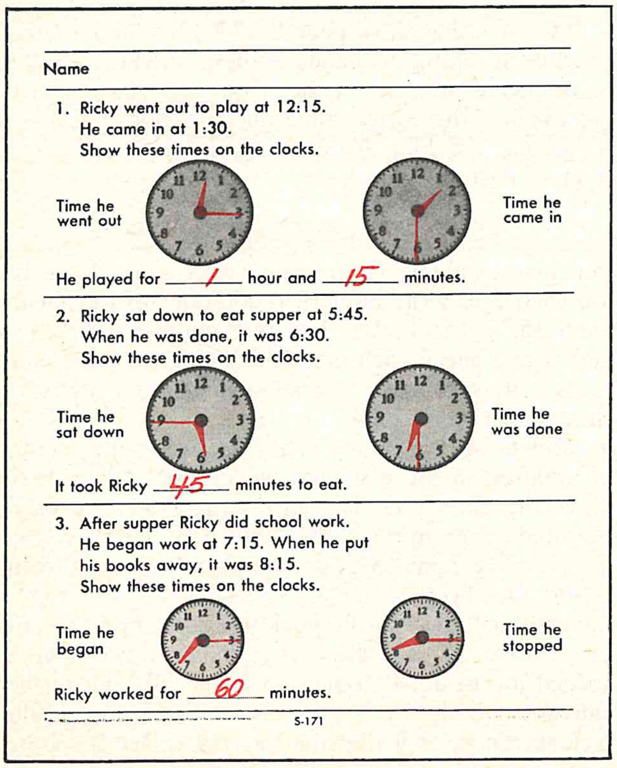
Readiness for Understanding

Ability to count by fives to sixty.

An understanding that there are sixty minutes in an hour.

Ability to read the time on the hour, half-hour, and quarter-hour.

Ability to write the numerals that represent the hour, half-hour, and quarter-hour.



Name

1. Ricky went out to play at 12:15.
He came in at 1:30.
Show these times on the clocks.

Time he went out

Time he came in

He played for 1 hour and 15 minutes.

2. Ricky sat down to eat supper at 5:45.
When he was done, it was 6:30.
Show these times on the clocks.

Time he sat down

Time he was done

It took Ricky 45 minutes to eat.

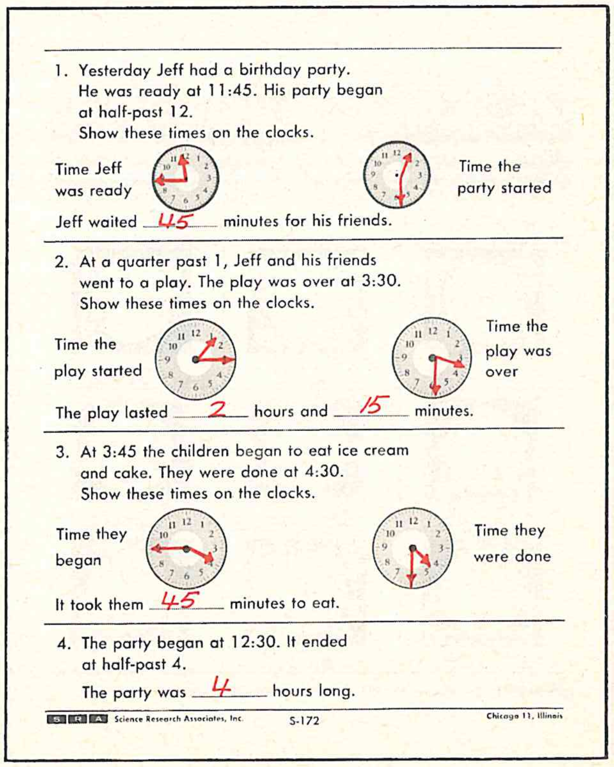
3. After supper Ricky did school work.
He began work at 7:15. When he put
his books away, it was 8:15.
Show these times on the clocks.

Time he began

Time he stopped

Ricky worked for 60 minutes.

S-171



1. Yesterday Jeff had a birthday party.
He was ready at 11:45. His party began
at half-past 12.
Show these times on the clocks.

Time Jeff was ready

Time the party started

Jeff waited 45 minutes for his friends.

2. At a quarter past 1, Jeff and his friends
went to a play. The play was over at 3:30.
Show these times on the clocks.

Time the play started

Time the play was over

The play lasted 2 hours and 15 minutes.

3. At 3:45 the children began to eat ice cream
and cake. They were done at 4:30.
Show these times on the clocks.

Time they began

Time they were done

It took them 45 minutes to eat.

4. The party began at 12:30. It ended
at half-past 4.
The party was 4 hours long.

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Developmental Experiences

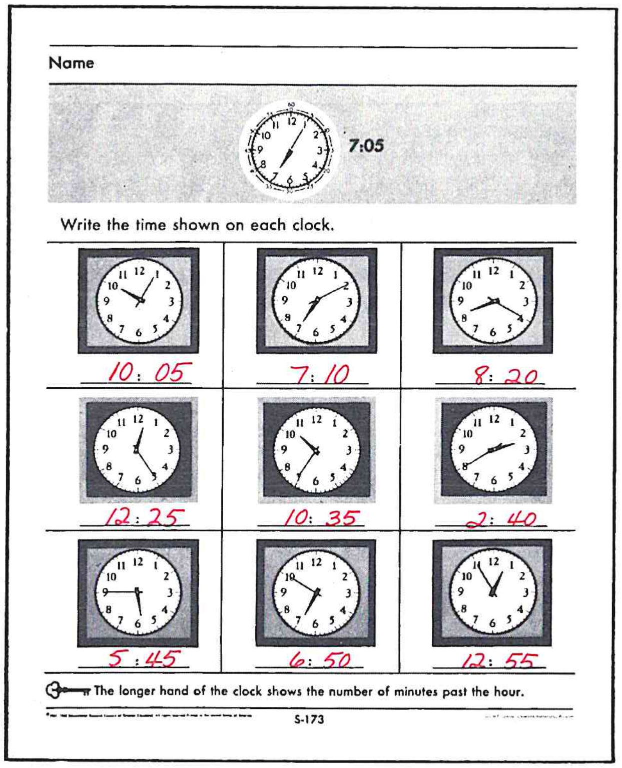
MATERIALS

- large demonstration clock
- individual clock models
- cards showing numerical notation of time

▶ Using the demonstration clock, review the counting of the minute marks between the numerals 12 and 1. Also review the fact that the children can count by fives to remind themselves that there are sixty minutes in an hour.

Set the clock to five o'clock. Then have a child read the hour and write it in numerical-notation form (5:00) on the chalkboard. Ask other children to read the times and write the numerical notations 5:15, 5:30, and 5:45.

Now set the clock to 5:10, and ask if any child can read or write the time indicated. Encourage the children to start at the hour and count by fives to the time indicated by the long hand. Write the numeral 5:10 on the chalkboard, explaining that it means ten minutes past five o'clock. Proceed with other designations of time, such as 5:20, 5:35, 5:55. Encourage the children to count by fives to find out how many minutes past the hour are represented. The time should be written on the chalkboard each time to emphasize the hour and the minutes past the hour. Remind the children that the short hand points to the hour while the long hand points to the number of minutes past the hour.



Name

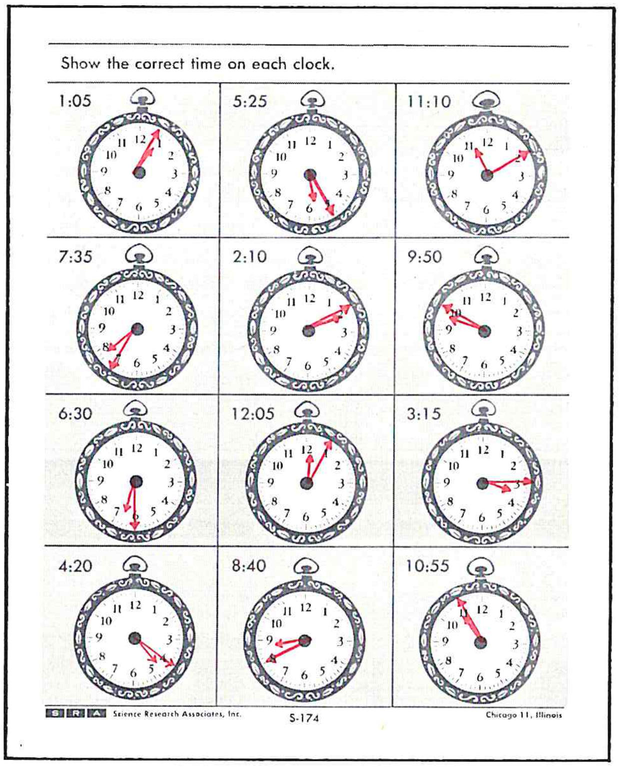
7:05

Write the time shown on each clock.

10: 05
7: 10
8: 20
12: 25
10: 35
2: 40
5 : 45
6: 50
12: 55

The longer hand of the clock shows the number of minutes past the hour.

S-173



Show the correct time on each clock.

1:05
5:25
11:10
7:35
2:10
9:50
6:30
12:05
3:15
4:20
8:40
10:55

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▶ Use work page S-173 to give the children practice in counting the number of minutes past the hour and in writing the indicated time. Talk about the clock in the illustration and, with the children, count by fives around the clock: "Five minutes past seven, ten minutes past seven . . . eight o'clock." Direct them to write the time shown on each of the remaining clocks.

Prior to using work page S-174, give the children practice in setting the hands of their individual clocks to designated times. On the work page the children are to show the correct time on each clock. Remind them that the long hand points to the number of minutes past the hour.

Supplementary Experiences

▶ Have available an envelope with a set of cards. On each card write numerical notations of time such as 6:45, 7:15, 10:05. Let two children work together and check each other's work. Have them take turns setting the demonstration clock to match the time shown on the card.

▶ Relate an imaginary boat trip, using the names of children in the class for the names of the crew. Have the class use their individual clocks to show the times given in the story. "Our boat will leave the dock at five minutes past nine for a trip to Green Island. One hour earlier, (clocks should show 8:05), the crew will meet at the dock to board the boat and prepare for the trip. Each crew member will have a special job to do. Patricia and John will remove the canvas covering by twenty-five minutes past eight o'clock. Diane will find the flag and place it in the holder by eight-thirty. At quarter to nine, Howard will help the captain check the engine and the radio. Marge will put the picnic basket on the boat at five minutes to nine. Marion, Ray, Jim and Jean will help the captain start the engines and untie the lines to the dock at nine o'clock. Our trip will begin as planned at five minutes past nine."

Time

Work Pages S-175 Through S-180

Objectives

To teach the children that there are different ways of writing the time past the half-hour.

Fundamentals

The child knows how to tell time by reading the number of minutes past the hour. He is now ready to learn how to tell the time past the half-hour by counting the number of minutes to the next hour. In this way, he will gain an understanding of the relation between the numerical notation, the number of minutes to the hour, and the number of minutes past the hour.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

Ability to count the number of minutes past the hour.

Developmental Experiences

MATERIALS

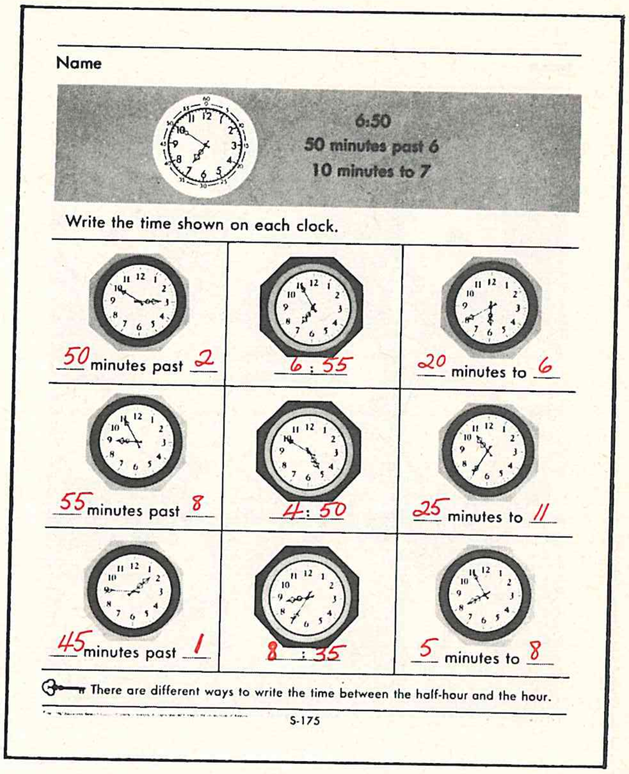
large demonstration clock

▶ Use the demonstration clock to show three o'clock. Have the class read the time, then move the minute hand slowly to the half-hour mark, having the time read as each numeral is approached. When the minute hand arrives at the half-hour mark, stop, and through class discussion help the children to discover that, from this mark on, the time is approaching the next hour, which is four o'clock. As the hand moves toward the twelve, we can speak of the time as so many minutes to the hour. Now move the minute hand to the numeral 11, and ask the children how many minutes are indicated between the numeral 11 and the numeral 12. Ask them to tell the time (five minutes to four) represented on the clock. Then on the chalkboard write:

____ minutes to ____
____ minutes past ____
____ : ____

Have a child fill in the blanks to show the three ways of expressing the time. Move the long minute hand to the ten, guiding the children to an understanding that since ten minutes are marked off between the numeral 10 and the numeral 12, we can say that it is ten minutes to four. Repeat this procedure with the other numerals between 6 and 12.

▶ Use work page S-175 to give the children practice in writing the time past the half-hour in differ-



Name

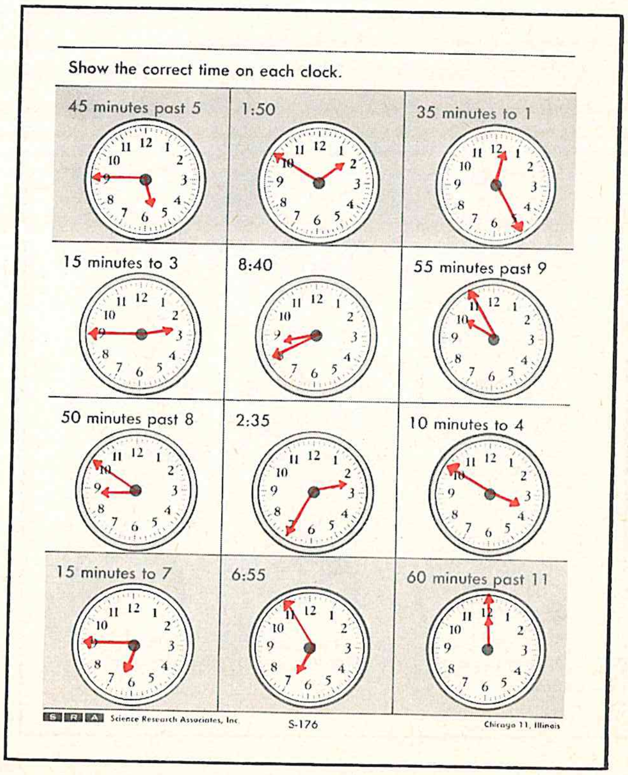
6:50
50 minutes past 6
10 minutes to 7

Write the time shown on each clock.

50 minutes past 2	6 : 55	20 minutes to 6
55 minutes past 8	4 : 50	25 minutes to 11
45 minutes past 1	8 : 35	5 minutes to 8

There are different ways to write the time between the half-hour and the hour.

S-175



Show the correct time on each clock.

45 minutes past 5	1:50	35 minutes to 1
15 minutes to 3	8:40	55 minutes past 9
50 minutes past 8	2:35	10 minutes to 4
15 minutes to 7	6:55	60 minutes past 11

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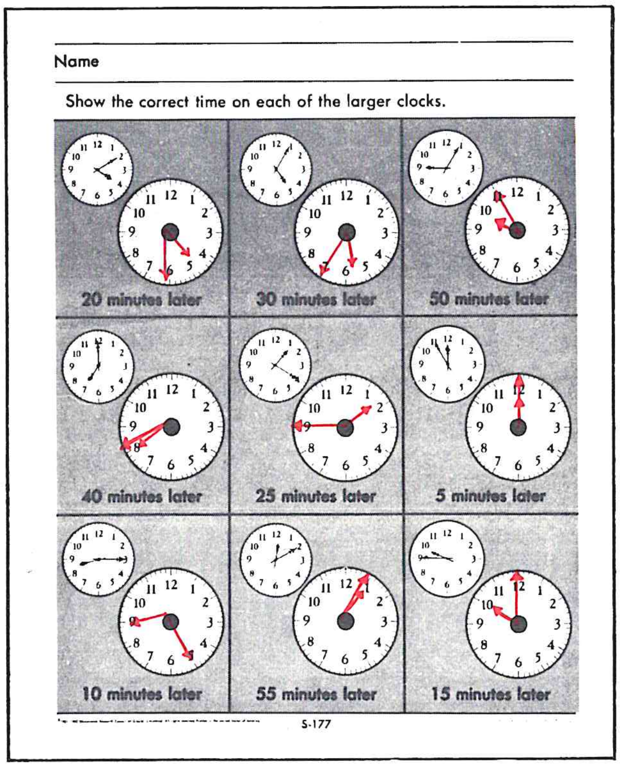
ent ways. Have the children write the time indicated by each clock. Caution them to look carefully so as to discover whether they are to show the number of minutes past the hour or the number of minutes to the next hour.

Prior to work page S-176, use a set of cards with designated times written on them. Include all the written forms. Give each child a card and have him set his model clock, or the demonstration clock, to the same time as the card indicates.

Give the children an opportunity to draw the hands on clocks at the chalkboard, too. Talk about the position of the hour and minute hands. Ask the children to show the correct time by drawing in the hands on each clock.

▶ Work pages S-177 and S-178 present problems

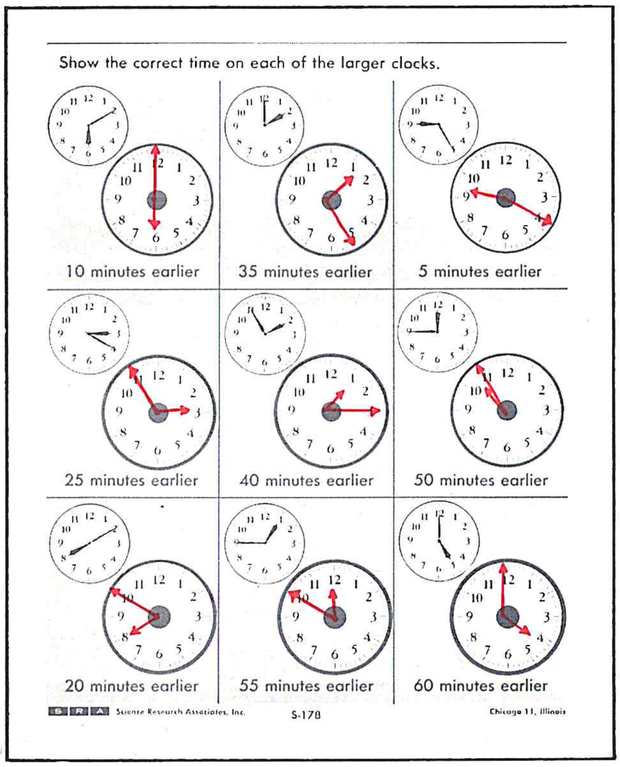


Name

Show the correct time on each of the larger clocks.

20 minutes later
30 minutes later
50 minutes later
40 minutes later
25 minutes later
5 minutes later
10 minutes later
55 minutes later
15 minutes later

S-177



Show the correct time on each of the larger clocks.

10 minutes earlier
35 minutes earlier
5 minutes earlier
25 minutes earlier
40 minutes earlier
50 minutes earlier
20 minutes earlier
55 minutes earlier
60 minutes earlier

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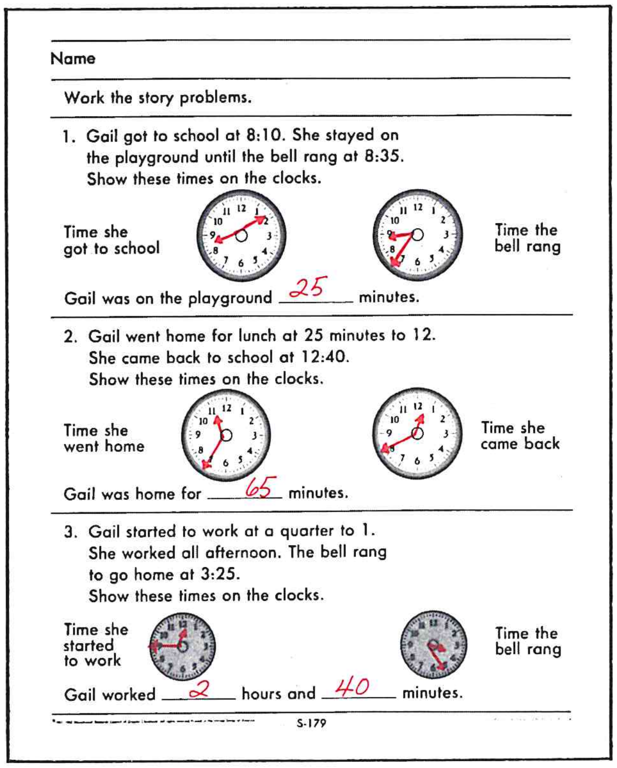


Name

Work the story problems.

1. Gail got to school at 8:10. She stayed on the playground until the bell rang at 8:35. Show these times on the clocks.

Time she got to school

Time the bell rang

Gail was on the playground 25 minutes.

2. Gail went home for lunch at 25 minutes to 12. She came back to school at 12:40. Show these times on the clocks.

Time she went home

Time she came back

Gail was home for 65 minutes.

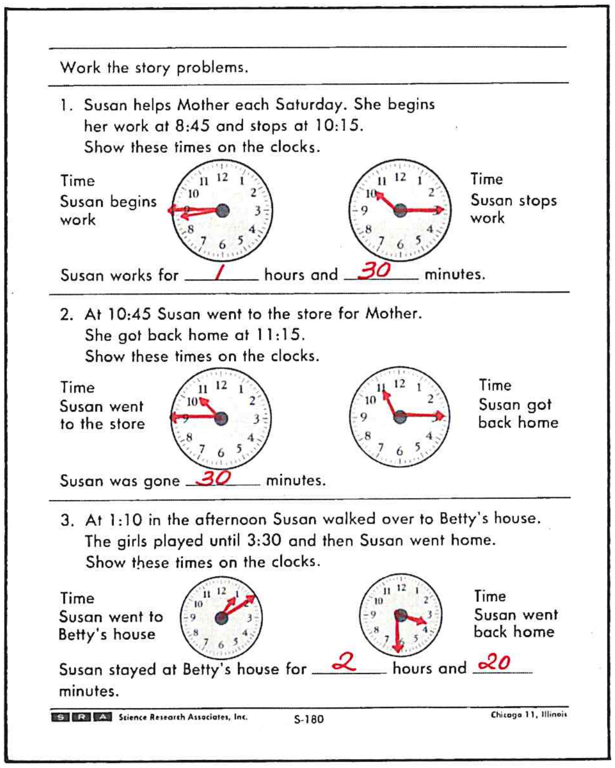
3. Gail started to work at a quarter to 1. She worked all afternoon. The bell rang to go home at 3:25. Show these times on the clocks.

Time she started to work

Time the bell rang

Gail worked 2 hours and 40 minutes.

S-179



Work the story problems.

1. Susan helps Mother each Saturday. She begins her work at 8:45 and stops at 10:15. Show these times on the clocks.

Time Susan begins work

Time Susan stops work

Susan works for 1 hours and 30 minutes.

2. At 10:45 Susan went to the store for Mother. She got back home at 11:15. Show these times on the clocks.

Time Susan went to the store

Time Susan got back home

Susan was gone 30 minutes.

3. At 1:10 in the afternoon Susan walked over to Betty's house. The girls played until 3:30 and then Susan went home. Show these times on the clocks.

Time Susan went to Betty's house

Time Susan went back home

Susan stayed at Betty's house for 2 hours and 20 minutes.

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involving indicated intervals of time, and asking the students to picture an earlier or later time. Give the children practice working with the demonstration clock and their model clocks before they attempt to show the earlier or later time on these work pages. It is suggested that each side of the work sheet be completed at a different time.

▶ On work pages S-179 and S-180 there are story problems to be read, discussed, and worked out with the children. Each problem involves picturing two indicated times, as well as finding a final answer.

Supplemental Experiences

▶ The children can play "Telling Time Relay." One child can operate the demonstration clock. Select three teams—A, B, and C. When the operator of the clock sets the hands, the first member of team A writes the numeral notation for the time indicated by the clock on the chalkboard. At the same time, the first member of team B goes to the chalkboard and fills in the blanks of "____ minutes to ____ ." Team C fills in the blanks on the chalkboard "____ minutes past ____ ." Each team receives one point for each correct answer. The operator resets the hands of the clock. The next member of each team goes to the chalkboard and writes, according to the method being used by his team, the time indicated by the clock. Continue in this way until each team member has had a turn. Then alternate each team's directions for writing the time so that all the children will have the experience of writing time in three ways.

Addition with Carrying

Work Pages S-181 Through S-188

Objectives

To introduce the children to addition problems involving carrying.

To build a foundation for an understanding of the rules for carrying in the addition algorism.

Fundamentals

The second-grade child has had many experiences with addition. He has become aware of the basic properties governing this operation—the commutative and the associative properties. He has learned to solve single-digit addition problems and to add ones and tens in two-digit problems. He has discovered how to move to ten and then go beyond, as when he adds seven and six. He is now ready to investigate the concept of carrying.

A three-step approach to addition that involves going from one decade to another will be adopted in the development of carrying. The first step can be thought of as finding the number of ones which can be added to ten to give the sum. In a problem such as $19 + 4 = \square$, $19 + 4$ can be regarded as $(10 + 9) + 4$. Because of the associative property, $(10 + 9) + 4$ becomes $10 + (9 + 4)$. Now we add ones, and $10 + (9 + 4)$ becomes $10 + 13$.

In the second step, tens will be grouped. $10 + 13$ can be regarded as $10 + (10 + 3)$. Again, because of the associative property, $10 + (10 + 3)$ becomes $(10 + 10) + 3$ or $(20 + 3)$. The number which will be added to twenty to get the correct sum has been found. Here the child is given an opportunity to see that he will take the ten from the thirteen and put it with the other ten; he finds that he has two tens and three ones, or twenty plus three.

The last step is a final deduction from the two previous steps: twenty is added to three to give twenty-three.

There is another approach which could be taken with problems in which a number represented by a single-digit numeral is added to a number represented by a two-digit numeral, with carrying involved. The child has learned to think of $9 + 4$ as $10 + 3$. It is also possible for him to think of $19 + 4$ as $20 + 3$ by moving first to twenty and then going beyond.

$$\begin{aligned} 19 + 4 &= 19 + (1 + 3) \\ &= (19 + 1) + 3 \\ &= 20 + 3 \\ &= 23 \end{aligned}$$

There is a distinct advantage, however, in placing the main emphasis on the three-step approach and using the second method as a supplemental approach. The first approach leads the child directly to the algorism. When he is faced with such problems as $19 + 4 = \square$, he will not learn to think "nineteen plus four equals twenty-three" as the first step. He will learn to think "nine plus four equals thirteen." He will regroup the thirteen ones as one ten and three ones; he will add the one ten to those already visible in the problem, and finally he will add the two tens to the three ones to get his answer.

Problems in which the second addend is a number represented by a single-digit numeral lend themselves more easily to the second approach. The teacher will want to take a long-range view, however, and be aware that the three-step approach will be more workable in complex problems involving carrying.

In the initial stages of the work with carrying, the child should be guided to discover the answer to problems through work with sets and the Counting-men, and then relate his discovery to an equation and the expanded notation in vertical form. The algorism constitutes the final step in the development; it should be introduced only after the child understands the three-step method of solving addition problems involving carrying.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

An understanding of the concept of place value.

An understanding of the concept of expanded notation.

An understanding of the associative property.

Ability to use different numerals for the same number.

Ability to work two-digit addition problems without carrying.

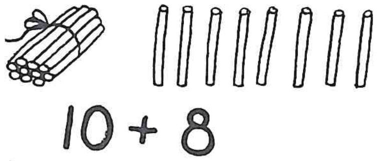
Ability to work with harder combinations.

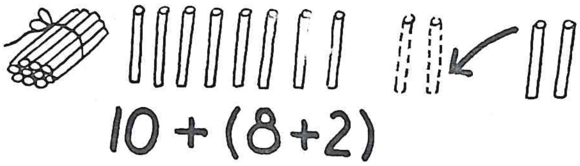
Ability to work with placeholder equations and vertical notation.

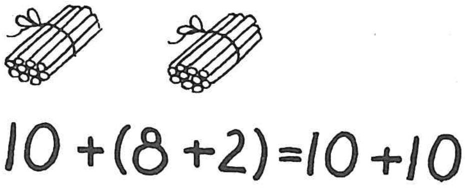
Developmental Experiences

MATERIALS	VOCABULARY
sticks	carrying
rubber bands	carry-digit
2 Countingmen	

▶ Write the equation 18 + 2 = ☐ on the chalkboard. Ask the children to place eighteen sticks (one bundle of ten sticks and eight single sticks) on their desks. Write the expanded-notation numeral for eighteen on the chalkboard: 10 + 8. Tell the children to join two more sticks to the set of ten plus eight. Write the symbolism related to this phase of the work, on the chalkboard: 10 + (8 + 2). The children will discover that they now have one bundle of ten sticks and ten single sticks. Ask them whether there is another way to think of ten ones. Have the children band the ten ones together as one ten. Ask how many bundles of sticks they now have. Complete the equation on the chalkboard: 10 + (8 + 2) = 10 + 10. Have someone tell a shorter way to say "ten plus ten." When the answer "twenty" is given, write the equation 10 + 10 = 20 on the chalkboard. Point out that 18 + 2 became 10 + 10, or 20. Complete the placeholder equation.

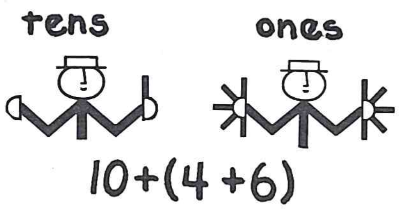






Continue in this way to work with sets and related equations in order to give the children an opportunity to add two numbers whose sum is the next multiple of ten.

▶ Write the equation 14 + 6 = ☐ on the chalkboard. Display two Countingmen. Have a child show the first addend in the equation, on the Countingmen.

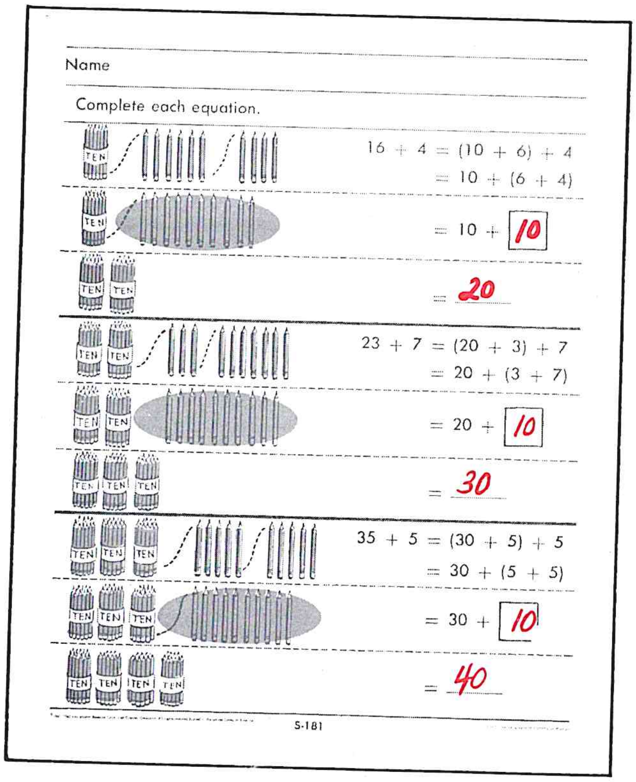


Have him join a set of six to the set of ten plus four. When he discovers he has ten fingers on the Ones-man, ask him what he must do next. He should record the ten ones as one ten by removing the fingers from the Ones-man and placing one more finger on the Tens-man, who will now show two tens, or twenty.

Write the symbolism 10 + 10 = 20 below the placeholder equation on the chalkboard. Have a child refer to the original equation and tell the class how many tens he visualizes. Have him summarize the thinking steps involved in going from 14 + 6 to 10 + 10.

Continue in this way to use the Countingmen to present the first step in the development of the concept of *carrying*. Relate appropriate equations to the activity each time. Encourage children to think about the ones and tens involved in the problems in terms of just tens.

▶ Use work pages S-181 and S-182 as a follow-up to the manipulation of concrete objects. When using work page S-181, discuss the steps in two-digit addition problems whose sums are multiples of ten. Discuss the use of the associative property. Work with the children as they complete each equation.



Name

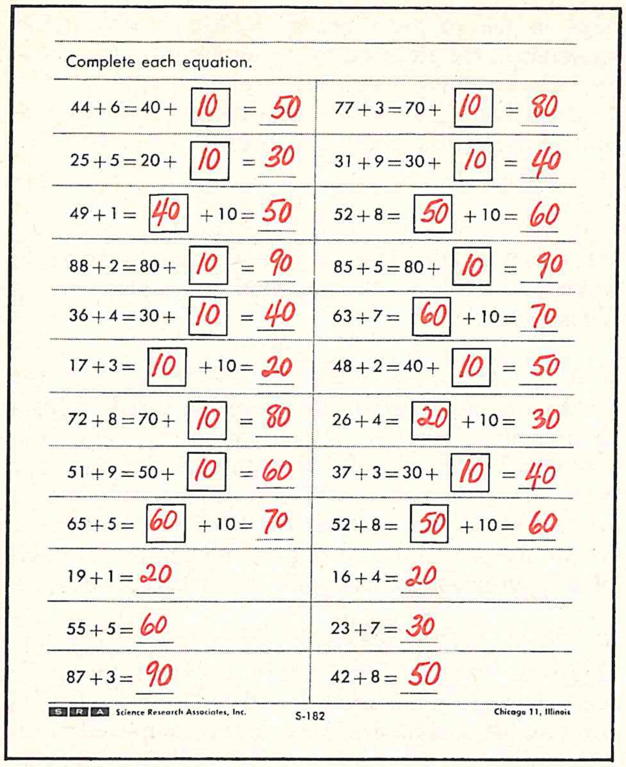
Complete each equation.

16 + 4 = (10 + 6) + 4
= 10 + (6 + 4)
= 10 + $\boxed{10}$
= 20

23 + 7 = (20 + 3) + 7
= 20 + (3 + 7)
= 20 + $\boxed{10}$
= 30

35 + 5 = (30 + 5) + 5
= 30 + (5 + 5)
= 30 + $\boxed{10}$
= 40

S-181



Complete each equation.

44 + 6 = 40 + $\boxed{10}$ = 50	77 + 3 = 70 + $\boxed{10}$ = 80
25 + 5 = 20 + $\boxed{10}$ = 30	31 + 9 = 30 + $\boxed{10}$ = 40
49 + 1 = $\boxed{40}$ + 10 = 50	52 + 8 = $\boxed{50}$ + 10 = 60
88 + 2 = 80 + $\boxed{10}$ = 90	85 + 5 = 80 + $\boxed{10}$ = 90
36 + 4 = 30 + $\boxed{10}$ = 40	63 + 7 = $\boxed{60}$ + 10 = 70
17 + 3 = $\boxed{10}$ + 10 = 20	48 + 2 = 40 + $\boxed{10}$ = 50
72 + 8 = 70 + $\boxed{10}$ = 80	26 + 4 = $\boxed{20}$ + 10 = 30
51 + 9 = 50 + $\boxed{10}$ = 60	37 + 3 = 30 + $\boxed{10}$ = 40
65 + 5 = $\boxed{60}$ + 10 = 70	52 + 8 = $\boxed{50}$ + 10 = 60
19 + 1 = 20	16 + 4 = 20
55 + 5 = 60	23 + 7 = 30
87 + 3 = 90	42 + 8 = 50

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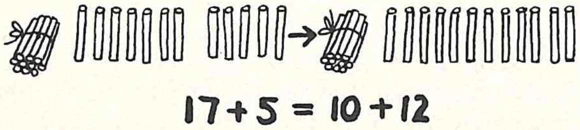
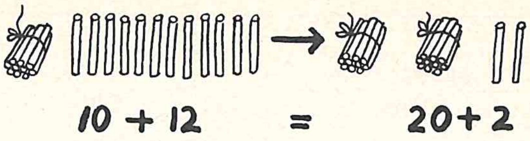
Work page S-182 condenses the steps presented in detail on work page S-181. Work the first problem with the children. Remind them to think of changing the ones in the problem to tens and then combining all of the tens for the final answer. Tell them to fill each placeholder either with the numeral for the number of tens given in the problems or the numeral for the number of tens that result after the ones have been added. Tell them that they must write the numeral for the total number of tens on the line at the end of each equation.

The last six problems on the page require only the standard place-value numeral for the number of tens.

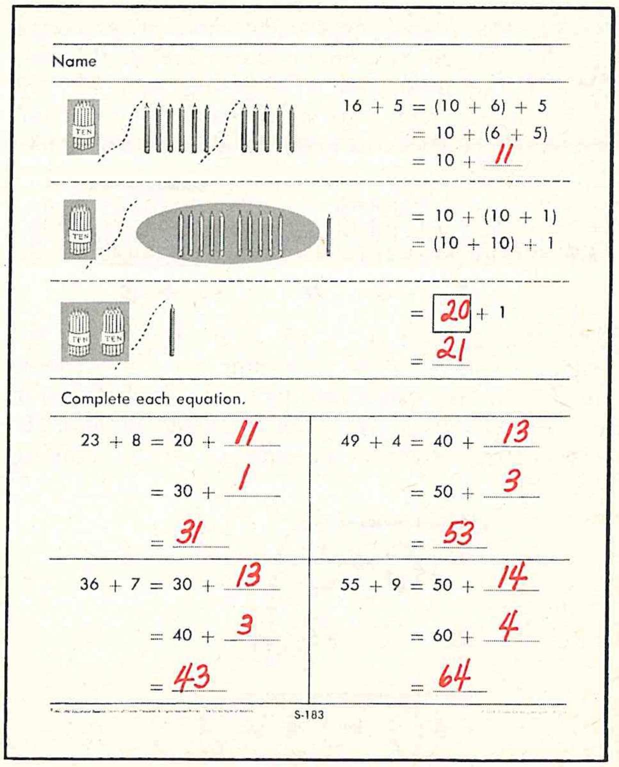
▶ Have the children lay out seventeen sticks (one bundle of ten sticks and seven single sticks) on their desks. Tell them to join five sticks to the set of ten plus seven sticks. The children will discover that they now have twelve single sticks and one bundle of ten sticks. Write on the chalkboard the symbolism related to this phase of the work with the sticks: 17 + 5 = 10 + 12. Ask the children for another way to think of twelve: twelve can be thought of as one ten and two ones. Tell them to band the ten ones together as one ten and to join the bundles of tens. Ask how many tens and how many ones they now have. Write on the chalkboard the symbolism related to this phase of the work: 10 + 12 = 20 + 2. Have some child tell a shorter way to say "twenty plus two." When the answer "twenty-two" is given, write the equation 20 + 2 = 22 on the chalkboard below the other equations. Continue in this way to work with sets and related equations in order to give the children an opportunity to understand what is behind the addition of two numbers when carrying is involved. Have the children take turns writing the symbolism related to each action so that they become thoroughly familiar with it.





▶ Use work pages S-183 and S-184 after the children have had a sufficient number of experiences with concrete objects to enable them to visualize the steps in the process of carrying. Work through the problem at the top of work page S-183 with the children: refer to the picture of sticks as each step of the problem is thought out. Ask the children to write the appropriate numerals as the discussion proceeds.

Work the first problem in the lower section of the page with the group. Remind them to first write a numeral for the ones and tens they see in the given problem. Tell them to regroup their tens and ones and write a numeral that shows how many are in the new grouping. Finally, tell them to write the standard place-value numeral for the sum. Have them complete the other equations on work pages S-183 and S-184 independently. Give help only if needed.



Name

16 + 5 = (10 + 6) + 5
= 10 + (6 + 5)
= 10 + 11

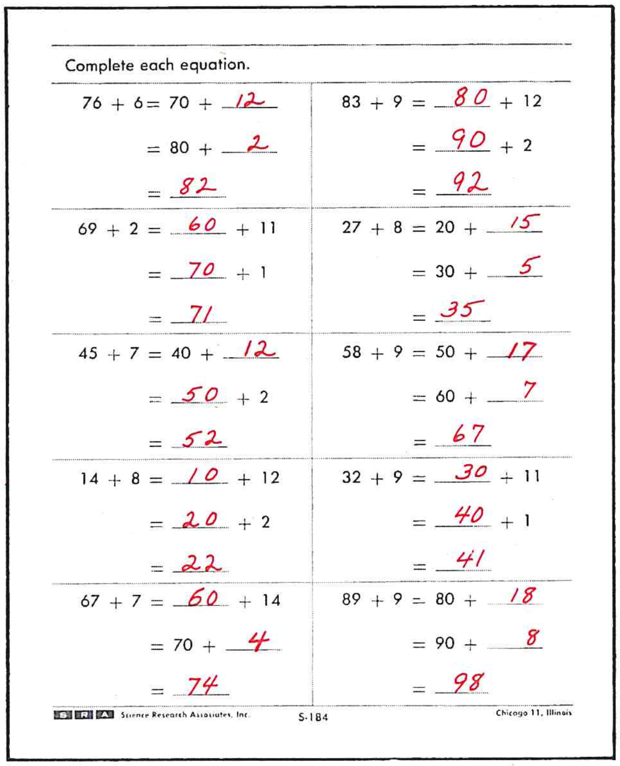
= 10 + (10 + 1)
= (10 + 10) + 1

= $\boxed{20}$ + 1
= 21

Complete each equation.

23 + 8 = 20 + 11 = 30 + 1 = 31	49 + 4 = 40 + 13 = 50 + 3 = 53
36 + 7 = 30 + 13 = 40 + 3 = 43	55 + 9 = 50 + 14 = 60 + 4 = 64

S-183


Complete each equation.

76 + 6 = 70 + 12
= 80 + 2
= 82

83 + 9 = 80 + 12
= 90 + 2
= 92

69 + 2 = 60 + 11
= 70 + 1
= 71

27 + 8 = 20 + 15
= 30 + 5
= 35

45 + 7 = 40 + 12
= 50 + 2
= 52

58 + 9 = 50 + 17
= 60 + 7
= 67

14 + 8 = 10 + 12
= 20 + 2
= 22

32 + 9 = 30 + 11
= 40 + 1
= 41

67 + 7 = 60 + 14
= 70 + 4
= 74

89 + 9 = 80 + 18
= 90 + 8
= 98

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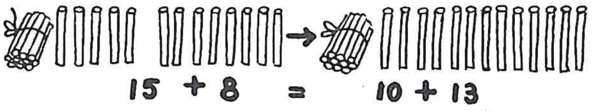
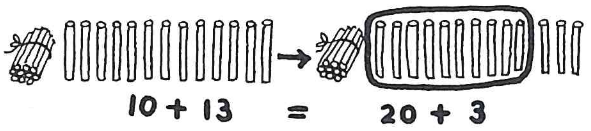
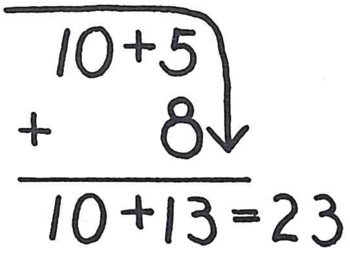
▶ Draw a picture of one bundle of ten sticks and five single sticks on the chalkboard. Ask the children to tell how many sticks are in the picture.



Next to this picture draw eight more sticks. Have the children tell how many sticks are in this set. Below the set illustration write the equation 15 + 8 = 10 + 13. Ask a child to draw a line around enough single sticks to make another group of ten.



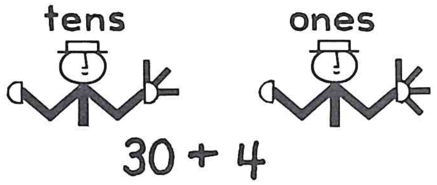
Write the equation 10 + 13 = 20 + 3 below the first equation. While the set picture and the equation are still on the chalkboard, write the symbolism for the ideas involved, using the expanded notation in vertical form. Refer to the equation as the thinking steps in the problem are summarized. Relate the numerals in the equation to the numerals in the vertical form involving expanded notation. Explain to the children that this form will help them quickly visualize and compute the number of ones and tens in the problem.

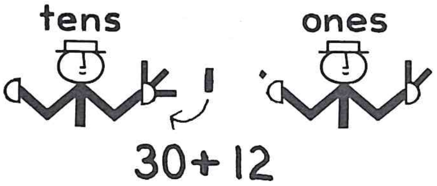


Continue in this way to have children work with sets, related equations, and expanded notation in order to establish a background for understanding the algorism.

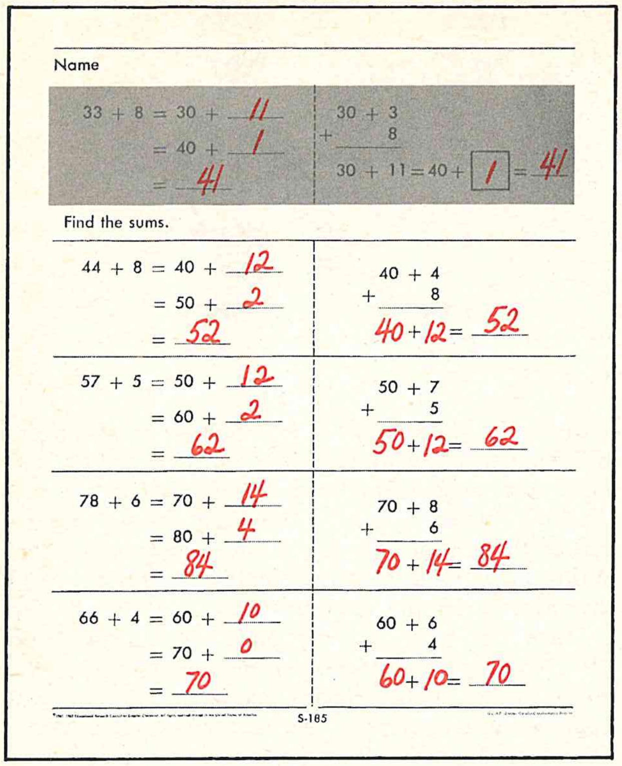
▶ Use work pages S-185 and S-186 to help the children make the transition from the equation form to expanded notation in vertical form. Work the problem at the top of work page S-185 with the class. Have them work the rest of the problems on work page S-185 and all of those on work page S-186 independently, writing the numerals that complete each problem. Give guidance wherever needed.

▶ Use the Countingmen to motivate work with the algorism. Ask the class to help you work the problem 34 + 8 = ☐ in expanded notation in vertical form on the chalkboard. Next to the completed problem, write the algorism for the addition. Display two Countingmen and have a child show the first addend in the given problem. Have the child join a set of eight to the set of thirty and four. He will discover that he reaches ten on the Ones-man before he has put all of the set of eight with the set of four ones. Ask him what he must do now. He should change the ten ones to one ten. At this point in the activity, hold up the one set of ten before the class while the remaining two ones are placed on the Ones-man. Now ask the child to place the one ten on the Tens-man as a record of the one ten "carried over" from the Ones-man to the Tens-man.





Refer to the algorism on the chalkboard. See that the children observe what happens as the addition takes place: four plus eight equals twelve; twelve means one ten and two ones. Explain that the two ones are recorded in the ones place and the one ten is carried over to the tens place. Say that the small numeral 1 placed above the digit 3 reminds us of the one ten carried over to the tens place. Ask the



Name

33 + 8 = 30 + 11
= 40 + 1
= 41

30 + 3
+ 8
30 + 11 = 40 + $\boxed{1}$ = 41

Find the sums.

44 + 8 = 40 + 12
= 50 + 2
= 52

40 + 4
+ 8
40 + 12 = 52

57 + 5 = 50 + 12
= 60 + 2
= 62

50 + 7
+ 5
50 + 12 = 62

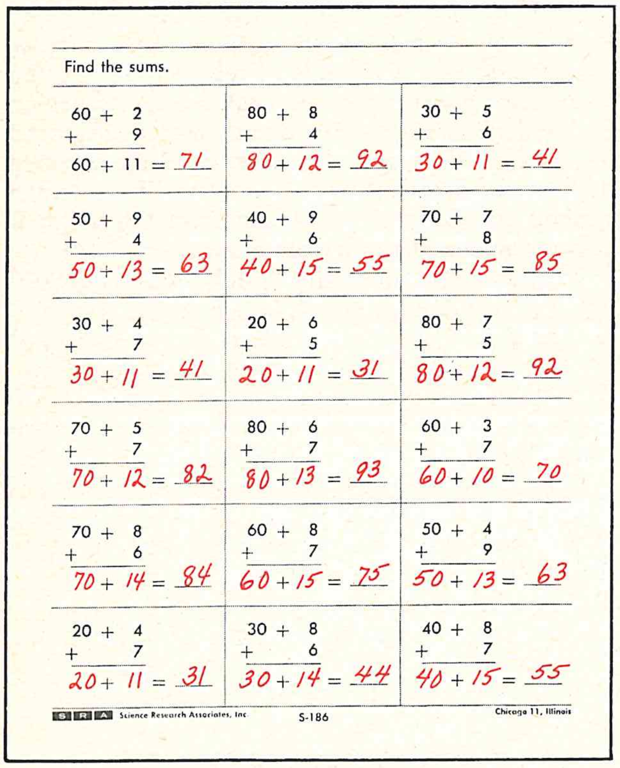
78 + 6 = 70 + 14
= 80 + 4
= 84

70 + 8
+ 6
70 + 14 = 84

66 + 4 = 60 + 10
= 70 + 0
= 70

60 + 6
+ 4
60 + 10 = 70

S-185



Find the sums.

60 + 2 + 9 60 + 11 = 71	80 + 8 + 4 80 + 12 = 92	30 + 5 + 6 30 + 11 = 41
50 + 9 + 4 50 + 13 = 63	40 + 9 + 6 40 + 15 = 55	70 + 7 + 8 70 + 15 = 85
30 + 4 + 7 30 + 11 = 41	20 + 6 + 5 20 + 11 = 31	80 + 7 + 5 80 + 12 = 92
70 + 5 + 7 70 + 12 = 82	80 + 6 + 7 80 + 13 = 93	60 + 3 + 7 60 + 10 = 70
70 + 8 + 6 70 + 14 = 84	60 + 8 + 7 60 + 15 = 75	50 + 4 + 9 50 + 13 = 63
20 + 4 + 7 20 + 11 = 31	30 + 8 + 6 30 + 14 = 44	40 + 8 + 7 40 + 15 = 55

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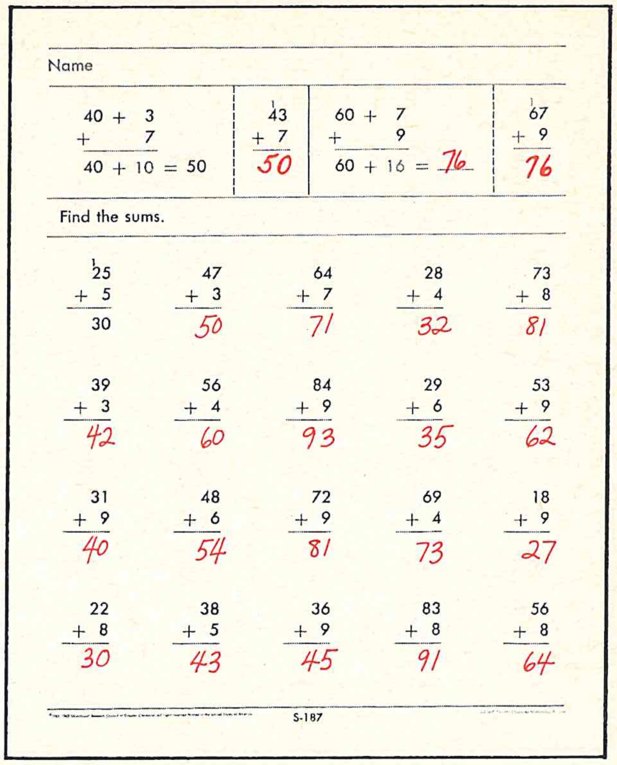


Name

40 + 3 + 7 40 + 10 = 50	$\overset{1}{4}3$ + 7 50	60 + 7 + 9 60 + 16 = 76	$\overset{1}{6}7$ + 9 76

Find the sums.

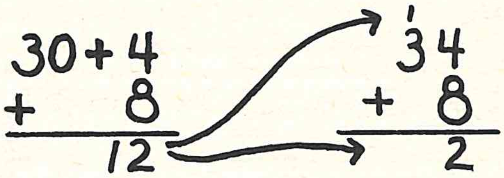
$\overset{1}{2}5$ + 5 30	47 + 3 50	64 + 7 71	28 + 4 32	73 + 8 81
39 + 3 42	56 + 4 60	84 + 9 93	29 + 6 35	53 + 9 62
31 + 9 40	48 + 6 54	72 + 9 81	69 + 4 73	18 + 9 27
22 + 8 30	38 + 5 43	36 + 9 45	83 + 8 91	56 + 8 64

S-187

$$\begin{array}{r} \overset{1}{3}4 \\ +\ 8 \\ \hline 2 \end{array}$$

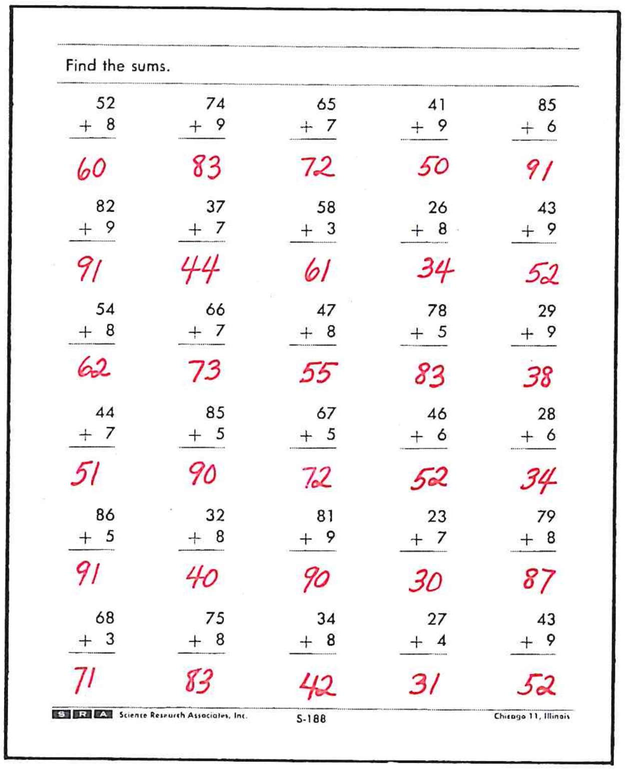
class to tell how many tens are on the Countingmen now. Again refer to the algorism. Say that one ten plus three tens equals four tens, or ten plus thirty equals forty. Record the digit 4 in the algorism. Explain that it stands for the four tens, or forty. After the class tells how many ones and tens are on the Countingmen, refer to the sum in the algorism and say that forty plus two equals forty-two.

Summarize by referring to the expanded notation and the algorism. Relate the ideas expressed by the symbols in the expanded notation to the ideas represented in the algorism. Point out how the numeral for the number twelve can be clearly seen both in the expanded notation and in the algorism.



Continue in this way to show the relation between the activity with the Countingmen and the work with the expanded notation on the one hand, and the algorism on the other. Each time a problem is demonstrated on the Countingmen, have one child record the ideas in expanded notation and have another use the algorism.

▶ Use work pages S-187 and S-188 for practice in using vertical notation when working addition problems with carrying. Work the two problems at the top of work page S-187 with the children. Encourage them to work independently in finding the sums for the rest of the problems on this page and all of the problems on work page S-188. Tell them to use the small *carry-digit* to help them remember how many tens were carried over to the tens place.



Find the sums.

52 + 8 60	74 + 9 83	65 + 7 72	41 + 9 50	85 + 6 91
82 + 9 91	37 + 7 44	58 + 3 61	26 + 8 34	43 + 9 52
54 + 8 62	66 + 7 73	47 + 8 55	78 + 5 83	29 + 9 38
44 + 7 51	85 + 5 90	67 + 5 72	46 + 6 52	28 + 6 34
86 + 5 91	32 + 8 40	81 + 9 90	23 + 7 30	79 + 8 87
68 + 3 71	75 + 8 83	34 + 8 42	27 + 4 31	43 + 9 52

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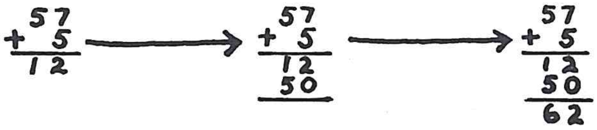
Supplemental Experiences

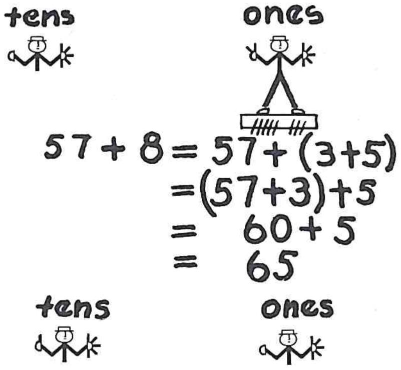
▶ The teacher may wish to show the children an intermediate step between the expanded notation and the algorism. This method can be used to help some children make the transition more easily. For others it will be an interesting new approach to the problem.

A problem such as 57 + 5 = ☐ would be worked out in three steps. First seven and five are added, and the twelve recorded under the line. Then the tens are added and recorded under the twelve. As a final step twelve and fifty are added to get the sum sixty-two.



▶ The teacher may wish to have some children try the second approach to carrying described in the "Fundamentals" for work pages S-181 through S-188. In developing this approach, Countingmen will prove helpful.

Display two Countingmen and call upon two children to operate them. Write the equation 57 + 8 = ☐ on the chalkboard. Have the children represent the first addend in the equation on their Countingmen. The tens-operator should place five fingers on his Countingman; the ones-operator should place seven fingers on his Countingman. Display eight fingers on the base of the Ones-man to show the set that will be joined to the set of fifty and the set of seven.



Have the children think first of how many fingers will be needed with seven to make a set of ten. Have the ones-operator rearrange the fingers on the Countingman's base to show that the set of eight can be thought of as a set of three and a set of five. Write the symbolism corresponding to this phase of the activity on the chalkboard: 57 + (3 + 5).

Have the ones-operator place three fingers on his Countingman to make ten ones. At this point the tens-operator should record one ten, and the ones-operator should remove the ten fingers. The ones-operator can now place the remaining five fingers on his Countingman.

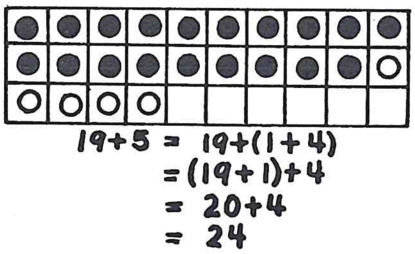
Write the equation that corresponds to this final phase of the activity, on the chalkboard: (57 + 3) + 5 = 60 + 5. Have the children give another name for 60 + 5. Conclude by referring back to the placeholder equation on the chalkboard and telling the children that the Countingmen helped them find the answer: 57 + 8 = 65. Summarize the thinking steps involved in solving the problem. Refer to the symbols on the chalkboard and have the children help review the ideas they represent.

This same procedure may be followed in developing other addition problems of this kind.

▶ The game "Operation Big Ten" can be expanded to "Operation Big Twenty." Place a latticework with three rows of ten openings on the flannel board. Place a set of nineteen counters in the first two rows. Place a set of five counters below the frame.

Remind the children that the aim of "Operation Big Ten" is to make a set of ten as the first step of the operation of joining two given sets. This time the aim is to make a set of twenty.

Call upon a child to carry out "Operation Big Twenty" with the sets on the flannel board. As the child gets ready to take one counter from the set of five and put it with the set of nineteen, stop the



action in order to "take a picture." Put the "picture" in the pocket chart: $19 + (1 + 4)$.

Now have the child resume the action and put the set of one with the set of nineteen just as he was starting to do before he was stopped. Then have him join the other four counters to the two sets of ten by placing them in the third row.

"Take a picture" of the sets that are now on the flannel board. Put the "picture" in the pocket chart: $20 + 4$. Place the card for the short way of writing $20 + 4$, in the chart: 24. Summarize the thinking steps involved in solving the problem. Refer to the cards in the pocket chart and have the children help review the ideas.

This same procedure can be adapted to "Operation Big Thirty" through "Operation Big Ninety."

Subtraction with Borrowing

Work Pages S-189 Through S-194

Objectives

To introduce subtraction problems involving borrowing.

To build a foundation for an understanding of the rules for borrowing in subtraction.

Fundamentals

The inverse concept is important in developing the idea of borrowing in subtraction. The child made use of different numerals for the same number in his work with carrying. He learned to rewrite a problem such as $25 + 8 = \square$ in the following ways:

$$\begin{aligned} 25 + 8 &= 20 + 13 \\ &= 30 + 3 \\ &= 33 \end{aligned} \qquad \begin{array}{r} 20 + 5 \\ +\underline{\qquad 8} \\ 20 + 13 = 33 \end{array}$$

To "undo" this problem, the child will learn to begin subtracting at the place where he finished adding. The equation $33 - 8 = \square$ can be written in expanded notation.

$$\begin{array}{r} 30 + 3 \\ -\underline{\qquad 8} \end{array}$$

The child will observe that it is impossible to subtract eight from three. Thus he will learn to think of another way to write $30 + 3$; he will use the numeral which was used in the addition problem: $20 + 13$. It is now possible for him to perform the subtraction. He is now back where he started in the

$$\begin{array}{rcr} 30 + 3 & = & 20 + 13 \\ -\underline{\qquad 8} & = & -\underline{\qquad 8} \\ & & 20 + \ \ 5 = 25 \end{array}$$

related addition problem.

The main emphasis in developing subtraction with borrowing will be placed on the approach just described. There is, however, a second method which can be used as a supplemental approach, just as there is a second approach in addition with carrying. This second method lends itself best to subtraction of a number represented by a one-digit numeral from a number represented by a two-digit numeral with borrowing involved. In doing problems such as $13 - 7 = \square$, the child learned to think of the problem as $(13 - 3) - 4 = \square$. It is possible for him to adopt an analogous approach for $23 - 7 = \square$. He can first subtract three from twenty-three to get twenty, then subtract four from twenty to get sixteen.

$$\begin{aligned} 23 - 7 &= (23 - 3) - 4 \\ &= 20 - 4 \\ &= 16 \end{aligned}$$

The advantage of using the first method is that it leads more directly to the algorism. When the child is ready to use the computational rule, for example in working the problem $32 - 19 = \square$, he will operate on the ones first. Rather than subtract nine ones from thirty-two and then subtract ten from this difference of twenty-three, he will think first of thirty-two as twenty plus twelve, subtract nine from twelve and ten from twenty, and finally combine the ten and three to give thirteen.

As a supplemental experience, however, the second approach will be explored. It has the advantage of challenging the child to think in different ways about a problem.

At the outset the child will work with sets and the Countingmen to discover the answer to subtraction problems involving borrowing. He will relate his discovery to equations and expanded notation in vertical form. Work with the algorism will constitute the final step in the development and should be introduced only after a background for understanding has been established.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concept of place value.

An understanding of the concept of expanded notation.

Awareness of the inverse relation between addition and subtraction.

Ability to work with two-digit subtraction without borrowing.

Ability to work with harder combinations related to sums of eleven through eighteen.

Ability to use different numerals for the same number.

Ability to work with placeholder equations and vertical notation.

Developmental Experiences

MATERIALS	VOCABULARY
sticks	borrowing
rubber bands	
2 Countingmen	

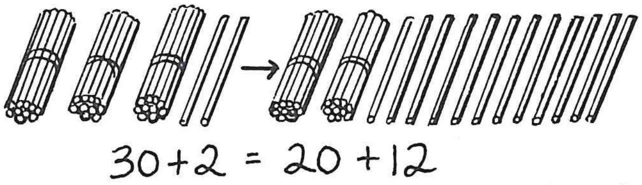
▶ Write the equation 28 + 4 = ☐ on the chalkboard. Have the children place two bundles of ten sticks and eight single sticks on their desks. Have them join four sticks to the set of eight. Write the equation related to this phase of the work on the chalkboard: $28 + 4 = 20 + 12$.

Next, have the children bundle ten of the twelve sticks and put this one set of ten with the two sets of ten. They now have three tens and two ones, or thirty-two. Write the equations related to this action, on the chalkboard: $20 + 12 = 30 + 2$ and $30 + 2 = 32$. Have someone write the expanded notation representing the activity. Leave this equation on the chalkboard as the development of subtraction proceeds.

Tell the children that you want them to "undo" what they just did. Ask what operation will "undo" addition of four. Start with the same sets of sticks: three bundles of ten sticks and two single sticks. Write the expanded notation for the problem, on the chalkboard.

$$\begin{array}{r} 30 + 2 \\ - \quad 4 \end{array}$$

Have the children explain what they must do first in this problem. They should say that four is to be subtracted from two. Tell them to remove four sticks from the two loose sticks on their desks. They will immediately discover that this is impossible. Ask how they can make this removal possible. Guide them to the discovery that the one ten "carried over" in solving the problem 28 + 4 = ☐ should be "brought back": $30 + 2$ can be thought of as $20 + 12$. One bundle of ten can be changed back to ten single sticks. It is now possible to remove four of the twelve sticks. Write the expanded notation related to the action, on the chalkboard. Have the subtraction completed and the standard place-value numeral for $20 + 8$ written as the final step.



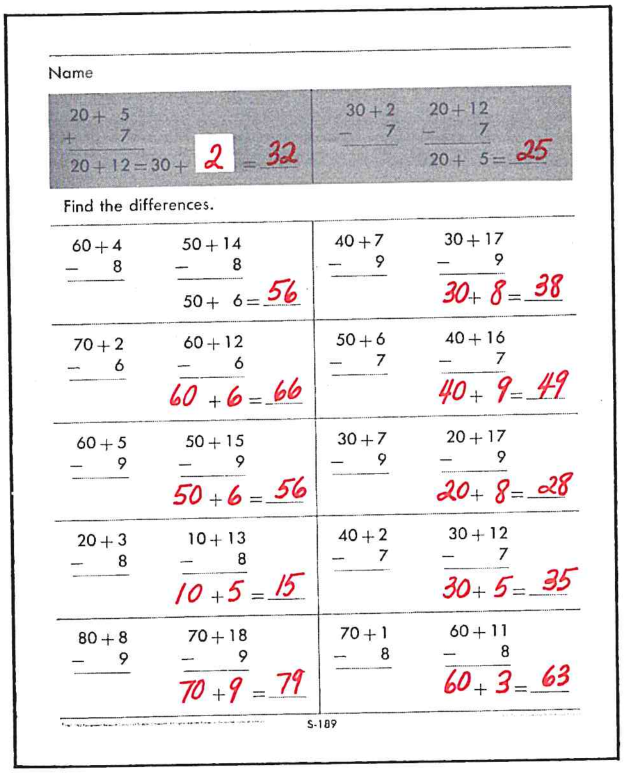
$$\begin{array}{rcr} 30 + 2 & = & 20 + 12 \\ - \quad 4 & = & - \quad\ 4 \\ \hline & & 20 + \ 8 = 28 \end{array}$$

Use arrows to show the corresponding parts of the addition and subtraction symbolism.

$$\begin{array}{rcr} 20 + 8 & & \\ + \quad 4 & & \\ \hline 20 + 12 & = & 30 + 2 \\ & & \\ 30 + \ 2 & = & 20 + 12 \\ - \quad 4 & = & - \quad\ 4 \\ \hline & & 20 + \ 8 \end{array}$$

Develop other subtraction problems of this kind in the same way. Emphasize the inverse concept. Have children help you write equations and expanded notations related to the action with sets.

▶ Use work pages S-189 and S-190 after children have had a sufficient number of experiences with concrete objects to enable them to visualize the steps in the process of *borrowing*. Work with the class through the problems at the top of the page. Stress the relation between addition and subtraction. Ask the children to write the appropriate numerals as



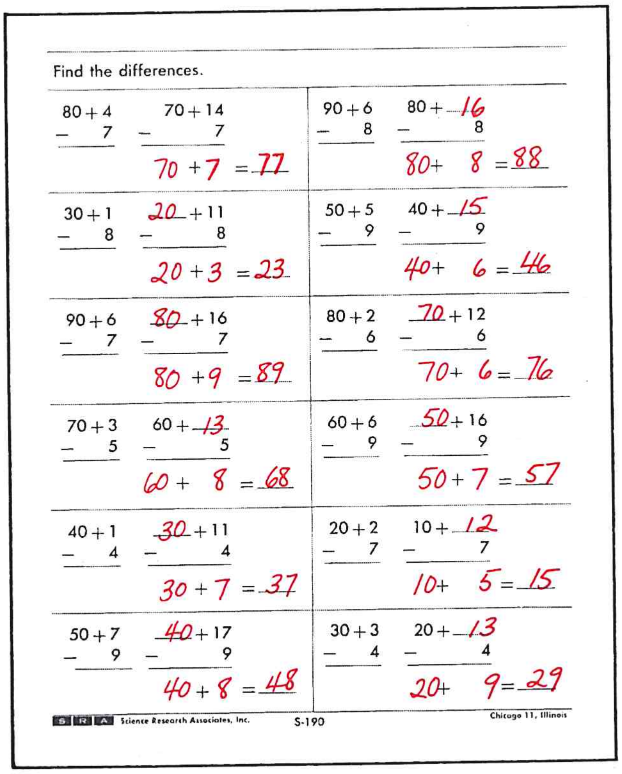
Name

$\begin{array}{r} 20+\ 5 \\ +\quad 7 \\ \hline 20+12=30+\boxed{2}=32 \end{array}$		$\begin{array}{r} 30+2 \\ -\quad 7 \\ \hline \end{array}$	$\begin{array}{r} 20+12 \\ -\quad 7 \\ \hline 20+\ 5=25 \end{array}$

Find the differences.

$\begin{array}{r} 60+4 \\ -\quad 8 \\ \hline \end{array}$	$\begin{array}{r} 50+14 \\ -\quad 8 \\ \hline 50+\ 6=56 \end{array}$	$\begin{array}{r} 40+7 \\ -\quad 9 \\ \hline \end{array}$	$\begin{array}{r} 30+17 \\ -\quad 9 \\ \hline 30+\ 8=38 \end{array}$
$\begin{array}{r} 70+2 \\ -\quad 6 \\ \hline \end{array}$	$\begin{array}{r} 60+12 \\ -\quad 6 \\ \hline 60+6=66 \end{array}$	$\begin{array}{r} 50+6 \\ -\quad 7 \\ \hline \end{array}$	$\begin{array}{r} 40+16 \\ -\quad 7 \\ \hline 40+\ 9=49 \end{array}$
$\begin{array}{r} 60+5 \\ -\quad 9 \\ \hline \end{array}$	$\begin{array}{r} 50+15 \\ -\quad 9 \\ \hline 50+6=56 \end{array}$	$\begin{array}{r} 30+7 \\ -\quad 9 \\ \hline \end{array}$	$\begin{array}{r} 20+17 \\ -\quad 9 \\ \hline 20+\ 8=28 \end{array}$
$\begin{array}{r} 20+3 \\ -\quad 8 \\ \hline \end{array}$	$\begin{array}{r} 10+13 \\ -\quad 8 \\ \hline 10+5=15 \end{array}$	$\begin{array}{r} 40+2 \\ -\quad 7 \\ \hline \end{array}$	$\begin{array}{r} 30+12 \\ -\quad 7 \\ \hline 30+\ 5=35 \end{array}$
$\begin{array}{r} 80+8 \\ -\quad 9 \\ \hline \end{array}$	$\begin{array}{r} 70+18 \\ -\quad 9 \\ \hline 70+9=79 \end{array}$	$\begin{array}{r} 70+1 \\ -\quad 8 \\ \hline \end{array}$	$\begin{array}{r} 60+11 \\ -\quad 8 \\ \hline 60+\ 3=63 \end{array}$

S-189



Find the differences.

$\begin{array}{r} 80+4 \\ -\quad 7 \\ \hline \end{array}$	$\begin{array}{r} 70+14 \\ -\quad 7 \\ \hline 70+7=77 \end{array}$	$\begin{array}{r} 90+6 \\ -\quad 8 \\ \hline \end{array}$	$\begin{array}{r} 80+16 \\ -\quad 8 \\ \hline 80+\ 8=88 \end{array}$
$\begin{array}{r} 30+1 \\ -\quad 8 \\ \hline \end{array}$	$\begin{array}{r} 20+11 \\ -\quad 8 \\ \hline 20+3=23 \end{array}$	$\begin{array}{r} 50+5 \\ -\quad 9 \\ \hline \end{array}$	$\begin{array}{r} 40+15 \\ -\quad 9 \\ \hline 40+\ 6=46 \end{array}$
$\begin{array}{r} 90+6 \\ -\quad 7 \\ \hline \end{array}$	$\begin{array}{r} 80+16 \\ -\quad 7 \\ \hline 80+9=89 \end{array}$	$\begin{array}{r} 80+2 \\ -\quad 6 \\ \hline \end{array}$	$\begin{array}{r} 70+12 \\ -\quad 6 \\ \hline 70+\ 6=76 \end{array}$
$\begin{array}{r} 70+3 \\ -\quad 5 \\ \hline \end{array}$	$\begin{array}{r} 60+13 \\ -\quad 5 \\ \hline 60+\ 8=68 \end{array}$	$\begin{array}{r} 60+6 \\ -\quad 9 \\ \hline \end{array}$	$\begin{array}{r} 50+16 \\ -\quad 9 \\ \hline 50+7=57 \end{array}$
$\begin{array}{r} 40+1 \\ -\quad 4 \\ \hline \end{array}$	$\begin{array}{r} 30+11 \\ -\quad 4 \\ \hline 30+7=37 \end{array}$	$\begin{array}{r} 20+2 \\ -\quad 7 \\ \hline \end{array}$	$\begin{array}{r} 10+12 \\ -\quad 7 \\ \hline 10+\ 5=15 \end{array}$
$\begin{array}{r} 50+7 \\ -\quad 9 \\ \hline \end{array}$	$\begin{array}{r} 40+17 \\ -\quad 9 \\ \hline 40+8=48 \end{array}$	$\begin{array}{r} 30+3 \\ -\quad 4 \\ \hline \end{array}$	$\begin{array}{r} 20+13 \\ -\quad 4 \\ \hline 20+\ 9=29 \end{array}$

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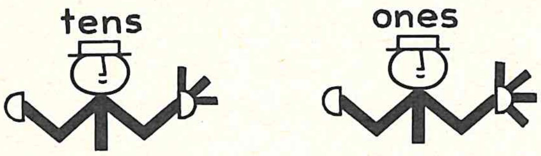
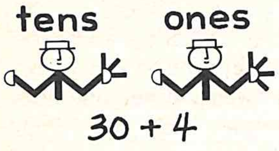
the discussion proceeds. Work with the class the first problem in the lower section of the page. Have them complete on their own the rest of the problems on work page S-189 and all of the problems on work page S-190. Give guidance wherever needed.

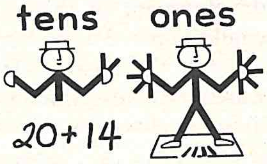
▶ Use the Countingmen to develop an understanding of the "borrowing" algorism. Ask the class to help you work the problem 34 − 8 = ☐ in expanded notation on the chalkboard. Next to this problem write the algorism for the subtraction.



$$\begin{array}{r} 34 \\ -\ 8 \\ \hline \end{array}$$

Have two children show the first number in the problem on two Countingmen. Tell the ones-operator that his job is to remove a set of eight ones from four ones. He will discover that this is impossible unless he gets one ten from the tens-operator, changes it back to ten ones, and joins the ten ones to the four ones on his Countingman to make fourteen. Have the ones-operator take the four ones from his Countingman's hands and put them on the base of the Countingman. Have the tens-operator give one ten to the ones-operator. As the tens-operator removes one finger from his Countingman, the ones-operator should put ten fingers on his. Ask the class to describe what happened: three tens and four ones is now shown as two tens and fourteen ones.

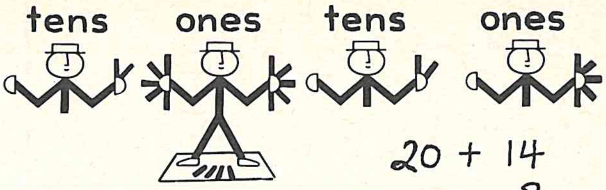




Refer to the algorism. Cross out the digit representing three tens, or thirty. Above it write the numeral 2 to represent two tens, or twenty. Place a small numeral 1 to the left of the digit 4 to represent fourteen.

$$\begin{array}{r} \overset{2}{\cancel{3}}{}^{1}4 \\ -\ 8 \\ \hline \end{array}$$

Have the ones-operator remove eight ones from the fourteen now on his Countingman. He will take away the four on the base first and then four more from the fingers on the Countingman's hand.



$$\begin{array}{r} 20 + 14 \\ -\qquad 8 \\ \hline 20 + \ 6 = 26 \end{array}$$

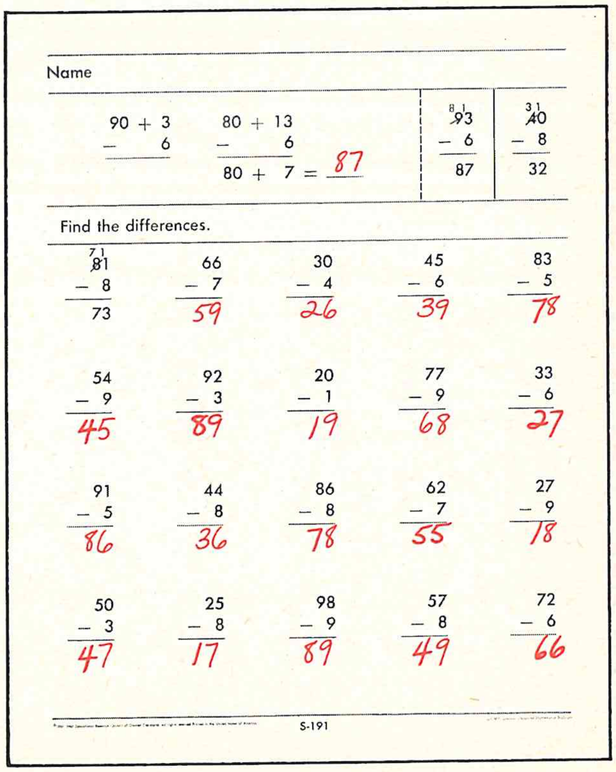
Complete the subtraction and write the related numerals in the algorism. Summarize the steps in this subtraction. Relate the algorism to the expanded notation.

$$\begin{array}{r} 30 + 4 \\ -\qquad 8 \\ \hline \end{array} \;\begin{array}{c} = \\ = \end{array}\; \begin{array}{r} 20 + 14 \\ -\qquad 8 \\ \hline 20 + \ 6 = 26 \end{array} \qquad \begin{array}{r} \overset{2}{\cancel{3}}{}^{1}4 \\ -\ 8 \\ \hline 2\,6 \end{array}$$

Continue in this way, developing other subtraction problems of this kind. Include problems with zero involved. For example, in a problem such as 30 − 8 = ☐, the expanded notation and the algorism would be set up in the following way:

$$\begin{array}{r} 30 + 0 \\ -\qquad 8 \\ \hline \end{array} \;\begin{array}{c} = \\ = \end{array}\; \begin{array}{r} 20 + 10 \\ -\qquad 8 \\ \hline 20 + \ 2 = 22 \end{array}$$

▶ Use work pages S-191 and S-192 for practice in using vertical notation when working subtraction problems with borrowing. Work the two problems at the top of work page S-191 with the children. Encourage them to work independently in completing the rest of the problems on this page and all of the



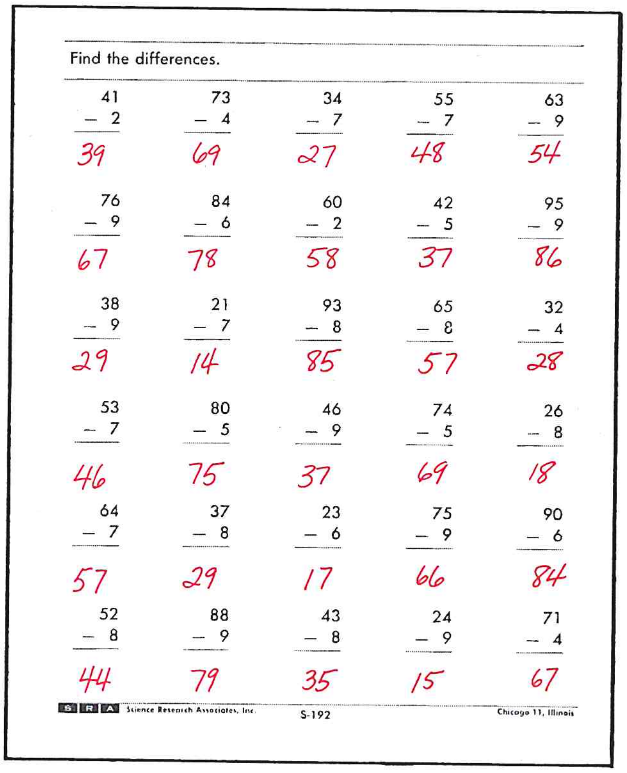
Name

$$\begin{array}{r} 90 + 3 \\ -\qquad 6 \\ \hline \end{array} \qquad \begin{array}{r} 80 + 13 \\ -\qquad 6 \\ \hline 80 + \ 7 = 87 \end{array} \qquad \begin{array}{r} {}^{8}\cancel{9}{}^{1}3 \\ -\ 6 \\ \hline 87 \end{array} \qquad \begin{array}{r} {}^{3}\cancel{4}{}^{1}0 \\ -\ 8 \\ \hline 32 \end{array}$$

Find the differences.

${}^{7}\cancel{8}{}^{1}1$ − 8 73	66 − 7 59	30 − 4 26	45 − 6 39	83 − 5 78
54 − 9 45	92 − 3 89	20 − 1 19	77 − 9 68	33 − 6 27
91 − 5 86	44 − 8 36	86 − 8 78	62 − 7 55	27 − 9 18
50 − 3 47	25 − 8 17	98 − 9 89	57 − 8 49	72 − 6 66

S-191



Find the differences.

41 − 2 39	73 − 4 69	34 − 7 27	55 − 7 48	63 − 9 54
76 − 9 67	84 − 6 78	60 − 2 58	42 − 5 37	95 − 9 86
38 − 9 29	21 − 7 14	93 − 8 85	65 − 8 57	32 − 4 28
53 − 7 46	80 − 5 75	46 − 9 37	74 − 5 69	26 − 8 18
64 − 7 57	37 − 8 29	23 − 6 17	75 − 9 66	90 − 6 84
52 − 8 44	88 − 9 79	43 − 8 35	24 − 9 15	71 − 4 67

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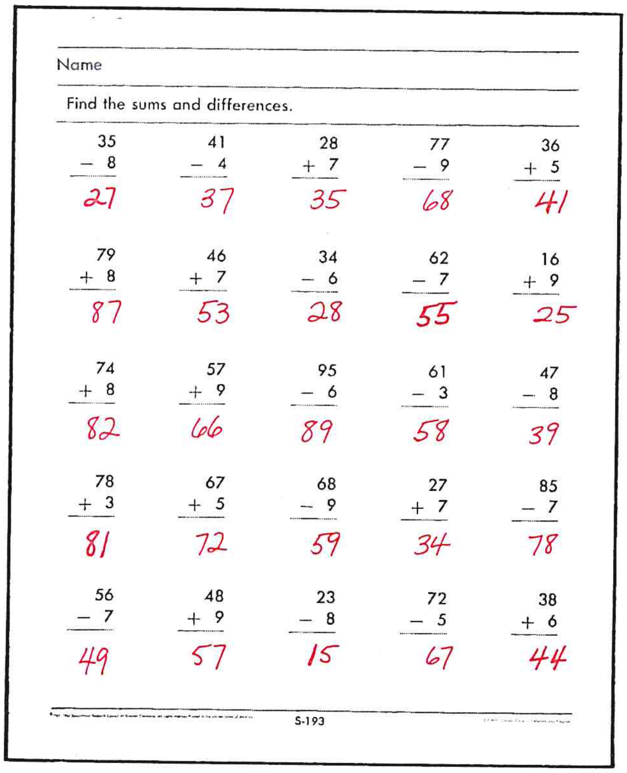
problems on work page S-192. Tell them to cross out the digit in the tens place and write another digit above it to help them remember how many tens are still represented in the problem after one ten has been changed to ten ones.

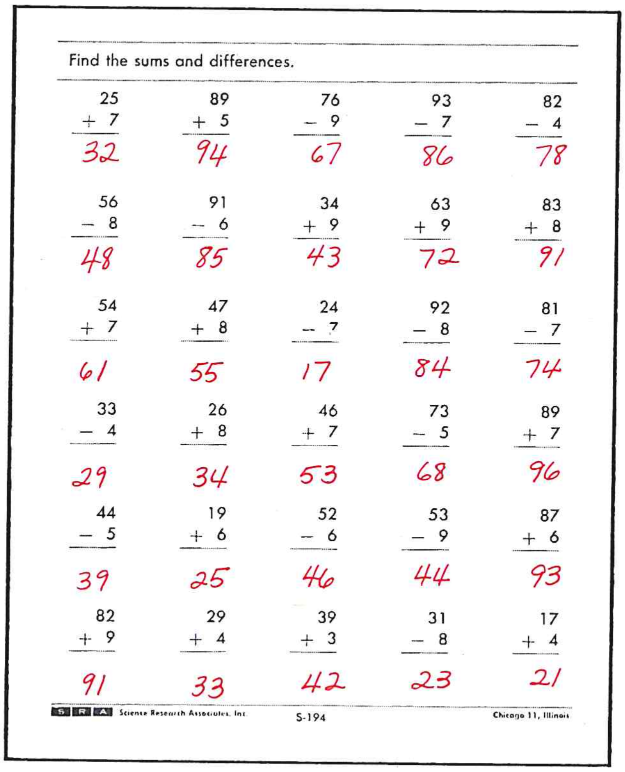


Name

Find the sums and differences.

35 − 8 27	41 − 4 37	28 + 7 35	77 − 9 68	36 + 5 41
79 + 8 87	46 + 7 53	34 − 6 28	62 − 7 55	16 + 9 25
74 + 8 82	57 + 9 66	95 − 6 89	61 − 3 58	47 − 8 39
78 + 3 81	67 + 5 72	68 − 9 59	27 + 7 34	85 − 7 78
56 − 7 49	48 + 9 57	23 − 8 15	72 − 5 67	38 + 6 44

S-193



Find the sums and differences.

25 + 7 32	89 + 5 94	76 − 9 67	93 − 7 86	82 − 4 78
56 − 8 48	91 − 6 85	34 + 9 43	63 + 9 72	83 + 8 91
54 + 7 61	47 + 8 55	24 − 7 17	92 − 8 84	81 − 7 74
33 − 4 29	26 + 8 34	46 + 7 53	73 − 5 68	89 + 7 96
44 − 5 39	19 + 6 25	52 − 6 46	53 − 9 44	87 + 6 93
82 + 9 91	29 + 4 33	39 + 3 42	31 − 8 23	17 + 4 21

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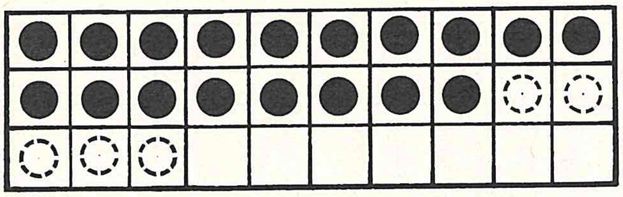
▶ Use work pages S-193 and S-194 for practice in addition and subtraction with carrying and borrowing. Tell the children to find the sums or differences.

Supplemental Experiences

▶ The teacher may wish to introduce to some children the second approach to subtraction that was described under Fundamentals. As a first step in the development of this method, put a problem on the chalkboard: 30 − 3 = ☐. Ask whether someone can tell the answer. Have the children count backward: one less than thirty is twenty-nine, one less than twenty-nine is twenty-eight, one less than twenty-eight is twenty-seven. Follow this procedure with several similar problems.

Have a child begin at twenty and count backward by ones: twenty, nineteen, eighteen, seventeen, and so on as far as seven. Have another child begin at ninety and count backward by tens: ninety, eighty, seventy, sixty, and so on, until ten is reached. Now tell a child to begin at ninety-three and count backward by tens: ninety-three, eighty-three, seventy-three, sixty-three, and so on down to three.

Review a familiar subtraction combination with the children: 13 − 5 = ☐. Use a latticework frame and felt disks. After the problem is completed, remove all felt disks from the flannel board and begin with a set of twenty-three disks in the latticework. Put the equation 23 − 5 = ☐ in the pocket chart. Tell the children that they must remove five disks from the set of twenty-three. Say that they will do it in pieces so that they can really see what happens. Tell them that they must remove as many objects as necessary to have a set of twenty. After the removal of three objects from the frame, place a card in the pocket chart that shows this much of the activity: 23 − 3 = 20.



$$\begin{aligned} 23-5 &= (23-3)-2 \\ &= 20-2 \\ &= 18 \end{aligned}$$

Ask the children how many they were supposed to remove according to the original equation. Ask whether any more objects must be removed besides the three that were taken away. After two more objects have been removed, place an equation related to the action in the pocket chart: 20 − 2 = 18.

Now remove the two cards, 23 − 3 = 20 and 20 − 2 = 18, from the pocket chart. Replace them with a card on which (23 − 3) − 2 = 18 is written. Remind the children that the parentheses are there to help them think which subtraction took place first, to help them think of (23 − 3) as representing the number twenty.

Summarize the thinking steps in solving the subtraction problem 23 − 5 = ☐. Refer to the cards in the pocket chart and have the children help review the ideas.

Follow the same procedure to develop similar subtraction problems.

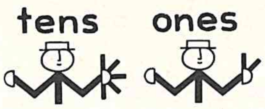
▶ The Countingmen can also be used to develop tne second approach to problems in subtraction. Place the equation 52 − 7 = ☐ on the chalkboard. Call on two children to act as operators of a Ones-man and a Tens-man. Have them show fifty-two on the Countingmen.

Tell ones-operator that his job is to remove seven fingers from his Countingman. He will find that it is impossible to remove seven ones from two ones. Have him transfer the two ones from the fingers to the base of his Countingman. Write an equation on the chalkboard for the action that has taken place: 52 − 2 = 50.

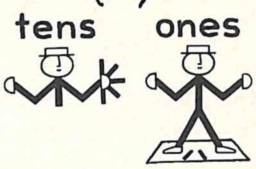
One of the five tens recorded on the Tens-man will have to be exchanged for ten ones if the complete removal of seven fingers is to be accomplished. As the one finger is removed from the Tens-man, ten fingers should be placed on the Ones-man.

The removal of the set of seven fingers can now be completed. Write the equation related to this action, on the chalkboard: 50 − 5 = 45.

(1)

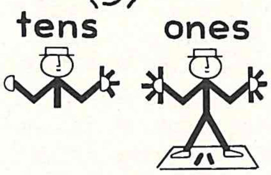


(2)

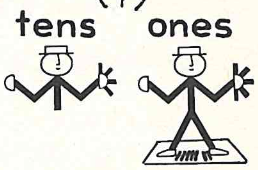


$$\begin{aligned} 52-7 &= (52-2)-5 \\ &= 50-5 \\ &= 45 \end{aligned}$$

(3)



(4)



Summarize the thinking steps in this problem. Continue in this way with similar subtractions.

Addition with Carrying

Work Pages S-195 Through S-204

Objectives

To extend the development of addition with carrying.

To strengthen the understanding of the concept of carrying in addition.

To help children develop skill in addition with carrying.

Fundamentals

Work with addition will now be extended to a pair of two-digit addends. After the child feels secure in this environment, the teacher will present column additions involving three to five addends.

The associative and the commutative properties will again justify the steps taken in solving a problem such as 27 + 35 = ☐.

$27 + 35 = (20 + 7) + (30 + 5)$
expanded notation

$= [(20 + 7) + 30] + 5$
associative property

$= [20 + (7 + 30)] + 5$
associative property

$= [20 + (30 + 7)] + 5$
commutative property

$= [(20 + 30) + 7] + 5$
associative property

$$\begin{aligned} &= (20 + 30) + (7 + 5) && \text{associative property}\\ &= 50 + 12 && \text{place-value notation}\\ &= 62 && \text{addition}\end{aligned}$$

The child will be asked to understand only the simplified version of this problem.

$$\begin{array}{r} 20 + \ \ 7\\ +\ (30 + \ \ 5)\\ \hline 50 + 12 \end{array} = 62$$

The teacher will want the child to be aware that there is no conceptual difference in the process of carrying between a problem such as 27 + 4 and a problem such as 27 + 14. One variation occurs: after the one ten is carried over to the tens place and added to the two tens or twenty already there, one more addition of tens must occur. In column additions, which the child will encounter in this phase of the development of addition with carrying, three or four additions of tens must be completed before the total number of tens is discovered.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Ability to work with addition combinations through eighteen.

Understanding of the concept of place value.

Understanding of the commutative and associative properties of addition.

Ability to work with expanded notation and vertical notation.

Developmental Experiences

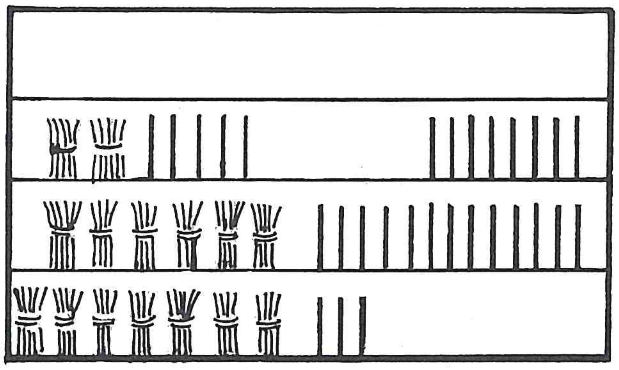
MATERIALS

pocket chart
strips of colored paper
rubber bands
2 Countingmen

▶ Using expanded notation, place the problem 25 + 48 = ☐ on the chalkboard. Have a child

$$\begin{array}{r} 20 + 5\\ +\ (40 + 8)\\ \hline \end{array}$$

place two bundles of ten paper straws each and five single straws in the pocket chart to represent the first addend in the problem. Have a second child place straws in the chart to represent the second addend. Ask still another child to join the sets of ones and then the sets of tens. When he gets thirteen ones and six tens, ask him to record this in the expanded-notation problem. Now have him bundle ten ones into one set of ten and tell how many ones and how many tens he has all together: seven tens and three ones are seventy-three. Have the child complete the expanded notation problem by recording the standard place-value numeral for the number 60 + 13.



$$\begin{array}{r} 20 + \ \ 5\\ +\ (40 + \ \ 8)\\ \hline 60 + 13 \ =\\ 70 + \ \ 3 \ =\\ 73 \end{array}$$

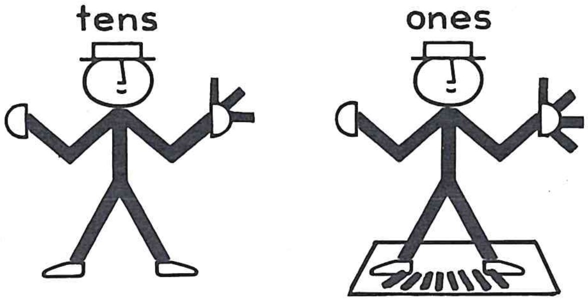
Continue in this way, having the children work with sets and expanded notation in similar problems of addition with carrying.

▶ Use two Countingmen to illustrate the algorism for addition with carrying. Write a problem on the chalkboard:

$$\begin{array}{r} 34\\ +\ 28\\ \hline \end{array}$$

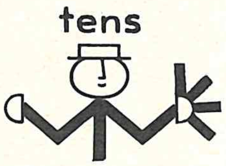
Have one child act as operator of the Ones-man; have another child act as operator of the Tens-man.

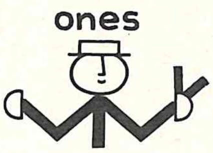
Have the addend thirty-four shown on the Countingmen. Have the children explain what the problem requires: twenty-eight is to be added to thirty-four. Have them tell how many ones should be added first.



The ones-operator should begin by placing six of the eight pins on his man. He will notice that his Countingman has all ten fingers before he has put the full set of eight with the set of four. Ask him what he must do. He should change ten ones to one ten. At this point in the activity, hold the set of one ten up before the class while the remaining two ones are placed on the Ones-man.

Now have the tens-operator place the one ten on his man as a record of the one ten "carried over" from the Ones-man to the Tens-man. Record this action on the chalkboard. Say that four plus eight

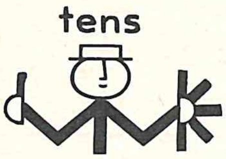


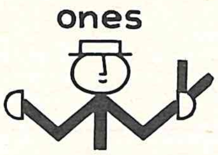


$$\begin{array}{r} {}^{1} \\ 34 \\ +\ 28 \\ \hline 2 \end{array}$$

equals twelve and that twelve is interpreted as one ten and two ones. Explain that the two is recorded in the ones place and that the one ten is carried over to the tens place.

Ask the class to tell how many tens are on the Countingman at this time. Have the tens-operator look at the original problem on the chalkboard and tell how many tens he must add. He should put two more fingers on the Tens-man. Ask the class to tell



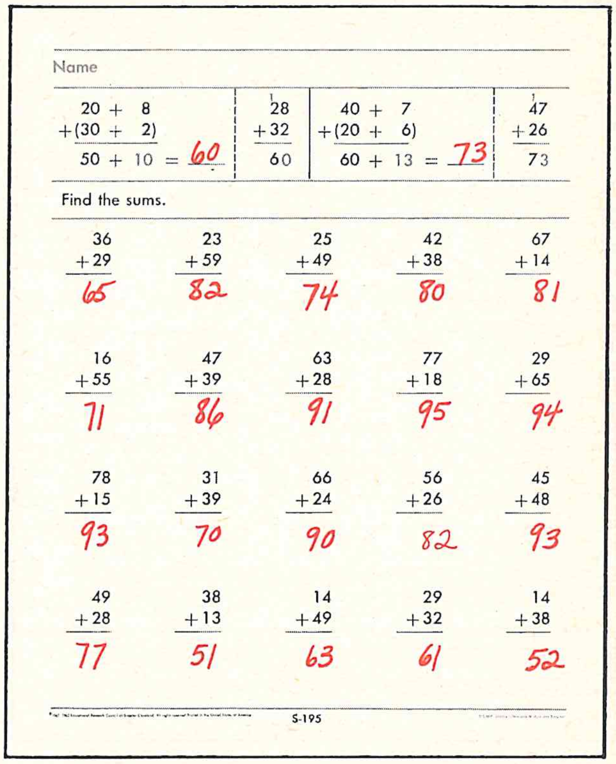


$$\begin{array}{r} {}^{1} \\ 34 \\ +\ 28 \\ \hline 62 \end{array}$$

how many tens the Countingman shows. Record this in vertical notation. Say, "Ten plus thirty equals forty; forty plus twenty equals sixty." Explain that the digit 6 in the algorism represents the six tens, or sixty, in the problem. Have the class tell how many ones and how many tens the Countingmen show. Refer to the vertical notation on the chalkboard and say, "Sixty plus two equals sixty-two. The sum of thirty-four and twenty-eight is sixty-two." Have the class summarize the steps in carrying as they refer to the vertical notation.

Continue this procedure with similar problems. Go through each step carefully; use the Countingmen and have a child write the vertical notation on the chalkboard for each problem. Include problems such as 56 + 14 in which the digit 0 must be recorded in the ones place to indicate there are no ones left after one ten has been carried over to the tens place.

▶ Use work pages S-195 and S-196 as a follow-up to the activity with the Countingmen. Work the two problems at the top of the page with the children. Have them complete these two work pages on their own. Tell them to find the sums; permit them to use



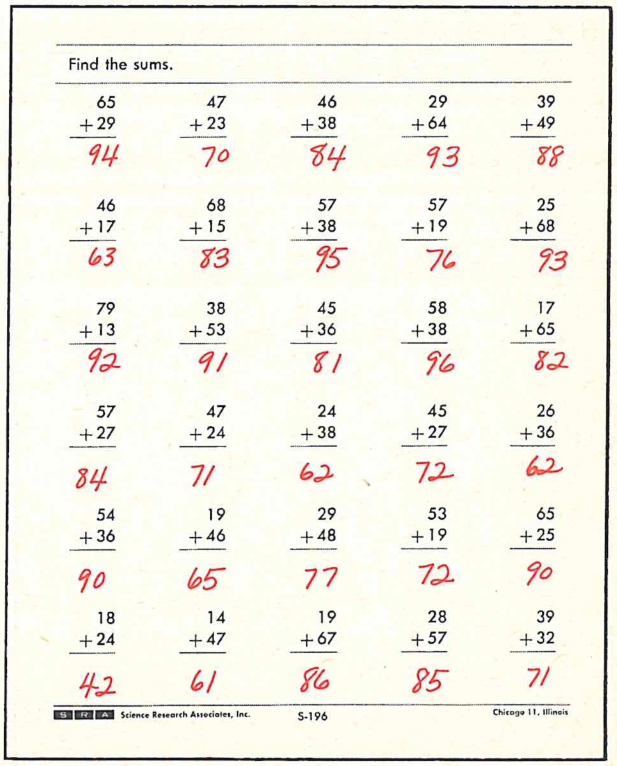
Name

20 + 8 +(30 + 2) 50 + 10 = 60	1 28 +32 60	40 + 7 +(20 + 6) 60 + 13 = 73	1 47 +26 73

Find the sums.

36 +29 65	23 +59 82	25 +49 74	42 +38 80	67 +14 81
16 +55 71	47 +39 86	63 +28 91	77 +18 95	29 +65 94
78 +15 93	31 +39 70	66 +24 90	56 +26 82	45 +48 93
49 +28 77	38 +13 51	14 +49 63	29 +32 61	14 +38 52

S-195



Find the sums.

65 +29 94	47 +23 70	46 +38 84	29 +64 93	39 +49 88
46 +17 63	68 +15 83	57 +38 95	57 +19 76	25 +68 93
79 +13 92	38 +53 91	45 +36 81	58 +38 96	17 +65 82
57 +27 84	47 +24 71	24 +38 62	45 +27 72	26 +36 62
54 +36 90	19 +46 65	29 +48 77	53 +19 72	65 +25 90
18 +24 42	14 +47 61	19 +67 86	28 +57 85	39 +32 71

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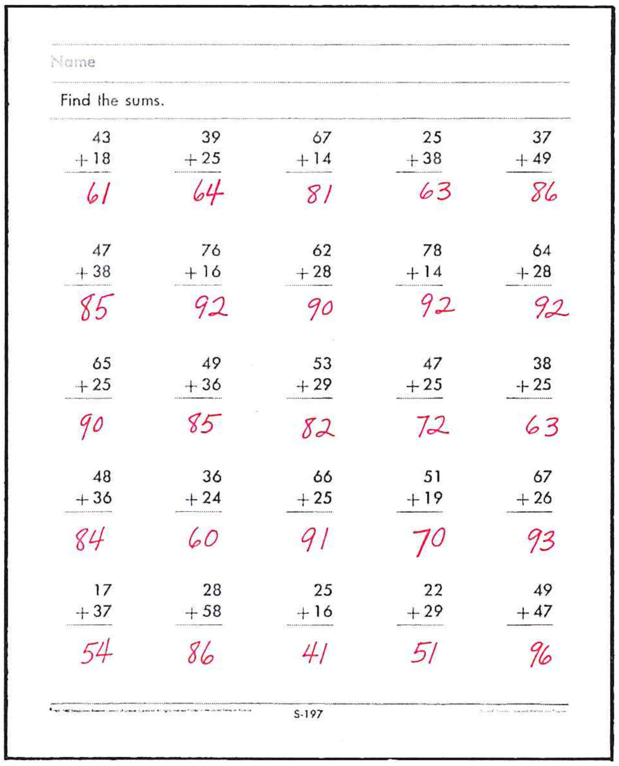
the carry-digit.

▶ Work pages S-197 through S-200 can be used to give the children the opportunity to grow more proficient in working with addition problems involving carrying. Direct the children to find the sums.

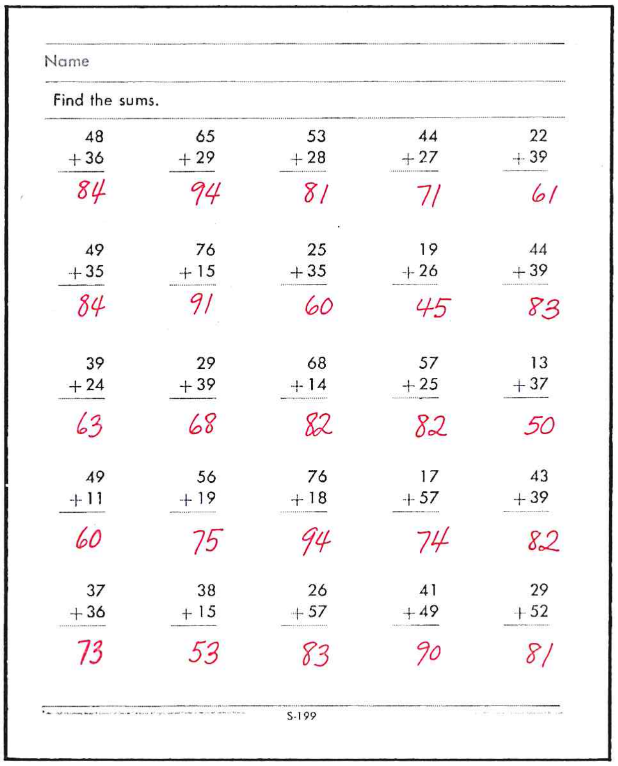


Name

Find the sums.

43	39	67	25	37
+18	+25	+14	+38	+49
61	64	81	63	86
47	76	62	78	64
+38	+16	+28	+14	+28
85	92	90	92	92
65	49	53	47	38
+25	+36	+29	+25	+25
90	85	82	72	63
48	36	66	51	67
+36	+24	+25	+19	+26
84	60	91	70	93
17	28	25	22	49
+37	+58	+16	+29	+47
54	86	41	51	96

S-197

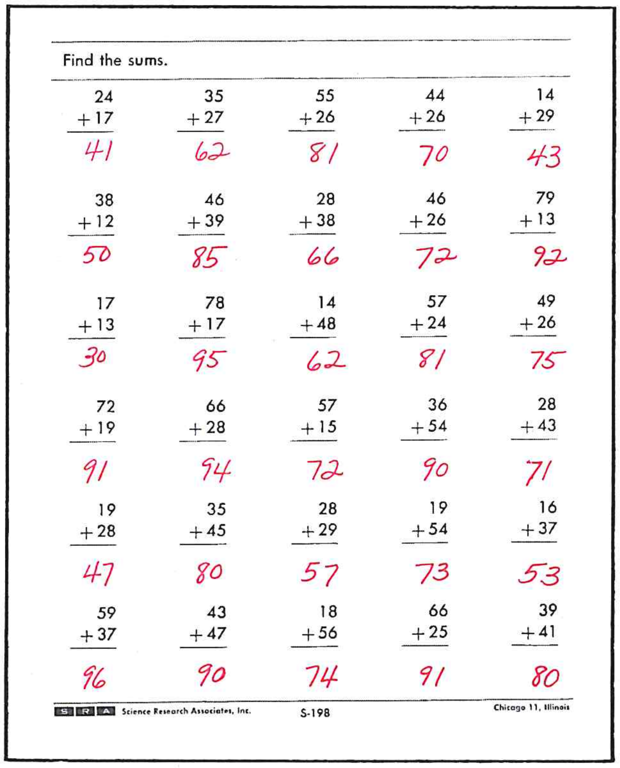


Name

Find the sums.

48	65	53	44	22
+36	+29	+28	+27	+39
84	94	81	71	61
49	76	25	19	44
+35	+15	+35	+26	+39
84	91	60	45	83
39	29	68	57	13
+24	+39	+14	+25	+37
63	68	82	82	50
49	56	76	17	43
+11	+19	+18	+57	+39
60	75	94	74	82
37	38	26	41	29
+36	+15	+57	+49	+52
73	53	83	90	81

S-199



Find the sums.

24	35	55	44	14
+17	+27	+26	+26	+29
41	62	81	70	43
38	46	28	46	79
+12	+39	+38	+26	+13
50	85	66	72	92
17	78	14	57	49
+13	+17	+48	+24	+26
30	95	62	81	75
72	66	57	36	28
+19	+28	+15	+54	+43
91	94	72	90	71
19	35	28	19	16
+28	+45	+29	+54	+37
47	80	57	73	53
59	43	18	66	39
+37	+47	+56	+25	+41
96	90	74	91	80

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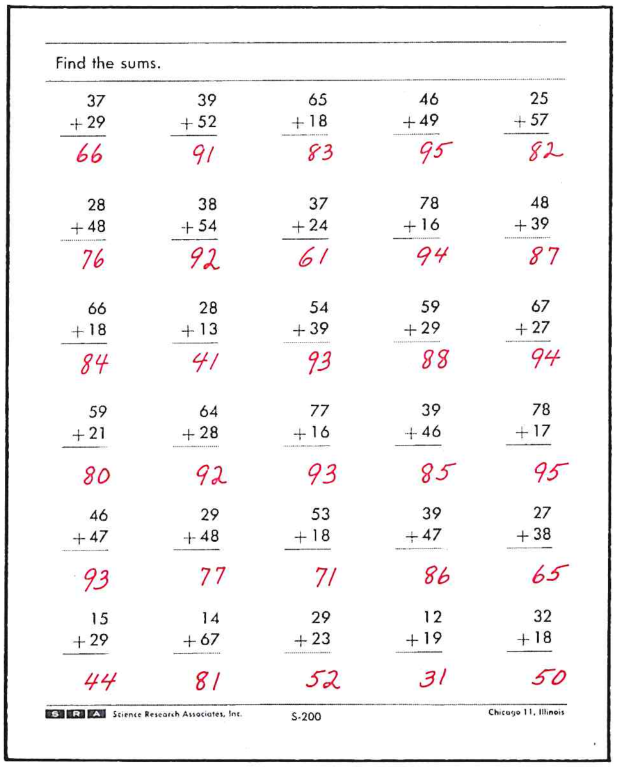

Find the sums.

37	39	65	46	25
+29	+52	+18	+49	+57
66	91	83	95	82
28	38	37	78	48
+48	+54	+24	+16	+39
76	92	61	94	87
66	28	54	59	67
+18	+13	+39	+29	+27
84	41	93	88	94
59	64	77	39	78
+21	+28	+16	+46	+17
80	92	93	85	95
46	29	53	39	27
+47	+48	+18	+47	+38
93	77	71	86	65
15	14	29	12	32
+29	+67	+23	+19	+18
44	81	52	31	50

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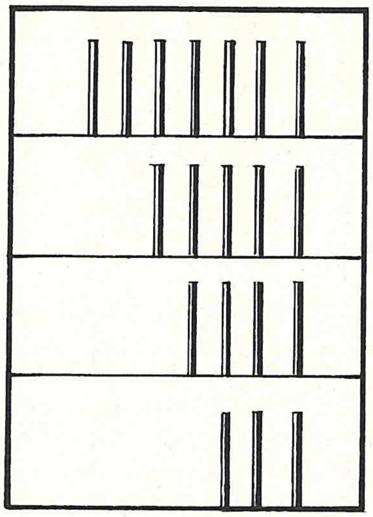
▶ Review the concepts involved in column addition. Place a set of seven objects, a set of five objects, a set of four objects, and a set of three objects in the pocket chart. Ask the children to tell how many are in each set. Have one child record the numerals for the given numbers in column-addition notation on the chalkboard. Ask how many there are all together. Some child may say, "Seven plus five equals twelve, twelve plus four equals sixteen, sixteen plus three equals nineteen."



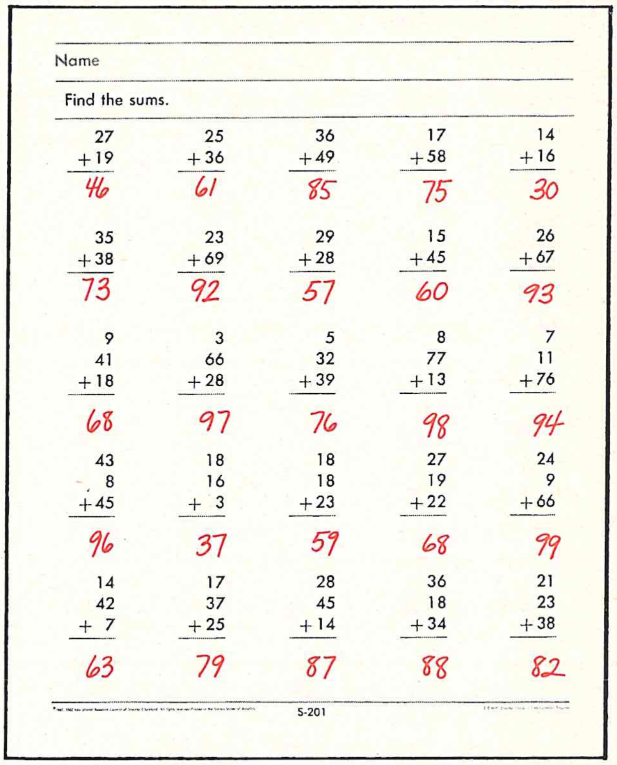
7
5
4
+3

Ask if there is an easier way to do this series of additions. Anticipate the answer: "Look first for combinations of ten." For this problem, children may think: 7 + 3 = 10, 10 + 5 = 15, 15 + 4 = 19. Discuss why it is possible to group the numbers in this way in addition. Remind the children that the problem involves both commutativity and associativity. Repeat this activity with other sets and related column additions.

▶ Put two-digit column-addition problems on the chalkboard. Have children work through these problems orally, in the most convenient way. Be sure they realize that they cannot always find combinations of ten in the problems, that they should also look for other convenient combinations.

24 32 + 16	15 27 6 + 50	37 12 28 + 11

▶ Work pages S-201 through S-204 can be used to give the children the opportunity to grow more proficient in working with column additions involving

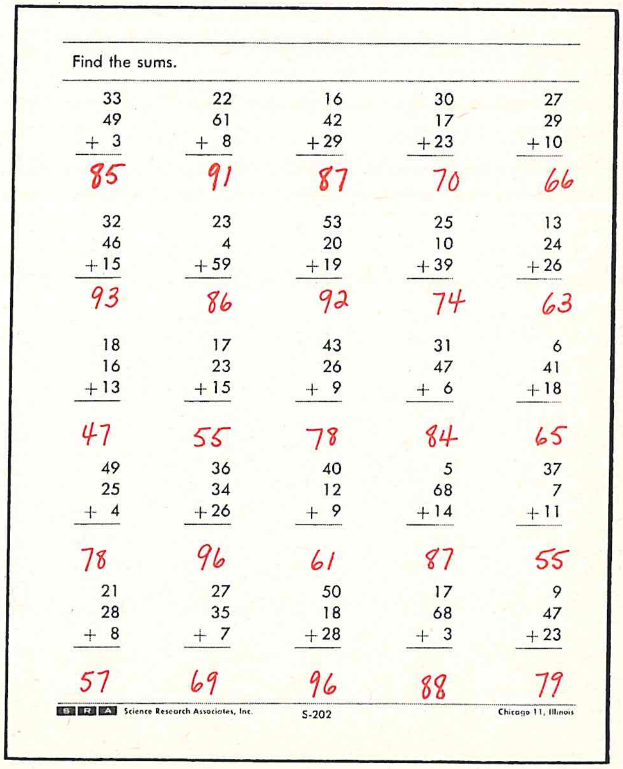


Name

Find the sums.

27 +19 46	25 +36 61	36 +49 85	17 +58 75	14 +16 30
35 +38 73	23 +69 92	29 +28 57	15 +45 60	26 +67 93
9 41 +18 68	3 66 +28 97	5 32 +39 76	8 77 +13 98	7 11 +76 94
43 8 +45 96	18 16 + 3 37	18 18 +23 59	27 19 +22 68	24 9 +66 99
14 42 + 7 63	17 37 +25 79	28 45 +14 87	36 18 +34 88	21 23 +38 82

S-201



Find the sums.

33 49 + 3 85	22 61 + 8 91	16 42 +29 87	30 17 +23 70	27 29 +10 66
32 46 +15 93	23 4 +59 86	53 20 +19 92	25 10 +39 74	13 24 +26 63
18 16 +13 47	17 23 +15 55	43 26 + 9 78	31 47 + 6 84	6 41 +18 65
49 25 + 4 78	36 34 +26 96	40 12 + 9 61	5 68 +14 87	37 7 +11 55
21 28 + 8 57	27 35 + 7 69	50 18 +28 96	17 68 + 3 88	9 47 +23 79

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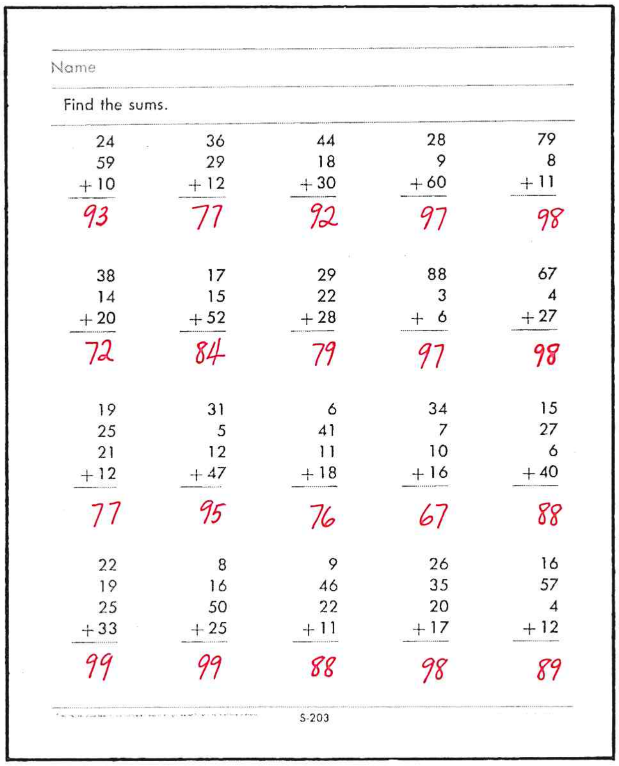
carrying. Remind the children that some of the addends will have no digits representing tens—that there are no tens in the numbers symbolized by these particular numerals. After working a sample with the children, tell them to find the sums. Permit the use of the carry-digit.

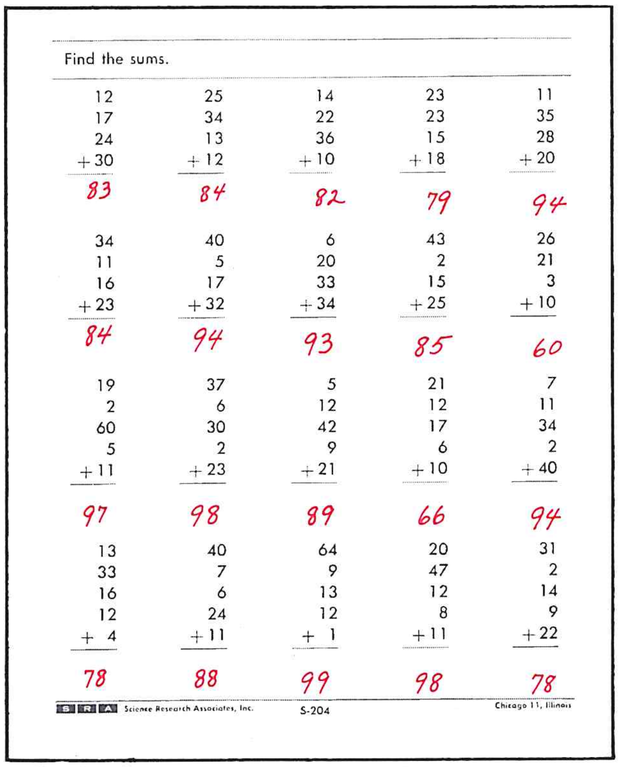


Name

Find the sums.

24 59 +10 93	36 29 +12 77	44 18 +30 92	28 9 +60 97	79 8 +11 98
38 14 +20 72	17 15 +52 84	29 22 +28 79	88 3 + 6 97	67 4 +27 98
19 25 21 +12 77	31 5 12 +47 95	6 41 11 +18 76	34 7 10 +16 67	15 27 6 +40 88
22 19 25 +33 99	8 16 50 +25 99	9 46 22 +11 88	26 35 20 +17 98	16 57 4 +12 89

S-203



Find the sums.

12 17 24 +30 83	25 34 13 +12 84	14 22 36 +10 82	23 23 15 +18 79	11 35 28 +20 94
34 11 16 +23 84	40 5 17 +32 94	6 20 33 +34 93	43 2 15 +25 85	26 21 3 +10 60
19 2 60 5 +11 97	37 6 30 2 +23 98	5 12 42 9 +21 89	21 12 17 6 +10 66	7 11 34 2 +40 94
13 33 16 12 + 4 78	40 7 6 24 +11 88	64 9 13 12 + 1 99	20 47 12 8 +11 98	31 2 14 9 +22 78

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Supplemental Experiences

▶ The teacher may wish to have the children use the vertical form and reduce a given problem to two simple problems. Have the children work problems such as the following, on the chalkboard, first in expanded notation and then in vertical form. As the

$$28 + 67 = \square$$

$$\begin{array}{rl} 20 + \ \ 8 & \\ + \ (60 + \ \ 7) & \\ \hline 80 + 15 & = 95 \end{array} \qquad \begin{array}{r} 28 \\ +\ 67 \\ \hline 15 \\ +\ 80 \\ \hline 95 \end{array} \qquad \begin{array}{r} \overset{1}{2}8 \\ +\ 67 \\ \hline 5 \\ \\ \overset{1}{2}8 \\ +\ 67 \\ \hline 95 \end{array}$$

children discuss the different forms for the problem, have them use arrows to show the corresponding parts.

▶ Give the children long equations and see if they can use commutativity and associativity to make the addition easier. Have the children use the commutative and associative properties to show that $5 + (6 + 7) = (7 + 5) + 6$ or that $(2 + 8) + 4 = 2 + (4 + 8)$. Present one example on the chalkboard and then have children attempt problems of this kind on their own.

$$\begin{aligned} 5 + 7 + 2 + 5 + 3 + 8 + 6 &= \\ (5 + 5) + (7 + 3) + (2 + 8) + 6 &= \\ 10 + 10 + 10 + 6 &= 36 \end{aligned}$$

$$\begin{aligned} 25 + 7 + 5 + 3 + 9 &= \\ (25 + 5) + (7 + 3) + 9 &= \\ 30 + 10 + 9 &= 49 \end{aligned}$$

Show that $(3 + 2) + 4 = (3 + 4) + 2$.

$(3 + 2) + 4 = 3 + (2 + 4)$
associative property
$= 3 + (4 + 2)$
commutative property
$= (3 + 4) + 2$
associative property

▶ Provide an opportunity for children to work with number sequences. The number sequence with which children are most familiar is the sequence of the counting numbers which begins with one and increases by one each time.

The numbers of a sequence are called terms. Each of the terms of a sequence corresponds to one of the ordinal numerals—there is a first term, a second term, a third term, and so on. There is usually some definite rule or pattern that determines the terms of the sequence. In work with sequences, the rule or pattern is established by listing some of the terms.

A standard exercise on sequences is to give the children a sequence with certain terms missing and have them fill in the missing terms. This means that they must discover the rule or pattern that was used when the sequence was constructed, and then apply this rule or pattern to find the missing terms. It is necessary to give a sufficient number of terms to establish the pattern.

Keep the sequences simple at first. The children will discover that the rule for obtaining each successive term is usually to add (or subtract) a certain number. In these sequences, children will recognize patterns of skip-counting and be able to complete the simple ones with ease.

5, 7, 9, ____, 13, ____, 17.
(Pattern: Skip-count by 2's, starting with 5.)
16, 19, ____, 25, 28, ____, 34, 37, ____.
(Pattern: Start at 16 and add 3 each time.)

Story Problems

Work Pages S-205 and S-206

Objectives

To develop skill in two-digit addition involving carrying.

To increase the ability to analyze and solve story problems.

Fundamentals

An understanding of the operation of addition has been established, and the process of carrying has been developed. Much practice has been given in computation. Now the child will be asked to apply his skills and understandings to solve problems. He should know that addition is the operation which relates to the joining or putting together of sets, and should be able to use this understanding in selecting an operation for solving story problems. He should also be able to apply his understanding of the commutative property in checking addition problems.

Readiness for Understanding

Understanding of place value.

Ability to add numbers represented by two-digit numerals with and without carrying.

Skill in working with combinations through eighteen.

Developmental Experiences

MATERIALS

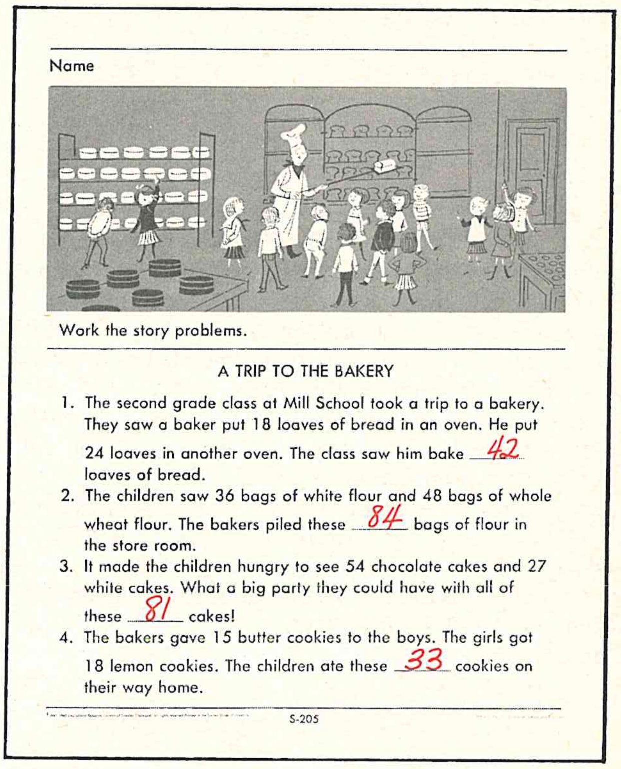
Countingmen
plastic numerals

▶ Distribute plastic numerals to each student.

Pose a problem about a familiar classroom experience. Say, for example:

> Johnnie has read thirty-five library books. Mary has read twenty-eight books. Together they have read how many books?

Ask what operation must be used to find the total number of books. Call on one child to work this problem at the chalkboard and another to show it on the Countingmen. Have the other children work at their desks with plastic numerals. Ask the child at the board and then the one with the Countingmen to explain the procedure they used.



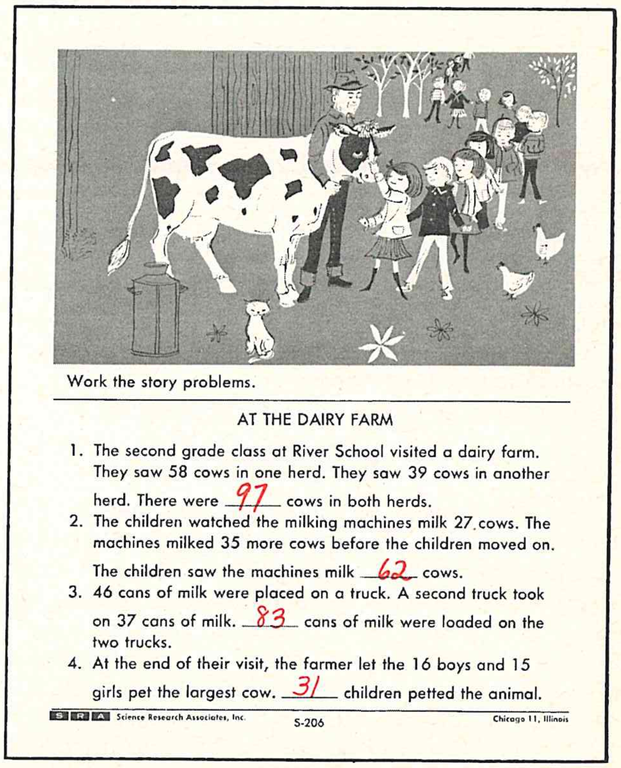
Name

Work the story problems.

A TRIP TO THE BAKERY

1. The second grade class at Mill School took a trip to a bakery. They saw a baker put 18 loaves of bread in an oven. He put 24 loaves in another oven. The class saw him bake 42 loaves of bread.
2. The children saw 36 bags of white flour and 48 bags of whole wheat flour. The bakers piled these 84 bags of flour in the store room.
3. It made the children hungry to see 54 chocolate cakes and 27 white cakes. What a big party they could have with all of these 81 cakes!
4. The bakers gave 15 butter cookies to the boys. The girls got 18 lemon cookies. The children ate these 33 cookies on their way home.

S-205



Work the story problems.

AT THE DAIRY FARM

1. The second grade class at River School visited a dairy farm. They saw 58 cows in one herd. They saw 39 cows in another herd. There were 97 cows in both herds.
2. The children watched the milking machines milk 27 cows. The machines milked 35 more cows before the children moved on. The children saw the machines milk 62 cows.
3. 46 cans of milk were placed on a truck. A second truck took on 37 cans of milk. 83 cans of milk were loaded on the two trucks.
4. At the end of their visit, the farmer let the 16 boys and 15 girls pet the largest cow. 31 children petted the animal.

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Repeat with other problems of a similar nature.

▶ Discuss any field trips the class has taken.

Distribute work pages S-205 and S-206 Tell the children that the stories on this sheet are about trips another second-grade class took. Discuss the pictures and read the stories. Have the children work independently to find the answers. Then have them check their answers by changing the position of the addends and adding again.

$$\begin{array}{r} 18 \\ +\,24 \\ \hline 42 \end{array} \qquad\qquad \begin{array}{r} 24 \\ +\,18 \\ \hline 42 \end{array}$$

Supplemental Experiences

▶ Encourage children to write a story problem, solve, and check by interchanging addends. Seasonal subjects, subjects studied in reading class, or school staff members make lively items for story problems.

▶ Provide plenty of oral practice to help the children maintain and extend their knowledge of addition and subtraction combinations.

Make use of patterns in helping children learn to think quickly. Ask them to give the sum as you call out number combinations: 9 + 6, 19 + 6, 29 + 6, 39 + 6, and so forth through 89 + 6. Again call out 9 + 6, but vary the pattern by following with 9 + 16, 9 + 26, 9 + 36, and so forth.

Call out subtraction combinations and have children give the differences: 12 − 8, 22 − 8, 32 − 8, through 92 − 8.

Subtraction—Addition

BORROWING AND CARRYING

Work Pages S-207 Through S-218

Objectives

To extend the development of subtraction with borrowing.

To strengthen understanding of the concept of borrowing in subtraction.

To emphasize the relation between addition and subtraction.

To develop skill in subtraction with borrowing and addition with carrying.

To increase ability to solve story problems.

Fundamentals

The development of subtraction will now be extended to work with numbers represented by two-digit numerals, with borrowing. The teacher will want to encourage the child to apply his understanding of the inverse relation between addition and subtraction as he works with both operations in expanded notation and vertical form. The child should be able to use addition to check subtraction problems.

Story problems will be presented to give the child an opportunity to apply his skills. To be able to choose the correct operation in solving story problems, the child should be aware of the different physical situations which relate to addition and subtraction. Addition is involved whenever sets are joined or put together. He should know that subtraction is involved whenever it is necessary to determine how many are left after some have been taken away from a given set; when the numbers of two sets are compared to determine how many more or how many less; and when it is necessary to find how many are needed to have a given number.

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Ability to work with number combinations through eighteen.

Understanding of the concept of place value.

Understanding of the concepts of carrying and borrowing.

Awareness of the inverse relation between addition and subtraction.

Ability to work with expanded notation and vertical notation.

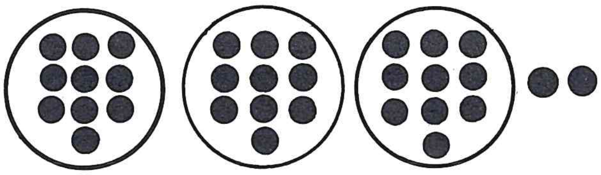
Ability to analyze story problems and select the correct operation.

Developmental Experiences

MATERIALS

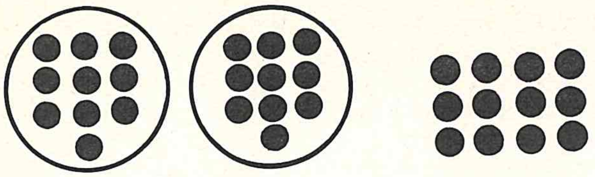
flannel board
yarn
felt counters
2 Countingmen
sticks

▶ On the chalkboard, write the expanded notation for the problem 32 − 17 = □. On the flannel board, place sets to represent the first number in this subtraction. Circle each set of ten with a piece of yarn.



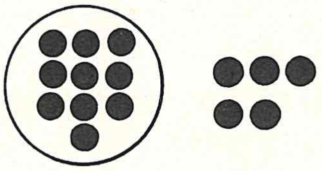
$$\begin{array}{r} 30 + 2 \\ -\ \underline{(10 + 7)} \end{array}$$

Have children explain what they must do first in this problem: they must try to subtract seven from two. Tell them to remove seven disks from the two loose disks on the board. They will immediately discover that this is impossible. Ask how they can make the removal possible. They should say that one of the sets of ten can be thought of as ten ones. Remove the yarn from around one set of ten. Explain that three tens and two ones can be thought of as two tens and twelve ones. It is possible to remove



$$\begin{array}{rcr} 30 + 2 & = & 20 + 12 \\ -\,(10 + 7) & = & -\,(10 + 7) \\ \hline \end{array}$$

seven from twelve. Help a child write the expanded notation related to this action, on the chalkboard. Have children observe that another numeral for 30 + 2 is 20 + 12. Have the subtraction of ones



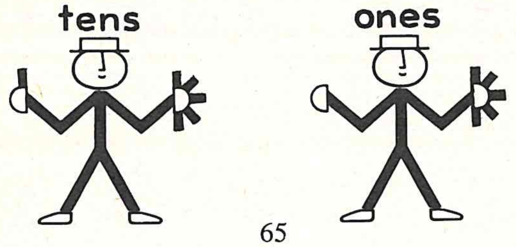
$$\begin{array}{rcrl} 30 + 2 & = & 20 + 12 & \\ -\,(10 + 7) & = & -\,(10 + 7) & \\ \hline & & 10 + 5 & = 15 \end{array}$$

completed in the expanded notation. Ask the children to decide if any more subtractions must be made according to the given problem. Now have one set of ten removed from the two sets of ten. Have a ten subtracted from twenty in the expanded notation. Ask how many sets of tens and ones remain on the flannel board. Have the expanded notation completed and the standard place-value numeral for 10 + 5, written as a final step.

Develop other subtraction problems of this kind, in the same way. Have children help write the expanded notation related to the action.

▶ Use the Countingmen to illustrate subtraction with borrowing. Write the problem 65 − 37 = ☐ in expanded notation on the chalkboard. Next to this problem, write the vertical form for the subtraction.

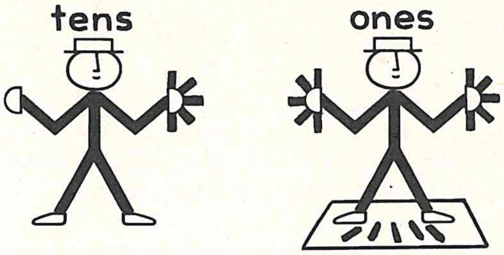
Have two children show the first number in the problem on the Countingmen. Tell the ones-operator



$$\begin{array}{r} 65 \\ -\,37 \\ \hline \end{array}$$

that his job is to remove a set of seven ones from six tens and five ones. He will discover that this is impossible, unless he gets one ten from the tens-operator and changes it to ten ones, which when joined to the five ones on his Countingman, will make fifteen. Have the ones-operator place the five ones, from his Countingman's hands on the base of the device. Have the tens-operator give one ten to the ones-operator. As the tens-operator removes one finger from his Countingman, the ones-operator should put ten fingers on his Countingman.

Ask the class to describe what happened in terms of ones and tens: six tens and five ones is now shown as five tens and fifteen ones. Refer to the

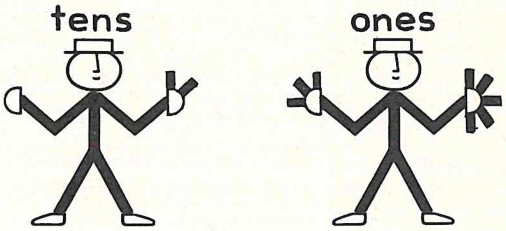


$$\begin{array}{r} \overset{5}{\cancel{6}}{}^{1}5 \\ -\,3\,7 \\ \hline \end{array}$$

algorism form of the problem. Cross out the digit representing six tens or sixty. Above it write the digit 5 to represent five tens or fifty. Place a small 1 to the left of the digit 5 to represent fifteen.

Have the ones-operator remove seven from the fifteen now shown on his Countingman. He should take away the five on the base first and then two more from the Countingman's hands. Subtract seven from fifteen in the vertical notation and write 8 in the ones place under the line.

Now that the ones-operator has completed his job, look to the tens-operator. The problem on the chalkboard reminds him that he must subtract three tens; he should remove three tens from the five tens on his Countingman.



$$\begin{array}{r} \overset{5}{\cancel{6}}{}^{1}5 \\ -\,3\,7 \\ \hline 2\,8 \end{array}$$

Complete the subtraction of tens in the algorism. Have the difference read. Summarize the steps involved in solving this subtraction problem. Relate the symbolism in the algorism notation to the symbolism in the expanded notation.

$$\begin{array}{rcrlr} 60 + 5 & = & 50 + 15 & & \overset{5}{\cancel{6}}{}^{1}5 \\ -\,(30 + 7) & = & -\,(30 + 7) & & -\,3\,7 \\ \hline & & 20 + 8 & = 28 & 2\,8 \end{array}$$

Continue in this way to develop other subtraction problems of the same kind. Include problems involving zero, such as 30 − 18 = ☐. In solving this problem, the expanded notation and the algorism would be set up in the following way:

$$\begin{array}{rcrlr} 30 + 0 & = & 20 + 10 & & \overset{2}{\cancel{3}}{}^{1}0 \\ -\,(10 + 8) & = & -\,(10 + 8) & & -\,1\,8 \\ \hline & & 10 + 2 & = 12 & 1\,2 \end{array}$$

Have children check their subtractions by setting up the inverse additions. The number subtracted

$$\begin{array}{r} \overset{5}{\cancel{6}}{}^{1}5 \\ -\,3\;7 \\ \hline 2\;8 \end{array} \qquad\qquad \begin{array}{r} \overset{1}{2}8 \\ +\,37 \\ \hline 65 \end{array}$$

should be the number added to the difference. Have children reverse the steps taken in the Countingman demonstrations of subtraction as each related addition is set up in vertical (algorism) form.

▶ Use work pages S-207 and S-208 after children have had sufficient experience with concrete objects to enable them to visualize the steps in subtraction with borrowing. Work through the problem at the top of the page with the children; ask them to write the needed numerals as the discussion proceeds. Work the first two problems in the lower section of the page with the group. Tell them to cross out the numeral in the tens place and write another numeral above it to help them remember how many tens are still represented in the problem after one ten was changed to ten ones. Have them write the digit 1 above and to the left of the digit in ones place to help them recall that one ten has been changed to ten ones and is to be included with the ones already represented in the numeral. Have children work independently to find the differences for the remainder of the problems on work page S-207 and all of the problems on work page S-208. Give help only if needed.

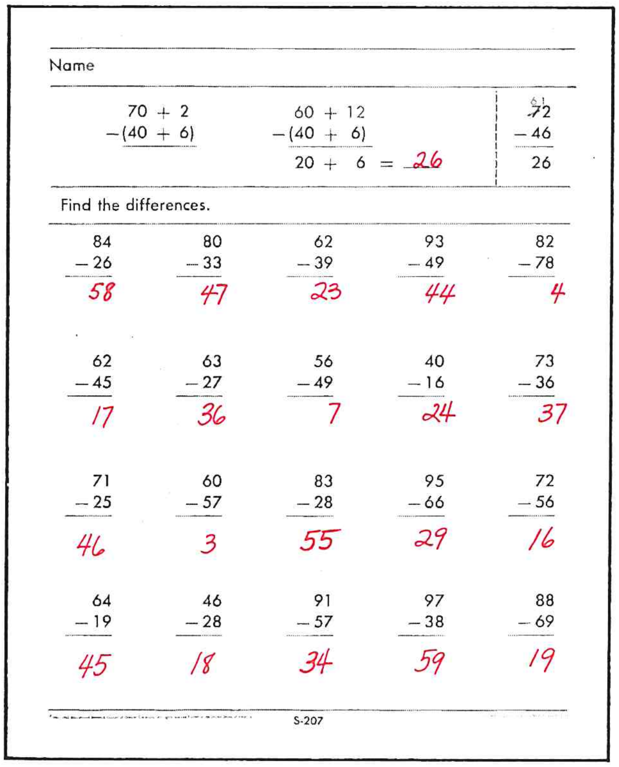
▶ Use work pages S-209 through S-211 to provide practice with problems involving borrowing. Tell the children to find the differences.

Work page S-212 will give children the opportunity to check their subtractions by using the inverse additions. Work the first pair of problems with the class. Have them complete the remainder of the page on their own.

▶ Pose three different problems without numbers and have the children decide what operation must be used to find the answers:

1. Mary had some lollipops. She gave some to her little brother. How many did she have left?
2. Mary had some red marbles. Her brother had some blue marbles. Who had more marbles? How many more? Who had fewer marbles? How many fewer?
3. Mary had some jacks. Her big sister had more jacks. Mary wished she had as many jacks as her sister. How many more did she wish to have?

Let the children supply different numbers for each story and find the specific answers. Lead them to see that no matter what the numbers, the situations described call for subtraction. Even if they assign the same number of marbles to Mary and her brother in problem 2, subtraction will tell them that the difference between the two numbers is zero.



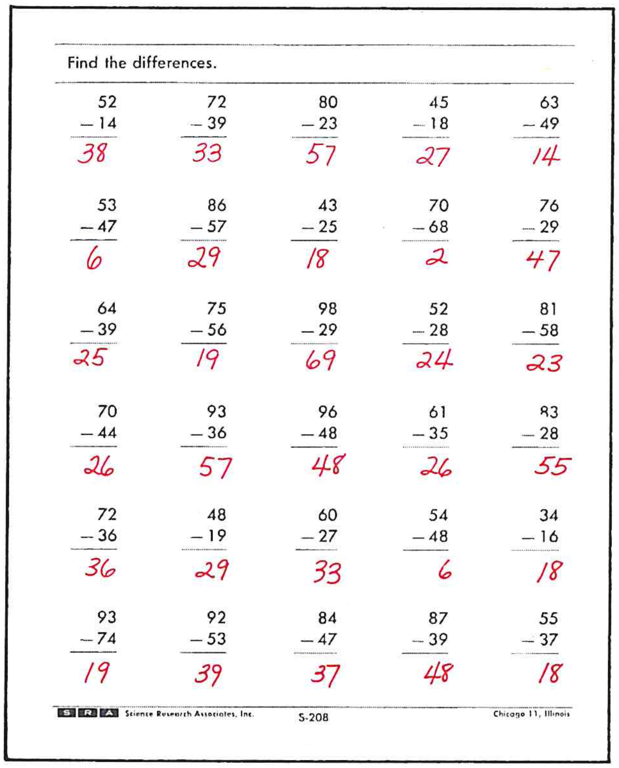
Name

$$\begin{array}{r} 70 + 2 \\ -(40 + 6) \\ \hline \end{array} \qquad \begin{array}{r} 60 + 12 \\ -(40 + \;\,6) \\ \hline 20 + \;\,6 \end{array} = \underline{26} \qquad \begin{array}{r} \overset{6}{\cancel{7}}{}^{1}2 \\ -\,46 \\ \hline 26 \end{array}$$

Find the differences.

84 − 26 58	80 − 33 47	62 − 39 23	93 − 49 44	82 − 78 4
62 − 45 17	63 − 27 36	56 − 49 7	40 − 16 24	73 − 36 37
71 − 25 46	60 − 57 3	83 − 28 55	95 − 66 29	72 − 56 16
64 − 19 45	46 − 28 18	91 − 57 34	97 − 38 59	88 − 69 19

S-207



Find the differences.

52 − 14 38	72 − 39 33	80 − 23 57	45 − 18 27	63 − 49 14
53 − 47 6	86 − 57 29	43 − 25 18	70 − 68 2	76 − 29 47
64 − 39 25	75 − 56 19	98 − 29 69	52 − 28 24	81 − 58 23
70 − 44 26	93 − 36 57	96 − 48 48	61 − 35 26	83 − 28 55
72 − 36 36	48 − 19 29	60 − 27 33	54 − 48 6	34 − 16 18
93 − 74 19	92 − 53 39	84 − 47 37	87 − 39 48	55 − 37 18

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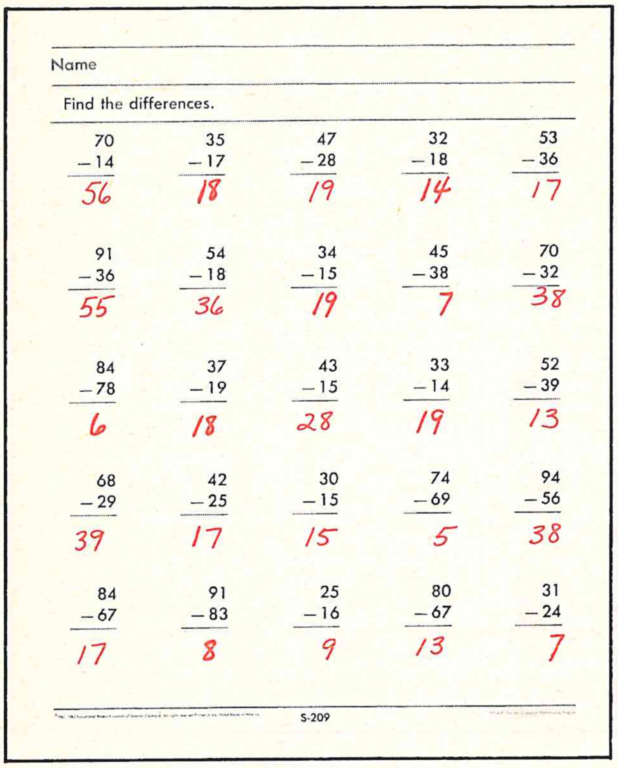
▶ Introduce the lesson on work page S-213 with a discussion of crops, animals, and work on a farm. Have one child read the first story problem and tell how the answer can be found. Have a second child show the problem on the Countingmen, and a third child use sticks to demonstrate it. Have a fourth child work the problem at the board, explaining each step in the computation. Ask a fifth child to check the

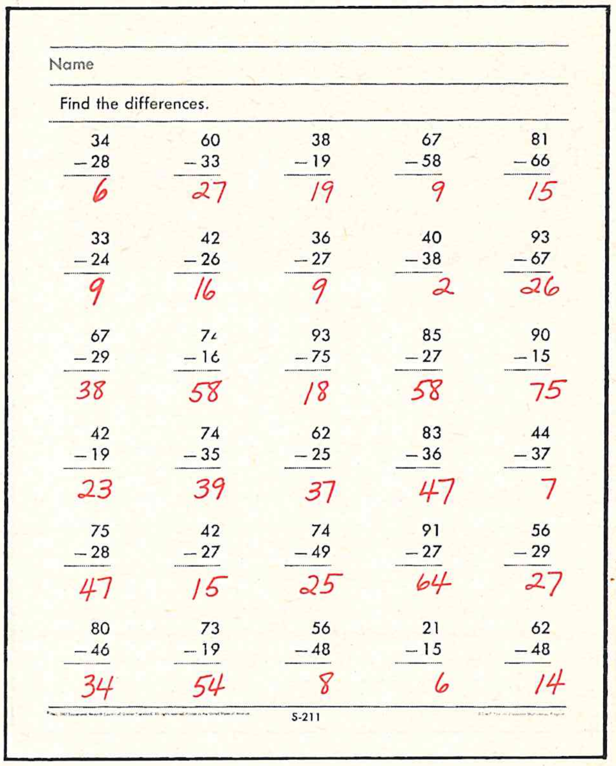


Name

Find the differences.

70 − 14 56	35 − 17 18	47 − 28 19	32 − 18 14	53 − 36 17
91 − 36 55	54 − 18 36	34 − 15 19	45 − 38 7	70 − 32 38
84 − 78 6	37 − 19 18	43 − 15 28	33 − 14 19	52 − 39 13
68 − 29 39	42 − 25 17	30 − 15 15	74 − 69 5	94 − 56 38
84 − 67 17	91 − 83 8	25 − 16 9	80 − 67 13	31 − 24 7

S-209

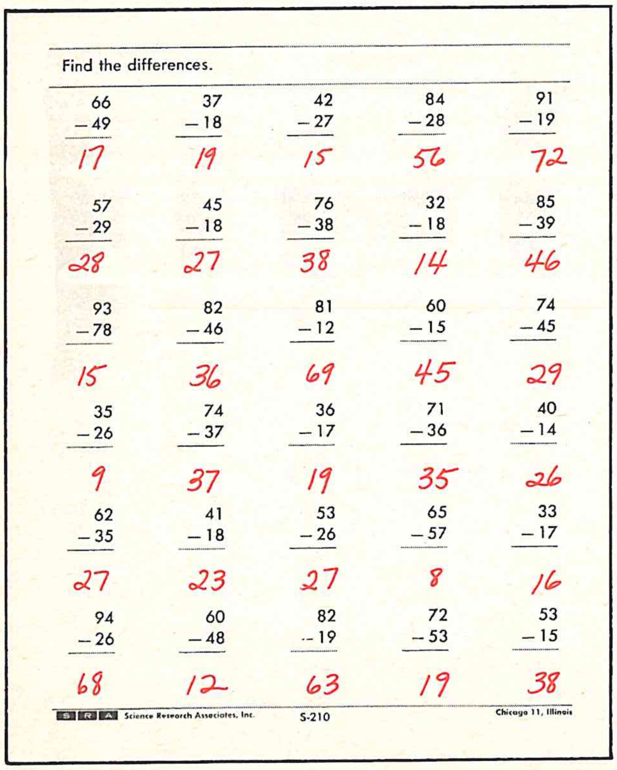


Name

Find the differences.

34 − 28 6	60 − 33 27	38 − 19 19	67 − 58 9	81 − 66 15
33 − 24 9	42 − 26 16	36 − 27 9	40 − 38 2	93 − 67 26
67 − 29 38	74 − 16 58	93 − 75 18	85 − 27 58	90 − 15 75
42 − 19 23	74 − 35 39	62 − 25 37	83 − 36 47	44 − 37 7
75 − 28 47	42 − 27 15	74 − 49 25	91 − 27 64	56 − 29 27
80 − 46 34	73 − 19 54	56 − 48 8	21 − 15 6	62 − 48 14

S-211



Find the differences.

66 − 49 17	37 − 18 19	42 − 27 15	84 − 28 56	91 − 19 72
57 − 29 28	45 − 18 27	76 − 38 38	32 − 18 14	85 − 39 46
93 − 78 15	82 − 46 36	81 − 12 69	60 − 15 45	74 − 45 29
35 − 26 9	74 − 37 37	36 − 17 19	71 − 36 35	40 − 14 26
62 − 35 27	41 − 18 23	53 − 26 27	65 − 57 8	33 − 17 16
94 − 26 68	60 − 48 12	82 − 19 63	72 − 53 19	53 − 15 38

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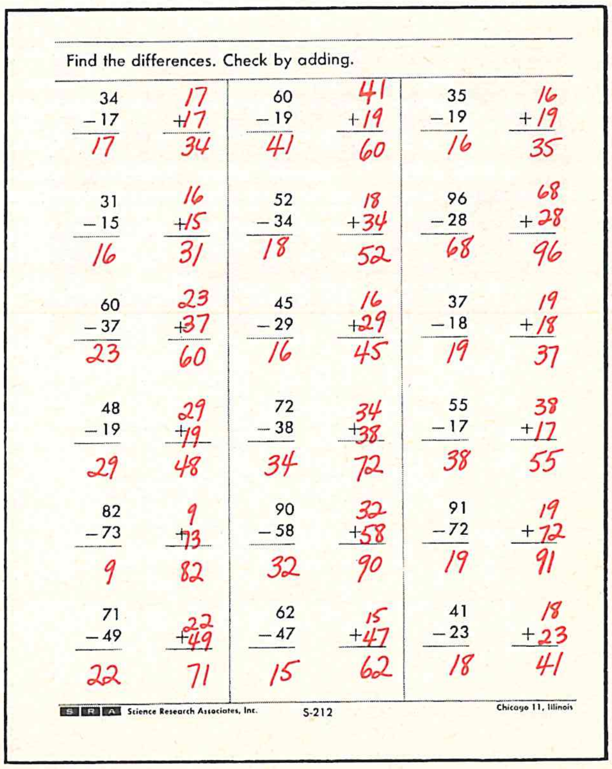

Find the differences. Check by adding.

34 − 17 17	17 + 17 34	60 − 19 41	41 + 19 60	35 − 19 16	16 + 19 35
31 − 15 16	16 + 15 31	52 − 34 18	18 + 34 52	96 − 28 68	68 + 28 96
60 − 37 23	23 + 37 60	45 − 29 16	16 + 29 45	37 − 18 19	19 + 18 37
48 − 19 29	29 + 19 48	72 − 38 34	34 + 38 72	55 − 17 38	38 + 17 55
82 − 73 9	9 + 73 82	90 − 58 32	32 + 58 90	91 − 72 19	19 + 72 91
71 − 49 22	22 + 49 71	62 − 47 15	15 + 47 62	41 − 23 18	18 + 23 41

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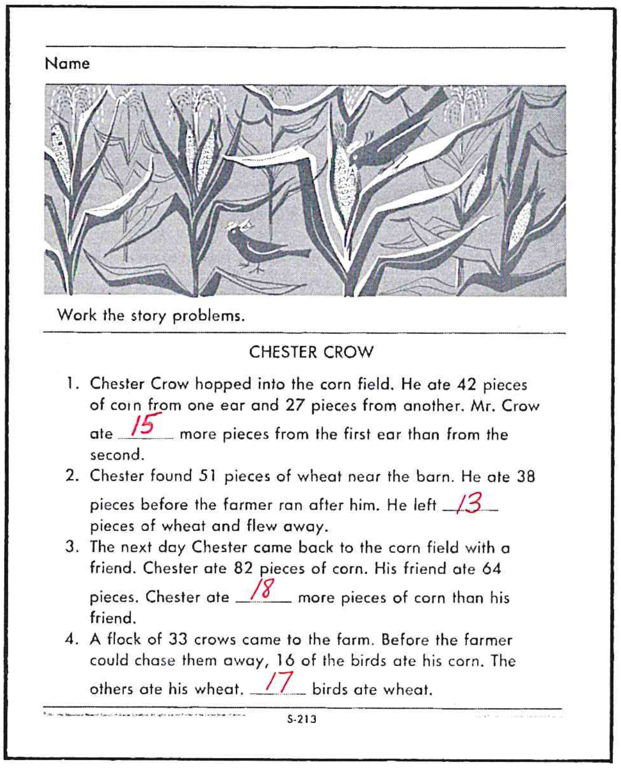
problem by addition. Some classes will be able to complete this page and work page S-214 independently; others should have teacher guidance.

▶ Use work pages S-215 through S-218 to give children the opportunity to test themselves on their ability to work with the carrying and borrowing con-



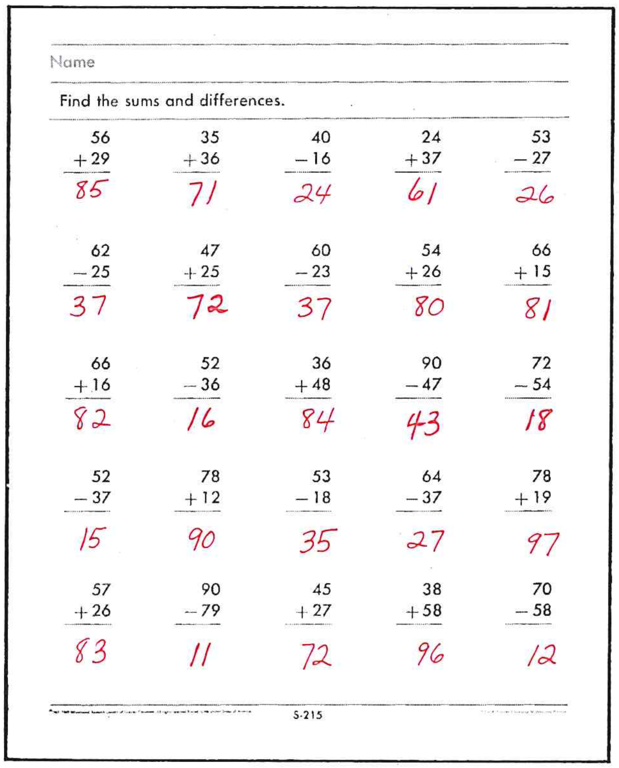
Name

Work the story problems.

CHESTER CROW

1. Chester Crow hopped into the corn field. He ate 42 pieces of corn from one ear and 27 pieces from another. Mr. Crow ate 15 more pieces from the first ear than from the second.
2. Chester found 51 pieces of wheat near the barn. He ate 38 pieces before the farmer ran after him. He left 13 pieces of wheat and flew away.
3. The next day Chester came back to the corn field with a friend. Chester ate 82 pieces of corn. His friend ate 64 pieces. Chester ate 18 more pieces of corn than his friend.
4. A flock of 33 crows came to the farm. Before the farmer could chase them away, 16 of the birds ate his corn. The others ate his wheat. 17 birds ate wheat.

S-213

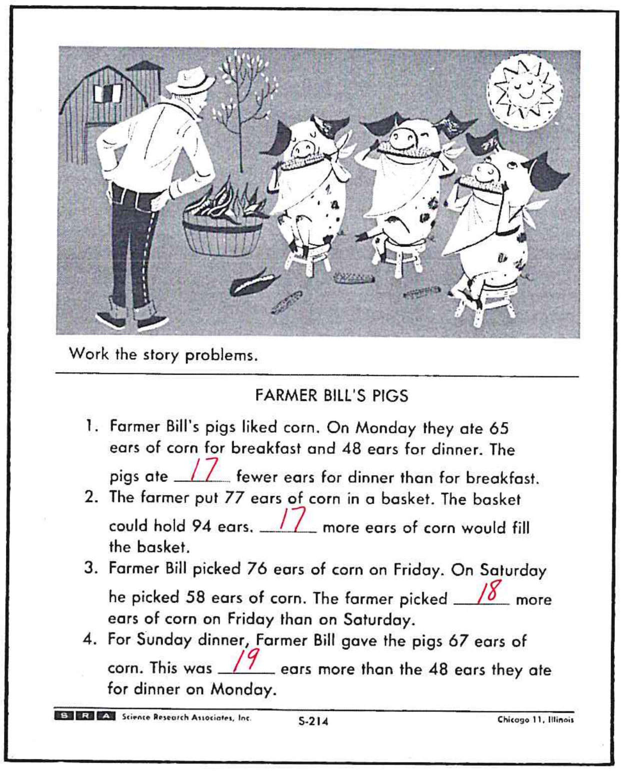


Name

Find the sums and differences.

56	35	40	24	53
+29	+36	−16	+37	−27
85	71	24	61	26
62	47	60	54	66
−25	+25	−23	+26	+15
37	72	37	80	81
66	52	36	90	72
+16	−36	+48	−47	−54
82	16	84	43	18
52	78	53	64	78
−37	+12	−18	−37	+19
15	90	35	27	97
57	90	45	38	70
+26	−79	+27	+58	−58
83	11	72	96	12

S-215



Work the story problems.

FARMER BILL'S PIGS

1. Farmer Bill's pigs liked corn. On Monday they ate 65 ears of corn for breakfast and 48 ears for dinner. The pigs ate 17 fewer ears for dinner than for breakfast.
2. The farmer put 77 ears of corn in a basket. The basket could hold 94 ears. 17 more ears of corn would fill the basket.
3. Farmer Bill picked 76 ears of corn on Friday. On Saturday he picked 58 ears of corn. The farmer picked 18 more ears of corn on Friday than on Saturday.
4. For Sunday dinner, Farmer Bill gave the pigs 67 ears of corn. This was 19 ears more than the 48 ears they ate for dinner on Monday.

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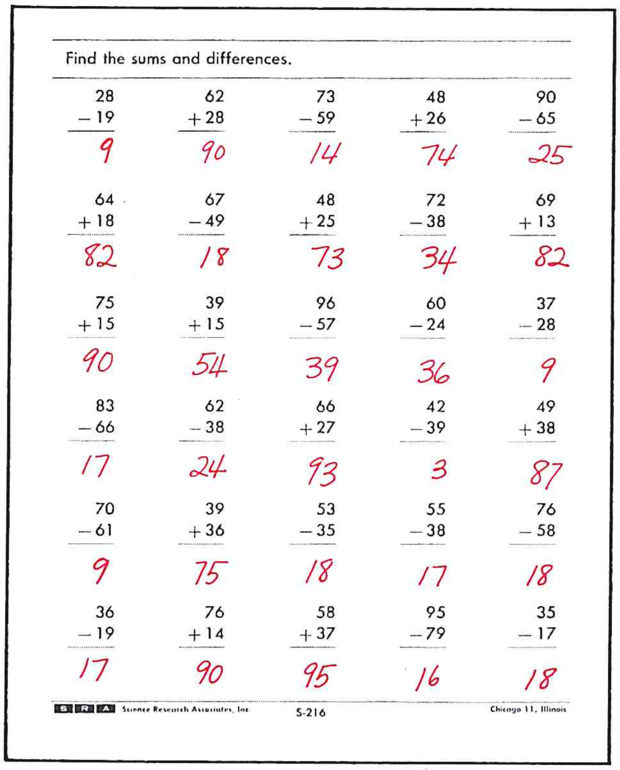

Find the sums and differences.

28	62	73	48	90
−19	+28	−59	+26	−65
9	90	14	74	25
64	67	48	72	69
+18	−49	+25	−38	+13
82	18	73	34	82
75	39	96	60	37
+15	+15	−57	−24	−28
90	54	39	36	9
83	62	66	42	49
−66	−38	+27	−39	+38
17	24	93	3	87
70	39	53	55	76
−61	+36	−35	−38	−58
9	75	18	17	18
36	76	58	95	35
−19	+14	+37	−79	−17
17	90	95	16	18

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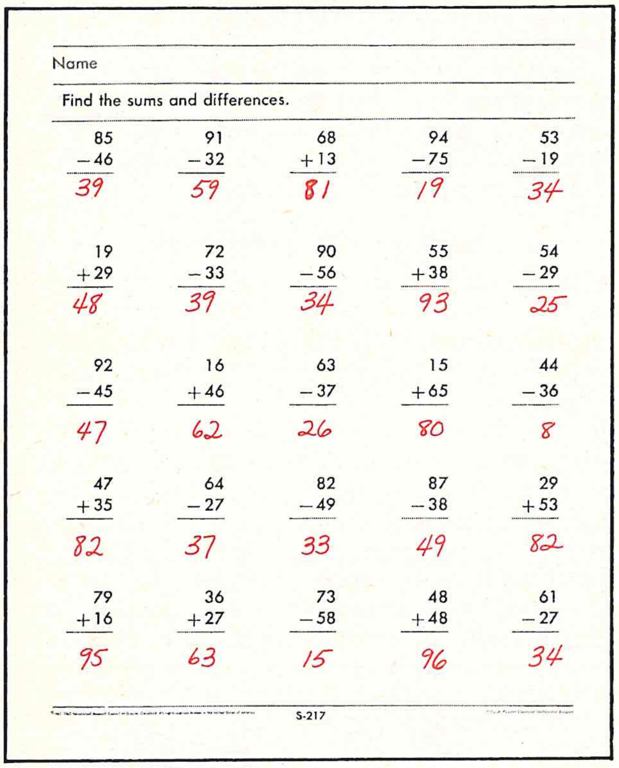
cepts developed up to this time. Tell children to find the sums and differences.

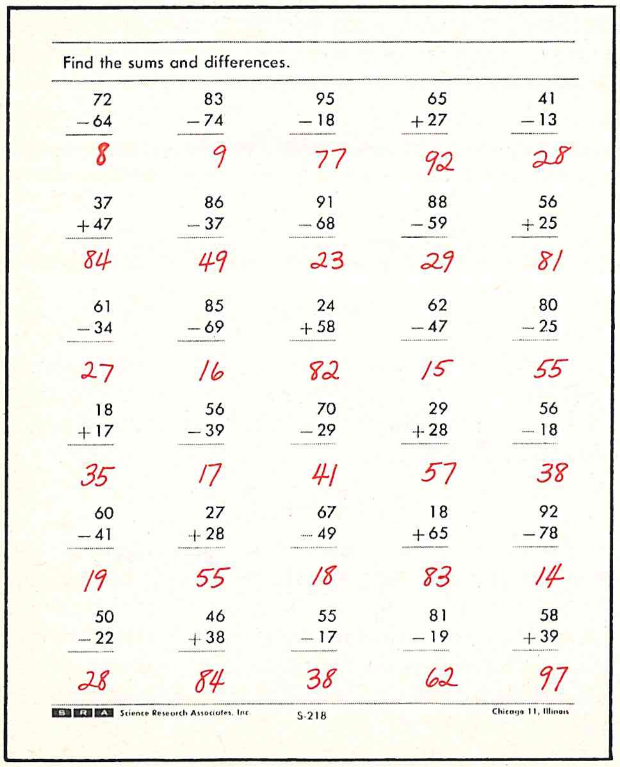


Name

Find the sums and differences.

85 −46 39	91 −32 59	68 +13 81	94 −75 19	53 −19 34
19 +29 48	72 −33 39	90 −56 34	55 +38 93	54 −29 25
92 −45 47	16 +46 62	63 −37 26	15 +65 80	44 −36 8
47 +35 82	64 −27 37	82 −49 33	87 −38 49	29 +53 82
79 +16 95	36 +27 63	73 −58 15	48 +48 96	61 −27 34

S-217



Find the sums and differences.

72 −64 8	83 −74 9	95 −18 77	65 +27 92	41 −13 28
37 +47 84	86 −37 49	91 −68 23	88 −59 29	56 +25 81
61 −34 27	85 −69 16	24 +58 82	62 −47 15	80 −25 55
18 +17 35	56 −39 17	70 −29 41	29 +28 57	56 −18 38
60 −41 19	27 +28 55	67 −49 18	18 +65 83	92 −78 14
50 −22 28	46 +38 84	55 −17 38	81 −19 62	58 +39 97

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Supplemental Experiences

▶ The teacher may wish to present some children with a variation in the writing and working of subtractions involving borrowing. She may show them how to subtract "in pieces" in problems involving numbers represented by two two-digit numerals. In

$$65 - 48 = \square$$

becomes

$$\begin{array}{r} 65\\ -\ 8\\ \hline 57\end{array} \qquad\qquad \begin{array}{r} 57\\ -\ 40\\ \hline 17\end{array}$$

this method ones are subtracted from the number represented by the first numeral; then tens are subtracted from this difference for the final result.

▶ The development of number sequences may be extended to the area of subtraction. Place a series

35, 33, 31,____, 27, 25, 23,____,____, 17.
71,____, 65, 62,____, 56, 53,____,____, 44.

of number sequences on the chalkboard. Have children take turns supplying the missing terms.

▶ As a variation of this activity begin with any chosen number and have children, in turn, count backward from this number by threes, by twos, or by fours.

Story Problems

Work Pages S-219 and S-220

Objectives

To increase the understanding of story problems and to develop a logical approach to the solution.

To review the processes involved in addition with carrying and subtraction with borrowing.

Fundamentals

Story problems should be used to make practical application of the algorisms learned previously.

Much emphasis should be given to the importance of finding the correct operation in solving such problems and in checking the answers.

Readiness for Understanding

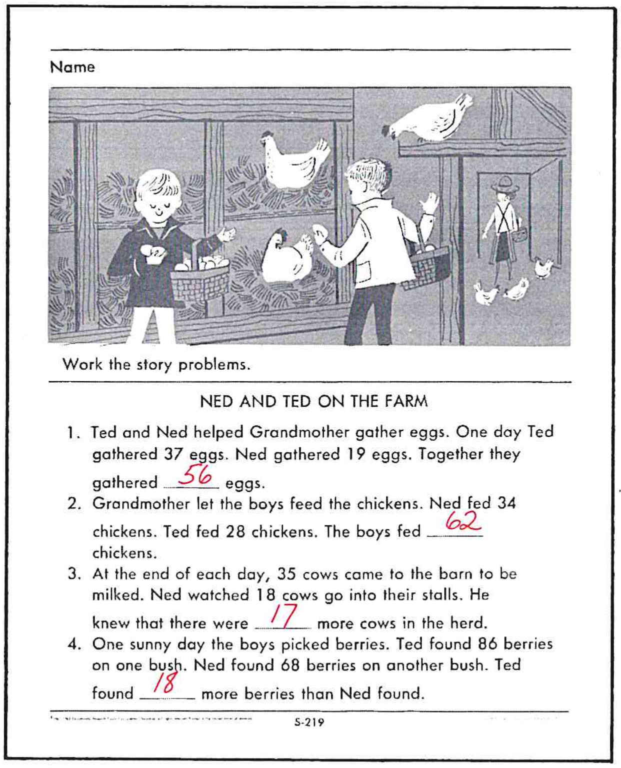
Skill in interpreting story problems.

An understanding of place value.

Ability to follow procedures involved in addition with carrying and subtraction with borrowing.

Developmental Experiences

▶ A discussion of farm work and activities on the farm will arouse interest in the story problems on work page S-219. Ask the children how the two boys in the picture at the top of the page are helping with the chores on the farm. Have the children work the first story problem together, first stating the problem and the known facts, then selecting the correct operations, and finally computing the answer. Ask if the answer is reasonable. Have the children check their addition by changing the order of the addends. Save problem two for solving independently and work together to solve problem three which requires subtraction. After the answer has been found, have the children check it by adding back what was subtracted.



Name

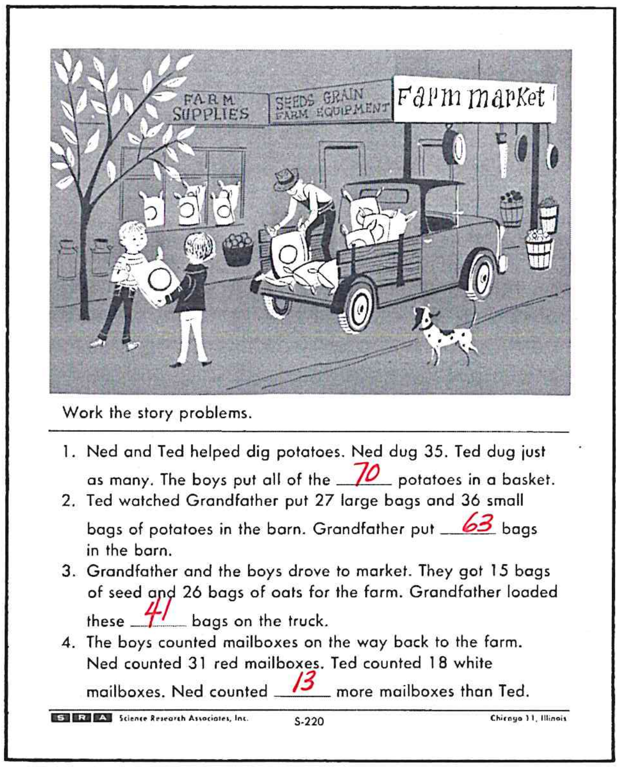
NED AND TED ON THE FARM

Work the story problems.

NED AND TED ON THE FARM

1. Ted and Ned helped Grandmother gather eggs. One day Ted gathered 37 eggs. Ned gathered 19 eggs. Together they gathered 56 eggs.
2. Grandmother let the boys feed the chickens. Ned fed 34 chickens. Ted fed 28 chickens. The boys fed 62 chickens.
3. At the end of each day, 35 cows came to the barn to be milked. Ned watched 18 cows go into their stalls. He knew that there were 17 more cows in the herd.
4. One sunny day the boys picked berries. Ted found 86 berries on one bush. Ned found 68 berries on another bush. Ted found 18 more berries than Ned found.

S-219



FARM SUPPLIES
SEEDS GRAIN FARM EQUIPMENT
Farm market

Work the story problems.

1. Ned and Ted helped dig potatoes. Ned dug 35. Ted dug just as many. The boys put all of the 70 potatoes in a basket.
2. Ted watched Grandfather put 27 large bags and 36 small bags of potatoes in the barn. Grandfather put 63 bags in the barn.
3. Grandfather and the boys drove to market. They got 15 bags of seed and 26 bags of oats for the farm. Grandfather loaded these 41 bags on the truck.
4. The boys counted mailboxes on the way back to the farm. Ned counted 31 red mailboxes. Ted counted 18 white mailboxes. Ned counted 13 more mailboxes than Ted.

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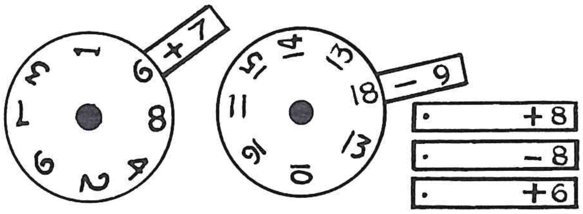
Have the children complete problems two and four independently. Let those who got the correct answers do the problems on work page S-220 by themselves. If some children have difficulty, work the problems with them in a small group.

Supplemental Experiences

▶ Have each of the faster workers write a story problem. Give them an opportunity to read these original stories. The author may request another child to tell the operation used in solving the story problem.

▶ Provide many opportunities for the children to maintain their skills in addition and subtraction combinations through eighteen. Make addition and subtraction wheels from heavy tagboard circles and strips. Write numerals around the outer edge of each circle. Write an operational sign and numeral, such as + 4 or − 8, on the end of each strip. Attach a strip beneath the circle with a paper fastener as illustrated. Have the child move the strip around the wheel and give each sum or difference. Provide extra strips so that the combinations can be changed frequently.



Money

Work Pages S-221 and S-222

Objectives

To review the concept of measurement of value.

To continue developing the understanding of the relations among the penny, nickel, and dime as units of measure of value.

Fundamentals

The second grade child has had the experience of spending pennies and nickels. In most instances, he knows that they can buy the same article with five pennies that they can buy with a nickel. The second-grade child is aware that these coins are not physically alike and that the set of five pennies contains a different number of objects than does a set of one nickel. He has been introduced to the idea that the penny has the value of one cent, the nickel has the value of five cents, and the dime has the value of ten cents. Many realize that the value rather than the number of coins determines buying power.

For many children, a thorough understanding of the concept of measurement of value will come only after many experiences with these ideas in concrete situations. Although the work with money is presented here as a unit, the teacher may find it advantageous to space the material out over the remainder of the year.

Finding the value of a set of coins begins with selecting the coin of the greatest value and involves counting by tens, fives, or ones.

See Key Topics in Mathematics for the Primary Teacher: Measurement.

Readiness for Understanding

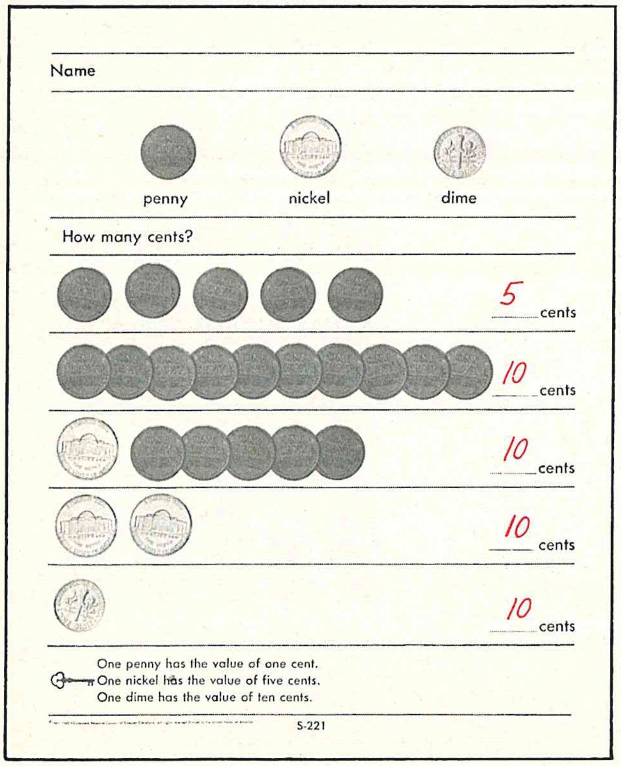
Ability to count to fifty by ones, fives, and tens.

Developmental Experiences

MATERIALS	VOCABULARY
10 pennies	penny
4 nickels	nickel
2 dimes	dime
	cent
	value

▶ Hold up a *penny*. Ask for the name of the coin. Establish that a penny has the *value* of one *cent*. Ask a child to find the words "one cent" on a penny. Hold up a *nickel*. Ask for the name of the coin. Have a child find the word "cents" on the coin. Establish that a nickel has the value of five cents. Hold up a *dime*. Investigate the coin to see if its value in cents is printed on the coin. Since it is not, encourage the children to use previous experience to find the value, ten cents.

Have a child choose two coins and count to arrive at their combined value. Begin counting with the coin of greater value. Extend this experience to ten coins with a sum less than fifty cents. Emphasize the value of the coins throughout this experience.


Name

penny nickel dime

How many cents?

5 cents

10 cents

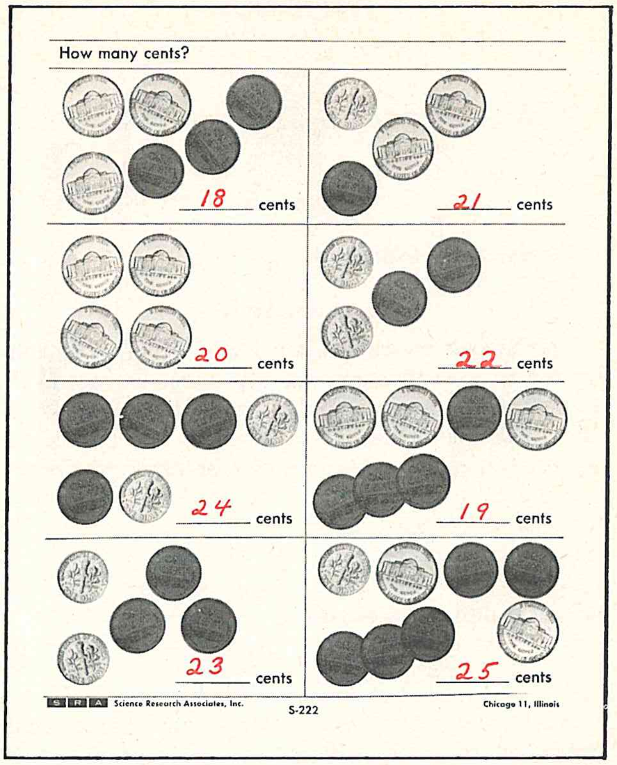
10 cents

10 cents

10 cents

One penny has the value of one cent.
One nickel has the value of five cents.
One dime has the value of ten cents.

S-221


How many cents?

18 cents

21 cents

20 cents

22 cents

24 cents

19 cents

23 cents

25 cents

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▶ Use work pages S-221 and S-222 to evaluate the children's ability to determine the value of a given set of coins. Direct the children to find the value of each set of coins, starting with the coin having the greatest value.

Supplemental Experiences

▶ Have each child use toy coins at his desk. If commercially made toy coins are not available, the child may make his own by coloring and cutting out cardboard disks. Each coin should be appropriately labeled. Direct the children to choose a set of coins which would have the value of sixteen cents and place them on a sheet of paper or a paper plate. As each child finishes the task, have him stand beside his seat. When all the children are standing, ask one child to identify the coins he chose for his set. Others may challenge him to count to determine the value if they do not agree with him. If he is correct, let all who chose the same set be seated. Repeat the procedure by having someone who is still standing identify the coins in his set. Continue until all children are seated.

Money

Work Pages S-223 and S-224

Objectives

To review the symbol $\overset{m}{=}$.

Fundamentals

The child is aware that ten pennies or two nickels are not exactly the same as one dime. He needs to be helped to understand that these different sets of coins have the same value. Often it is desirable to express this relationship in a concise manner. Therefore the $\overset{m}{=}$ symbol will now be reviewed. The child has had experience in using the $\overset{m}{=}$ symbol with measurement of various types. When used with money, it indicates that one set of coins has the same value as another set of coins.

1 dime $\overset{m}{=}$ 2 nickels
1 nickel $\overset{m}{=}$ 5 pennies

Work with money, particularly with pennies and dimes, offers an opportunity to reinforce the children's understanding of place value. Just as the number ten can be thought of as one ten or as ten ones, the value of ten cents can be thought of in terms of one dime or ten pennies. The value of one dime and three pennies then can be quickly calculated by thinking of ten cents and three cents as thirteen cents. Since the child knows that five plus five is ten, he will have no difficulty in determining that the value of 2 nickels is ten cents. In terms of place-value ideas, he can then think of four nickels as two nickels and two nickels, or as ten cents and ten cents.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

Ability to count to fifty by ones, fives, and tens.

Ability to determine the value of a set of coins whose value does not exceed fifty cents.

Developmental Experiences

MATERIALS	VOCABULARY
10 pennies	$\overset{m}{=}$
4 nickels	
2 dimes	

▶ Ask a child to choose two coins and give their combined value. Continue this with varying numbers of coins to review finding the value of a set of coins. Then ask a child to state a value less than twenty-five cents and pick up a set of coins of that value. Continue this with several children to develop the ability to choose a set of coins of a given value.

▶ Pick up a nickel and ask the value of it. Direct a child to pick up a set of coins of the same value as the nickel. On the board illustrate the nickel and the set of coins with the symbol $\overset{m}{=}$ between them.

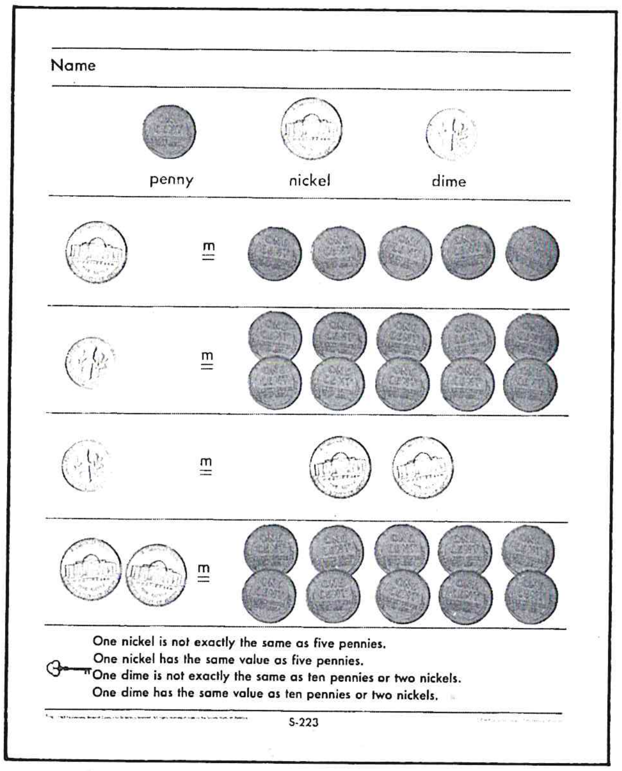
1 nickel $\overset{m}{=}$ 5 pennies

Explain to the children that this is read as "one nickel has the same value as five pennies." Tell them that since it is incorrect to say 1 = 5, we must use the symbol $\overset{m}{=}$. Use the same procedure with one dime and ten pennies. Stress the proper reading of the symbol $\overset{m}{=}$.

▶ Distribute work page S-223. Direct the children to look at the second box and find the symbol which is read "has the same value as." Ask: "What coins do you see in the set of coins just before it?" "What is the value of this set?" "What coins do you see in the set after the $\overset{m}{=}$?" "What is the value of this set?" "Do the two sets have the same value?" Read the second box as "One nickel has the same value as five pennies." Continue in a similar manner with the other three boxes on this page.

▶ Play "I Have." Let a child pick up some coins secretly. Then he should turn to the group and say, "I have three coins and they have the same value as fifteen pennies. What coins do I have?" The child who answers correctly then chooses some coins and repeats the procedure.

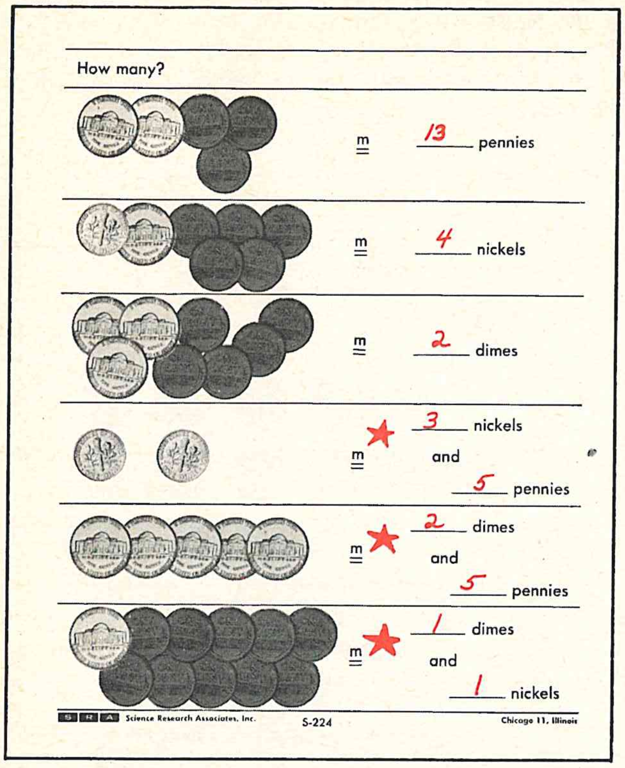
Use work page S-224 Direct the children to find the value of the first set of coins. Have them look at the words on the other side of the $\overset{m}{=}$. Ask: "How many pennies would be needed to have the same value as the first set of coins?" After the answer is given, have the children write the correct numeral on the line. Let the children continue to work in the same way with the other problems.



Name

One nickel is not exactly the same as five pennies.
One nickel has the same value as five pennies.
One dime is not exactly the same as ten pennies or two nickels.
One dime has the same value as ten pennies or two nickels.

S-223



How many?

$\overset{m}{=}$ 13 pennies

$\overset{m}{=}$ 4 nickels

$\overset{m}{=}$ 2 dimes

$\overset{m}{=}$ ★ 3 nickels and 5 pennies

$\overset{m}{=}$ ★ 2 dimes and 5 pennies

$\overset{m}{=}$ ★ 1 dimes and 1 nickels

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★ More than one correct answer is possible for each problem.

Supplemental Experiences

▶Provide a table with many coins on it and a sign with $\overset{m}{=}$ on it. Have three children stand in front of the class. Give the first child a certain amount of money, such as fifteen cents. The middle child holds the sign; the third child stands next to the table. Ask the first child to tell the class that he has "fifteen cents." Direct the third child to pick up enough coins to make this amount. Then each child will say what he is holding—first child, "fifteen cents"; second child, "has the same value as"; third child, "one dime and five pennies." Let them change places and try another problem; then repeat with different children.

Money

Work Pages S-225 Through S-228

Objectives

To introduce the quarter.

To review and extend the meaning of the value of a set of coins.

To review and extend the meaning of the $\overset{m}{=}$ symbol.

Fundamentals

Continued practice with money is provided to strengthen the child's ability to determine the value of a given set of coins and to find different sets of coins of the same value. The symbol ¢ will be reviewed as an abbreviation for the word "cent."

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

Ability to use the symbol $\overset{m}{=}$ and understand its use with coins.

Ability to determine the value of pennies, nickels, and dimes.

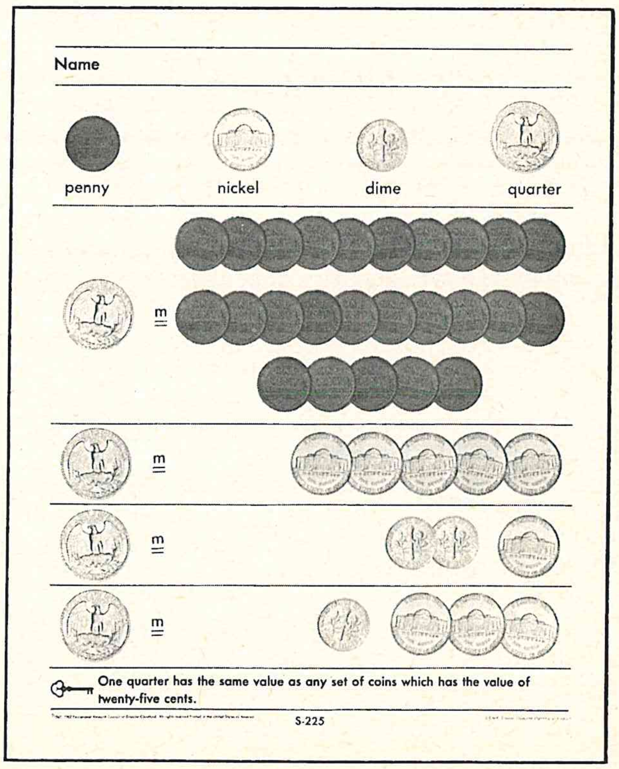
Developmental Experiences

MATERIALS	VOCABULARY
real or play money —pennies, nickels, dimes, quarters	quarter ¢

Hold up a *quarter* for identification. Write the word "quarter" on the chalkboard. Have the children look at the quarter closely. Ask: "Does the word 'quarter' appear on the coin?" "Does the word 'cents' appear on the coin?" Ask if anyone knows the value of a quarter. When it has been established that one quarter has the value of twenty-five cents, ask a child to choose a set of three or more coins that have the same value as a quarter. Have other children find different coin combinations that have the same value as a quarter. Let a child record these choices

1 quarter $\overset{m}{=}$ 2 dimes and 1 nickel
1 quarter $\overset{m}{=}$ 1 dime and 3 nickels
1 quarter $\overset{m}{=}$ 2 dimes and 5 pennies
1 quarter $\overset{m}{=}$ 1 dime and 15 pennies

on the chalkboard. As the list grows, lead the chil-



Name

penny
nickel
dime
quarter

$\overset{m}{=}$

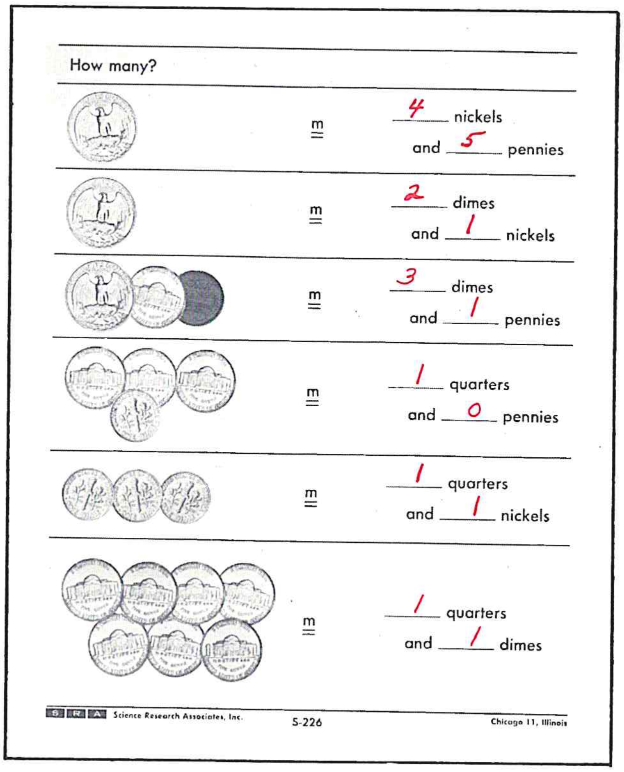
$\overset{m}{=}$

$\overset{m}{=}$

$\overset{m}{=}$

One quarter has the same value as any set of coins which has the value of twenty-five cents.

S-225


How many?

$\overset{m}{=}$ 4 nickels and 5 pennies

$\overset{m}{=}$ 2 dimes and 1 nickels

$\overset{m}{=}$ 3 dimes and 1 pennies

$\overset{m}{=}$ 1 quarters and 0 pennies

$\overset{m}{=}$ 1 quarters and 1 nickels

$\overset{m}{=}$ 1 quarters and 1 dimes

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★ More than one correct answer is possible for each problem.

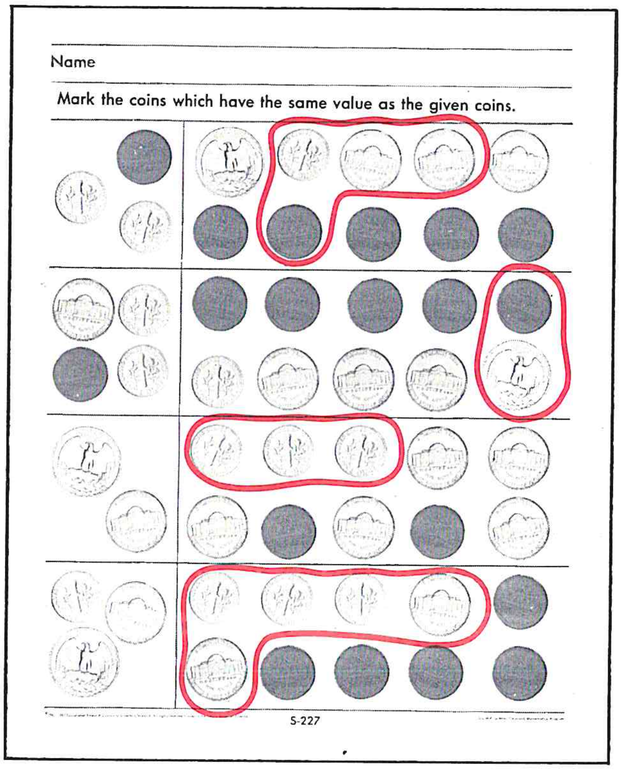
dren to conclude that one quarter has the same value as any group of coins which has the value of twenty-five cents.

▶ Distribute work page S-225 and call the children's attention to the second row. Ask for the value of the coin at the left of the $\underline{m}$. Then ask for the value of the set of coins at the right. Tell the children that since the two sets have the same value, the arithmetic statement is read, "One quarter has the same value as twenty-five pennies." Discuss the other rows in the same manner.

Direct the children's attention to the first row on work page S-226. Ask for the value of the coin on the left in the first row. Instruct the children to use coins at their desks to make a set of nickels and pennies that has the same value. Have them first count by fives. There are six correct answers, ranging from five nickels and zero pennies to zero nickels and twenty-five pennies. Direct the children to check by counting. For example, four nickels and five pennies can be counted five, ten, fifteen, twenty, twenty-one, twenty-two, twenty-three, twenty-four, twenty-five. The quick child will soon shorten this procedure. Many experiences of this type will be needed to speed the responses.

▶ Using work page S-227, direct the children to look at the coins in the first set. Ask what value they have. Direct them to find a set of coins in the same row which has the same value and draw a ring around this set of coins. Now ask the children to look at the coins that remain. Ask if another set with the same value can be made. If so, tell them to draw a ring around that set too. Have them work the rest of this page in the same way.

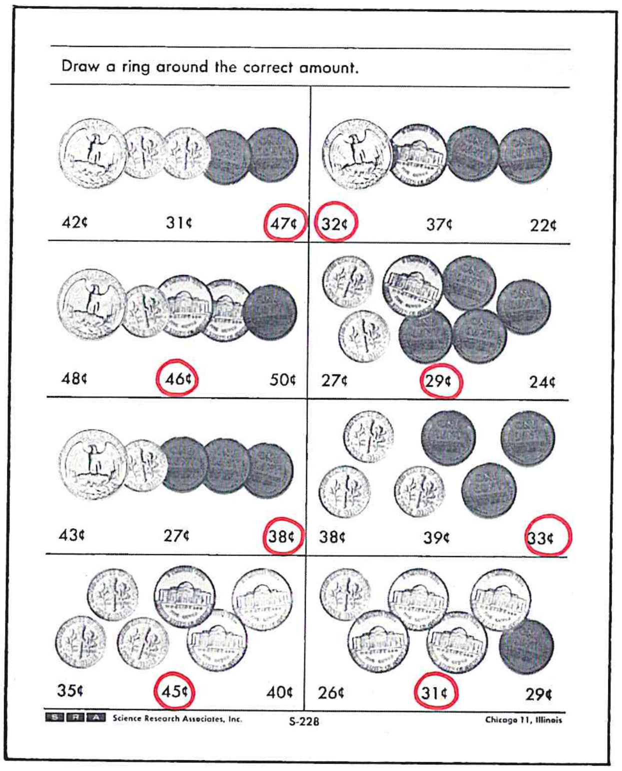
▶ Use work page S-228 to check on the children's ability to determine the value of a set of coins. Help the children recall that the symbol ¢ stands for the word "cent." Remind them to begin counting with the coin having the greatest value.


Name

Mark the coins which have the same value as the given coins.

S-227

★ More than one correct answer is possible for each problem.


Draw a ring around the correct amount.

42¢ 31¢ [✓ 47¢]

[✓ 32¢] 37¢ 22¢

48¢ [✓ 46¢] 50¢

27¢ [✓ 29¢] 24¢

43¢ 27¢ [✓ 38¢]

38¢ 39¢ [✓ 33¢]

35¢ [✓ 45¢] 40¢

26¢ [✓ 31¢] 29¢

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Supplemental Experiences

▶ Prepare cards picturing sets of coins whose value ranges from one cent to forty-nine cents. Hold up the cards one at a time. Have the children determine the value measured by each set.

▶ Use the coin cards to play "Conductor." Choose one child to be "conductor" and have him stand beside the seat of a second child who is the passenger. Display one of the coin cards. Both children must attempt to tell the value of the pictured coins. If the conductor answers correctly first, he may remain the conductor. If, however, the "passenger" answers correctly first, he becomes the new conductor. The new or the continuing conductor then moves on to the next passenger.

Money

Work Pages S-229 and S-230

Objectives

To introduce the idea of the purchasing power of money.

To strengthen the concept of measurement of value.

Fundamentals

The teacher will want to plan a variety of activities to strengthen the concept of measurement of value. She should be alert to the everyday experiences of the child which contribute to his understanding and growing skill in the handling of money —for example, collecting of milk or lunch money, health drives, school banking, and school-supply stores.

The teacher will want to lead children to set realistic prices for articles used in classroom activities. See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

Ability to determine the value of two sets of coins, compare them, and find the difference between them. Ability to understand and use the sign $\overset{m}{=}$.

Developmental Experiences

MATERIALS

10 pennies
5 nickels
3 dimes
1 quarter
pictures of toys or small actual toys
string or Scotch tape
paper for making price tags

VOCABULARY

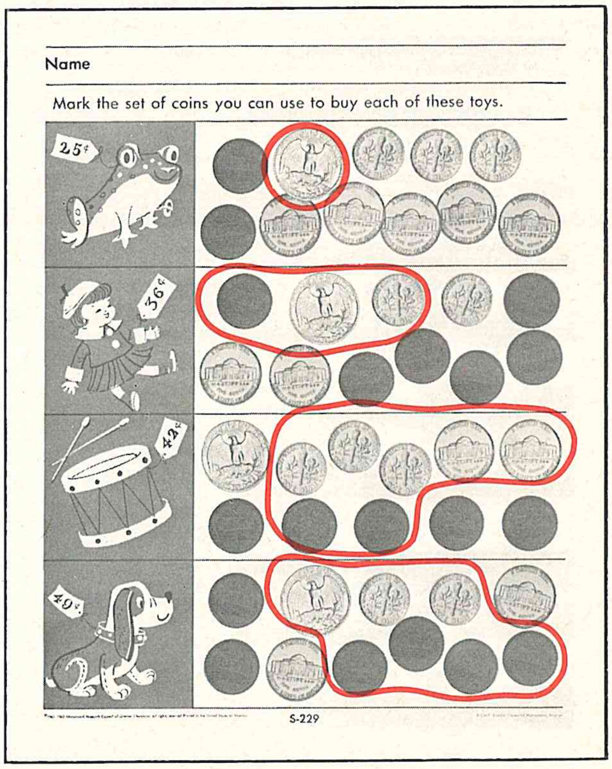
cost

▶ Ask ten children to bring in small toys that *cost* less than 50¢. If necessary, use pictures of inexpensive toys. Assign a toy to each child and ask him to make a price tag. Attach the tags prominently. Arrange the toys on a table. Provide another "money table" with coins on it.

Let one child be the storekeeper and another child be a customer. The customer must choose one toy and ask the cost. When the storekeeper replies, the customer must go to the table to get a set of coins which has the same value as the cost of the toy and present the coins to the storekeeper. Have the storekeeper orally count the value of the coins and accept or reject the sale. Choose other pairs of children to buy and sell other toys.

▶ Direct the children to discuss the price tag on the frog in the first row on work page S-229. Ask the value of the toy. Direct the children to draw a ring around a set of coins which has this value. Then direct them to look at the remaining coins. Ask if there is another set of coins which has the same value. If there is, direct the children to draw a ring around the second set of coins. Do the rest of the page in the same way.

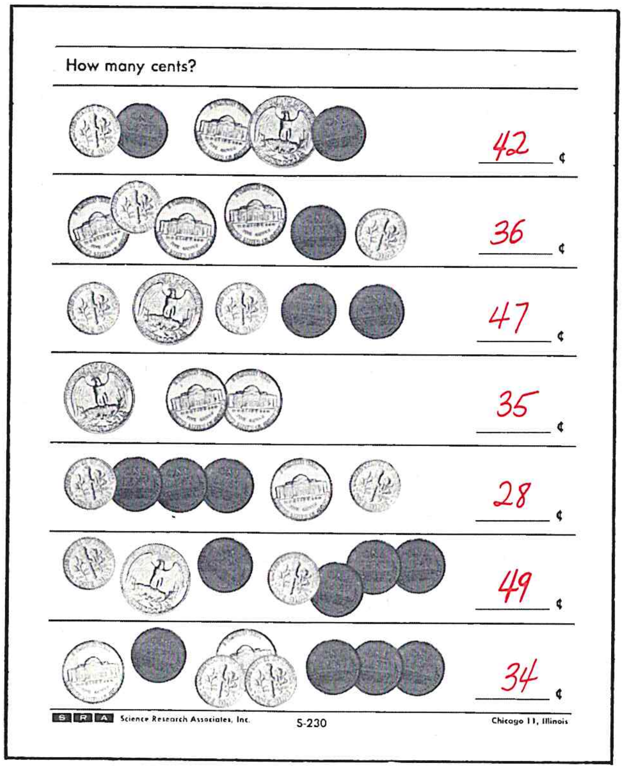
On work page S-230, ask the children to look at the first row and find the coin with the greatest value. Direct them to begin with this coin and find the value of the set of coins and then to write the numeral for the value on the line at the right. Have them complete the page independently.


Name

Mark the set of coins you can use to buy each of these toys.

S-229

★ More than one correct answer is possible for each problem.



How many cents?

42 ¢

36 ¢

47 ¢

35 ¢

28 ¢

49 ¢

34 ¢

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Supplemental Activities

▶ Set up a store which has toys, food, or candy. This can be done with objects or pictures of objects to which price tags have been attached. Pictures could be displayed in a pocket chart. Keep the number of items small so selection will not become the major emphasis. Change the items from time to time to vary the amounts of money involved. Let the children choose the coins necessary for a purchase. Later they could make transactions involving change.

▶ Draw four price tags on the chalk board. Ask 35¢ 19¢ 43¢ 28¢ the children to draw a picture of four toys and put one of these price tags on each picture. Direct them to draw under each picture a set of coins which could be used to purchase the toy.

Money

Work Pages S-231 Through S-236

Objectives

To introduce the half-dollar.

To help children understand the relations among the penny, nickel, dime, quarter, and half-dollar.

To extend the concept of measurement of value.

Fundamentals

The child has reviewed the value of the penny, the nickel, the dime, and the quarter. He has had practice in determining the value of a given set of coins and in choosing a set of coins with a given value. Continued practice will strengthen these understandings and help the child gain skill in handling money. The half-dollar will be introduced and included in the activities.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Measurement.

Readiness for Understanding

Ability to find the value of a set of coins, including the penny, nickel, dime, and quarter.

Ability to select a set of coins which have a given value.

Developmental Experiences

MATERIALS	VOCABULARY
pennies	half-dollar
nickels	
dimes	
quarters	
half-dollar	
toys or pictures of toys	
price tags for the toys	
envelopes for coins	

▶ Hold up a *half-dollar* for identification. Establish its value as fifty cents. Have children choose other sets of coins that have the same value as the half-dollar. Let the children list these sets on the chalkboard, using the symbol $\overset{m}{=}$. Note that the list of sets of coins with a value of fifty cents is much longer than the list of sets of coins with a value of twenty-five cents.

▶ Use work page S-231 as a review. Read the page with the class. The second line is read, "One half-dollar has the same value as two quarters."

▶ Ask children to use the toy coins at their desks to make different sets of coins which have the same value as one half-dollar. Call on several children to tell the class what set of coins they have chosen.

Draw a picture of a half-dollar and a dime on the chalkboard. Ask the children to place on their desks a set of coins which has the same value as the set pictured. Tell them they should use only quarters and nickels. Record the choices on the chalkboard:

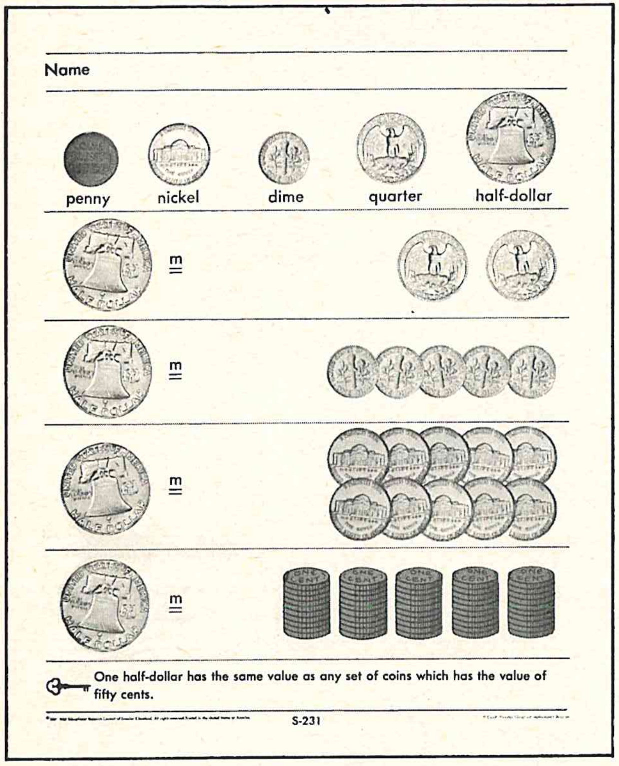
2 quarters and 2 nickels
1 quarter and 7 nickels
0 quarters and 12 nickels

Leave the list on the chalkboard and ask children to use quarters and pennies to make a set which has the same value. Again let the children record their sets on the board.

0 quarters	60 pennies
1 quarter	35 pennies
2 quarters	10 pennies

Direct the attention of the children to work page S-232 and ask them to use the coins on their desks to make a set of coins which has the same value as the first set pictured on the work page. Ask them to look at the words on the right to determine what coins may be used to make the set. As soon as their set has been made, have them write the numerals on the blanks and go on to the next row.

▶ Evaluate the progress of the children in determining the value of a set of coins and choosing another set which has the same value by using work page S-233 Direct the children to draw a ring around a set of coins which has the same value as the first set in the row. Tell them to look at the remain-



Name

penny nickel dime quarter half-dollar

=

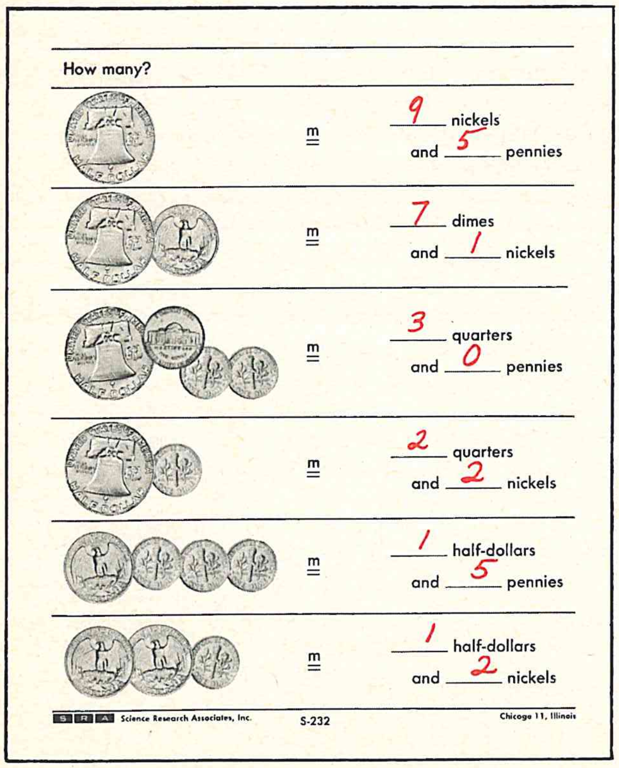
=

=

=

One half-dollar has the same value as any set of coins which has the value of fifty cents.

S-231



How many?

= 9 nickels and 5 pennies

= 7 dimes and 1 nickels

= 3 quarters and 0 pennies

= 2 quarters and 2 nickels

= 1 half-dollars and 5 pennies

= 1 half-dollars and 2 nickels

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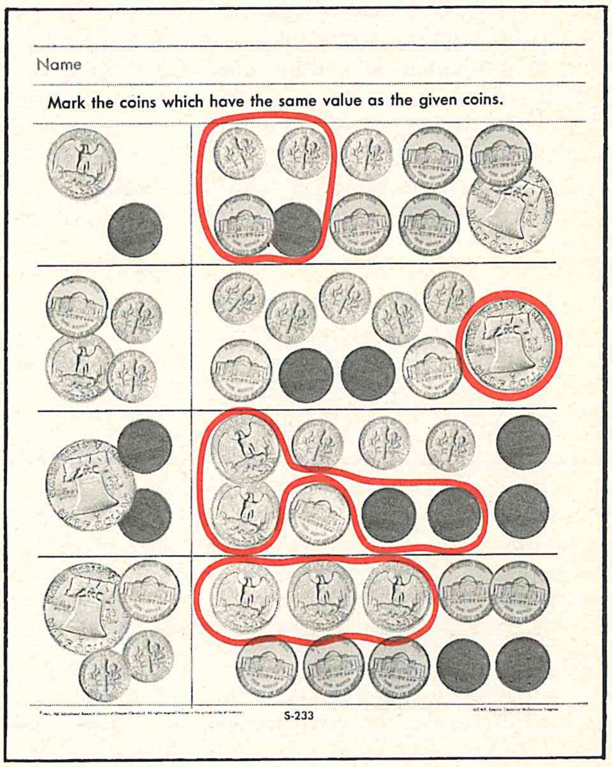
★ More than one correct answer is possible for each problem.

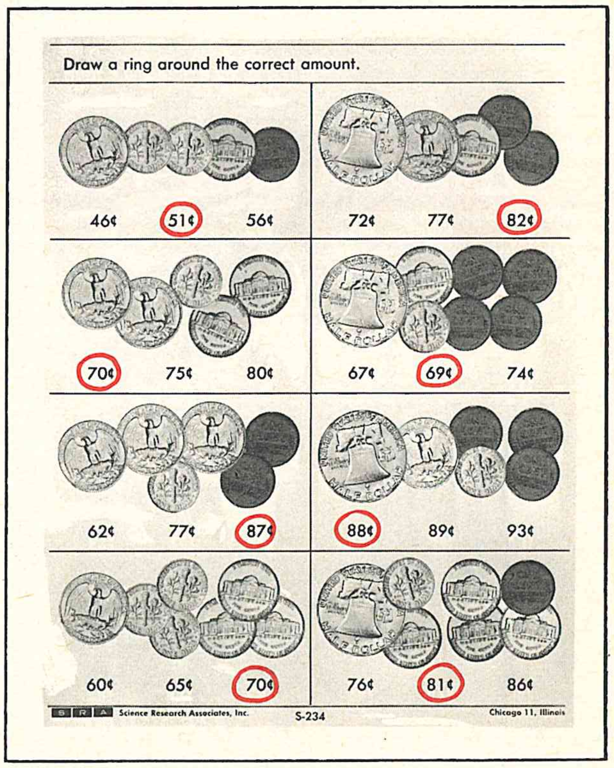


Name

Mark the coins which have the same value as the given coins.

S-233

★ More than one correct answer is possible for each problem.



Draw a ring around the correct amount.

46¢ [✓ 51¢] 56¢

72¢ 77¢ [✓ 82¢]

[✓ 70¢] 75¢ 80¢

67¢ [✓ 69¢] 74¢

62¢ 77¢ [✓ 87¢]

[✓ 88¢] 89¢ 93¢

60¢ 65¢ [✓ 70¢]

76¢ [✓ 81¢] 86¢

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ing coins and make another set if it is possible.

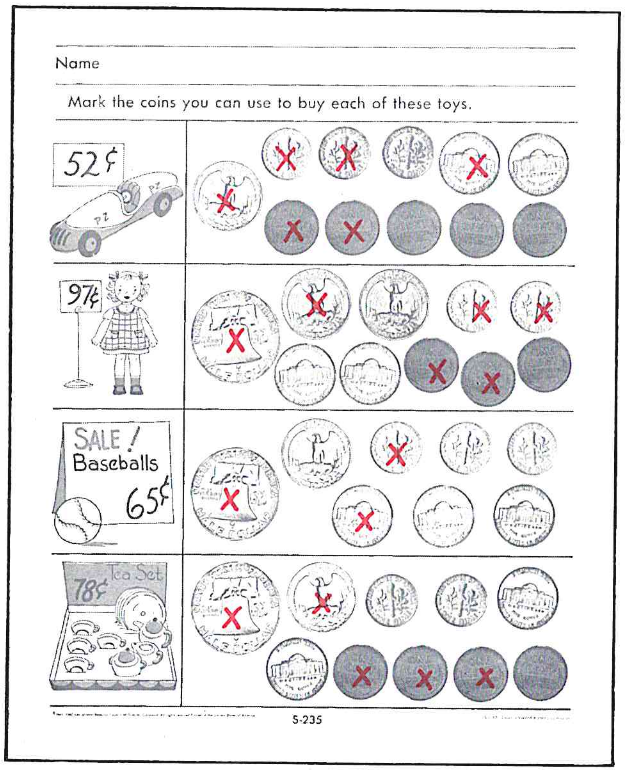
Use work page S-234 to evaluate the children's ability to determine the value of a set of coins. Direct the children to find the value of the set and draw a ring around the correct amount.

▶ Use toys having a value of less than one dollar. These may be real toys brought in by the children or pictures which were drawn by the children. Have a large price tag displayed on each. Let two children participate at a time, one as a shopkeeper and the other as a customer. Let the customer choose a toy and select a set of coins which has the same value as the price on the tag. Have him hand the set of coins to the shopkeeper who will determine the value of the set of coins orally. Continue until all children have had a turn.

▶ Use work page S-235 to check on the children's ability to choose a set of coins which has the same value as the price of a given article.

▶ Put sets of coins into envelopes. Each set should contain six or more coins with a total value of less than one dollar. Let one child choose an envelope and empty its contents on a table. Ask what coins are in his set and with which coin would he begin to determine the value of the set. Let him count orally and write the value on the chalkboard using the symbol ¢. Continue this activity with several different envelopes.

▶ Draw a set of coins on the chalkboard. Do not arrange them in the order of their values. Have a child come to the chalkboard, point to the coin with the greatest value, the next greatest. . . . Then ask him to follow the same order in finding the value of the set. Continue this activity using different combinations of coins.



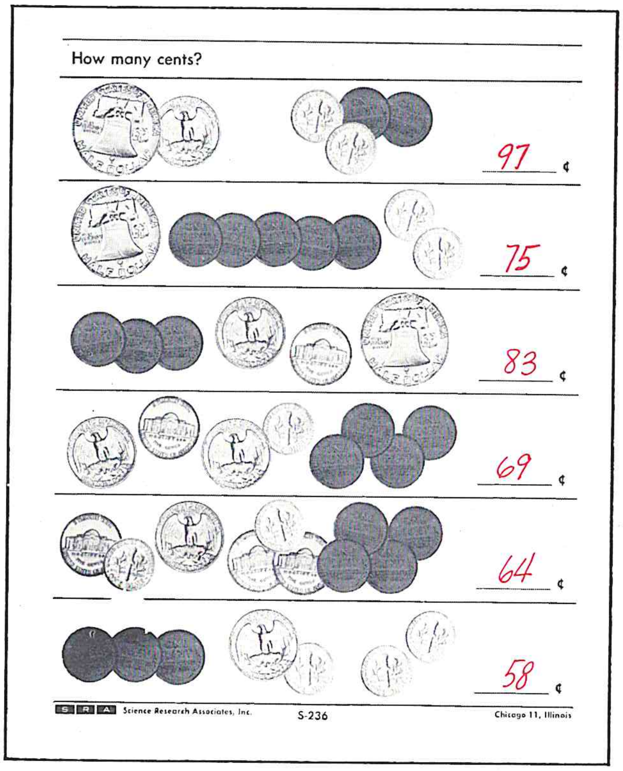
Name

Mark the coins you can use to buy each of these toys.

52¢
97¢
SALE!
Baseballs
65¢
Tea Set
78¢

S-235

★ More than one correct answer is possible for each problem.



How many cents?

97 ¢

75 ¢

83 ¢

69 ¢

64 ¢

58 ¢

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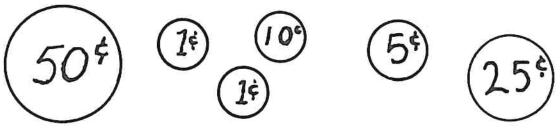


Use work page S-236 to determine the children's ability to find the value of a set of coins which fall in a random arrangement. If a child has difficulty, tell him to place a mark on each coin as he determines the total value of the set.

Supplemental Experiences

▶ Play a variation of "I Have." Let a child secretly choose a set of coins and count the value of the set. While the other children cover their eyes, the first child says, "I have twenty-five cents and there are this many coins." He drops them one by one into a container in such a way that each one makes a sound. As he asks the question "What coins do I have?" the others uncover their eyes and may identify the coins in the set.

▶ Prepare a set of cards, each containing pictures of coins with a combined value of less than one dollar. Also prepare a set of pictures of toys, games, or food with price tags. Display the coin-cards on the chalk tray. Have a child pick up the first picture card, go to the chalk tray to find the coin card containing a set of coins which has the same value as the price tag on the picture-card, and turn around to display the two cards. If his selection is correct, the next child may continue.

Money

Work Pages S-237 and S-238

Objectives

To develop an understanding of the comparative value of different sets of coins.

To evaluate the children's ability to determine the value of a given set of coins.

Fundamentals

The child has learned the value of the penny, nickel, dime, quarter, and half-dollar. Now he will be given an opportunity to compare different sets of coins and determine which set has the greater value and which has the lesser value.

See Key Topics in Mathematics for the Primary Teacher: Measurement.

Readiness for Understanding

Ability to find the value of a given set of coins which may include pennies, nickels, dimes, quarters, and half-dollars whose value is less than one dollar.

Ability to compare two numbers less than one hundred and to determine which is greater and which is lesser.

Developmental Experiences

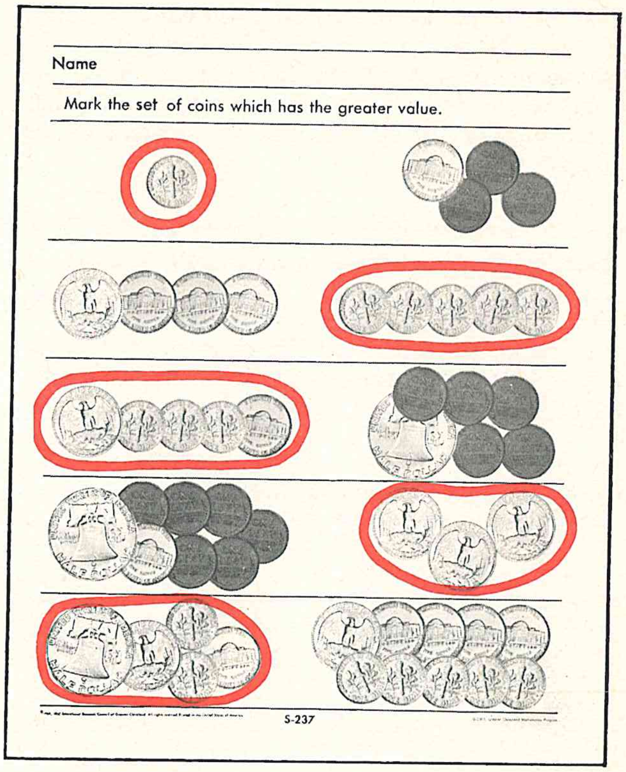
MATERIALS	VOCABULARY
pennies	greater value
nickels	greatest value
dimes	lesser value
quarters	least value
half-dollars	

▶ To review the value of the penny, nickel, dime, quarter, and half-dollar, hold up two coins of different value. Ask which coin has the *greater value.* The child who answers correctly then chooses a pair of coins to continue the game.

Vary the game by holding up two coins in each hand or by having two children each hold two coins. Direct the children to find the value of each set of coins and compare the two sets. Use a similar procedure to develop the meaning of *greatest value, lesser value,* and *least value.*

▶ Let one child choose a set of coins. Ask a second child to choose a set of coins with a greater value than the first set. The second child may call on another to choose a set of coins which has a lesser value. The third child may call on another child to identify the set which has the least value, and so forth.

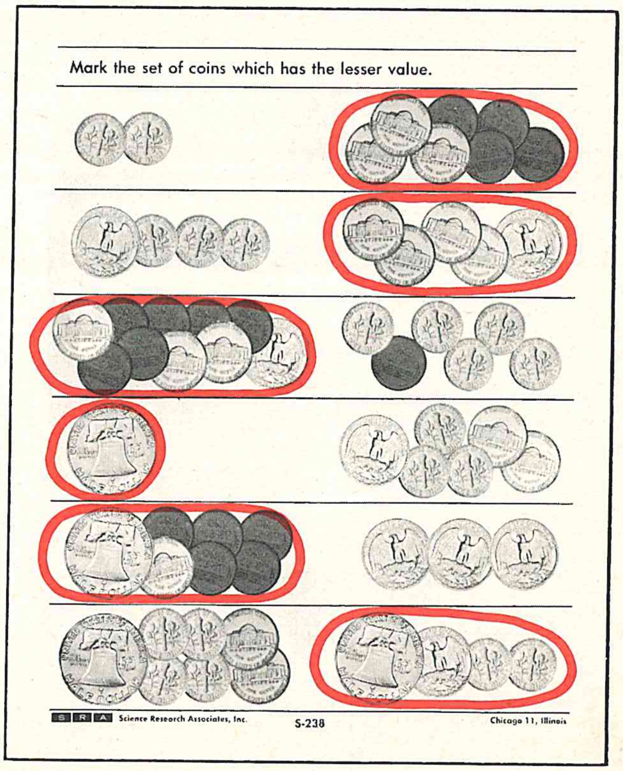
▶ Use work pages S-237 and S-238 to develop the children's understanding of comparative value. On work page S-237 direct the children to think of the value of the two sets of coins in each row and to draw a ring around the one which has the greater value. On work page S-238 have them mark the set of coins in each row which has the lesser value.



Name

Mark the set of coins which has the greater value.

S-237



Mark the set of coins which has the lesser value.

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S-238

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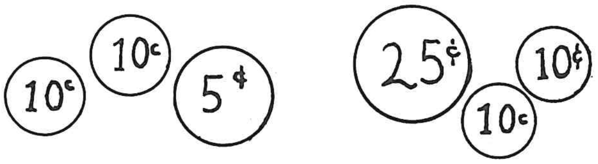
Supplemental Experiences

▶ "I'm Going Shopping" involves coins and a knowledge of comparative value. To begin, say, "I'm going shopping with a quarter and a dime and I'm going to buy a book that costs 40¢. Have I enough money?" If the child who answers the question says "no," he should determine how much more is needed and say, "I'll give you a nickel." Let him ask another "shopping" question. If the child answers in the affirmative, he should state that the other child has exactly enough or that he has ____ more than he needs.

▶ Use cards on which sets of coins each of total value less than one dollar are pictured. Let two or three children compete at one time. Hold up two cards. Ask the children to determine the value of each set of coins and tell the class which has the greater value. The one who answers first may choose the next children to compete with him.

▶ Draw a picture on the chalkboard of a set of coins, such as two dimes and a nickel. The children must try to "beat the teacher" by placing next to them a set of coins which is equal in number but which has a greater value than the set drawn by the teacher. (Set a limit of fifty cents.) Ask a child to tell the value of both sets of coins. Change the coins pictured and continue the game.



Money

Work Pages S-239 and S-240

Objectives

To introduce story problems involving money.

To strengthen the concept of measurement of value.

To develop the ability to make change.

Fundamentals

The work with money is continued in a functional setting. As he thinks through everyday situations involving money, the child will begin to develop the ability to make change.

As the teacher introduces story problems, she will want to remind the child to ask himself four questions:

1. What must I find?
2. What facts do I know?
3. How can I use these facts to find the answer?
4. Is my answer reasonable?

After the problem has been identified, the child must analyze the known facts, determine which facts are needed, and apply these facts to find the correct solution. Finally, he should check to see if his answer makes sense.

See Key Topics in Mathematics for the Primary Teacher: Measurement.

Readiness for Understanding

Ability to determine the value of a set of coins whose value is less than one dollar.

Ability to determine relations between the values of two sets of coins.

Developmental Experiences

MATERIALS	VOCABULARY
pennies	change
nickels	
dimes	
quarters	
half-dollars	
toys or pictures of toys with price tags	

▶ Let a child choose a coin and identify it. Ask a second child to pick up another coin, identify it, and give it to the first child. Ask a third child to repeat the process.

Write the story of what just happened on the chalkboard. For example, "Mary picked up a dime. Nancy gave her a nickel, and Joe gave her a penny." Next, let the children suggest a question about Mary and the money she has. For example, they may ask "How much money does Mary have now?" Write it on the chalkboard. At the end of the question, leave a place for the answer, ____¢. Call on a child to fill in the blank.

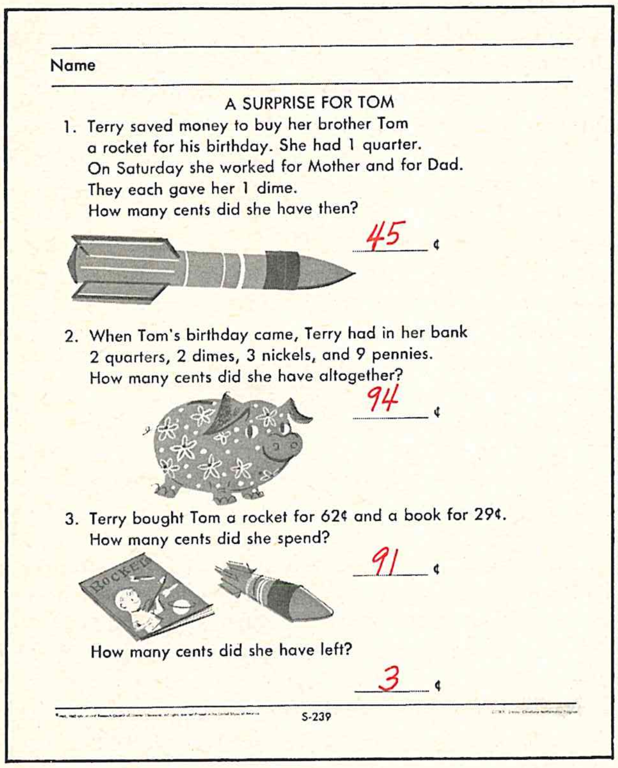
Continue acting out events of receiving, saving, and earning. Write stories to describe what happened. Lead children to determine the questions which could be asked.

Incorporate events of receiving, saving, and earning with buying an article which has a price tag attached. Have the children compare the value of the set of coins with the value of the article. "How much more must he save (earn)?" "How much will he have left?" Continue writing and solving stories as a group. Encourage small groups of children to dramatize stories for the class to write about.

Develop the idea of *change* as the difference in value between a set of coins and the price of something which is purchased.

Have children write individual stories about change. Each story must ask a question. Let them read their stories for the group to solve.

▶ Use work pages S-239 and S-240 to evaluate the children's ability to solve story problems involving money. Read the stories together, but have the children find the answers by themselves. After each page is completed, check the answers by calling on different children to tell how they solved the problems and what answers they got.



Name

A SURPRISE FOR TOM

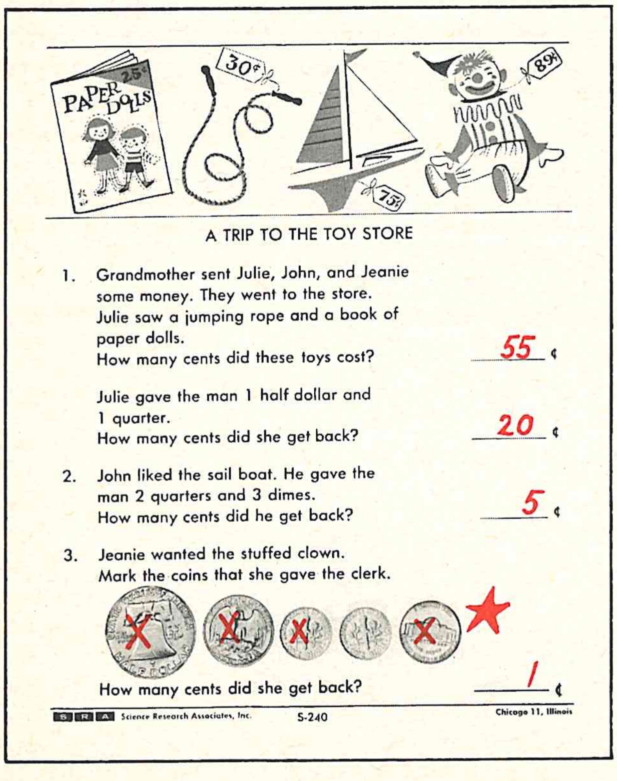
1. Terry saved money to buy her brother Tom a rocket for his birthday. She had 1 quarter. On Saturday she worked for Mother and for Dad. They each gave her 1 dime. How many cents did she have then?
45 ¢

2. When Tom's birthday came, Terry had in her bank 2 quarters, 2 dimes, 3 nickels, and 9 pennies. How many cents did she have altogether?
94 ¢

3. Terry bought Tom a rocket for 62¢ and a book for 29¢. How many cents did she spend?
91 ¢

How many cents did she have left?
3 ¢

S-239



PAPER DOLLS
25¢
30¢
75¢
85¢

A TRIP TO THE TOY STORE

1. Grandmother sent Julie, John, and Jeanie some money. They went to the store. Julie saw a jumping rope and a book of paper dolls. How many cents did these toys cost?
55 ¢

Julie gave the man 1 half dollar and 1 quarter. How many cents did she get back?
20 ¢

2. John liked the sail boat. He gave the man 2 quarters and 3 dimes. How many cents did he get back?
5 ¢

3. Jeanie wanted the stuffed clown. Mark the coins that she gave the clerk. ★

How many cents did she get back?
1 ¢

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★ More than one correct answer is possible for each problem.

Supplemental Experiences

▶ Have the children bring pictures of toys, clothes, or sports goods and write prices less than fifty cents on each picture. Have each child put out coins which measure fifty cents. Then have them exchange pictures with their neighbors, read the prices, and put the correct amount of change from fifty cents on the picture. Have each child check his own picture to see that his neighbor gave the correct change. If he is correct, he should change with another neighbor; if not, he should try again.

▶ Have each child lay out fifty cents on his desk using different coins. As he is called he is to tell a short story about change he has made. One child might say, "I had fifty cents. I bought a ball for forty-three cents and I have seven cents left." A child in one row checks the story of the child in the next row, and vice versa. Give them time to get stories and change ready while others are telling stories and checking answers.

Place Value

Work Pages S-241 Through S-244

Objectives

To extend the children's understanding of the place-value concept.

To develop an understanding of the numbers one hundred through nine hundred ninety-nine.

To introduce the numerals 100 through 999.

Fundamentals

If the child understands the meaning of place value in two-digit numerals, the introduction of the hundreds place will create no problems. Just as the child learned to think of ten as ten ones, or as one ten and zero ones, he will also learn to think of one hundred as ten tens, or as one hundred, zero tens, and zero ones. He will learn that the digit 3, for example, in the hundreds place has a different meaning from the digit 3 in the tens place or in the ones place.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Systems of Numeration.

Readiness for Understanding

Complete understanding of the meaning of each digit in a two-digit numeral.

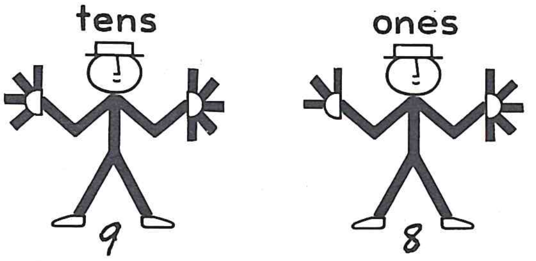
Ability to represent numbers zero through ninety-nine on the Countingmen.

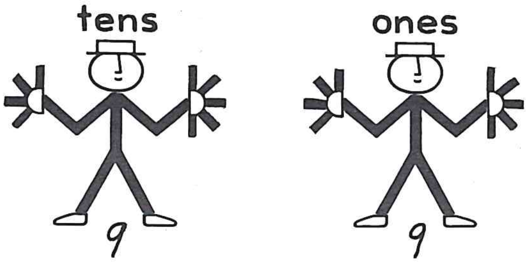
Developmental Experiences

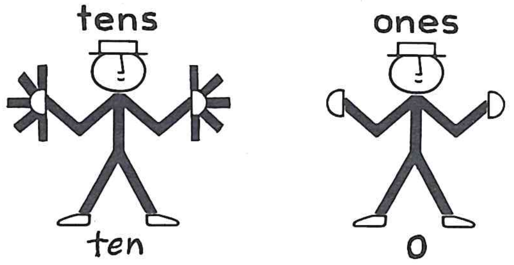
MATERIALS

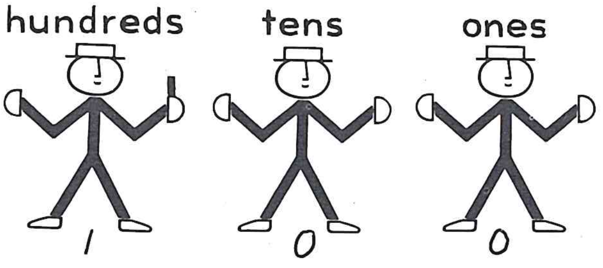
3 Countingmen
bead frame
beads
numeral cards

▶ Have a child show 98 on the Countingmen. Ask the class to tell how many tens and how many ones they see. Ask another child to show 99. He should place one more finger on the Ones-man. Ask a third child to show one more using these two Countingmen. He should place one more finger on the Ones-man. After he notices that the Ones-man has ten fingers he should put one reminder finger on the Tens-man and remove the ten fingers from the Ones-man. If none of the children immediately notice that a third Countingman is needed, lead them to discover this by asking pertinent questions: "What did we do when the Ones-man had all his fingers out? How many fingers does the Tens-man have? What should we do now?" Help the class decide that in order to continue the count a third Countingman will be needed to keep track of how many times the Tens-man had ten fingers out. Introduce a third Countingman to keep track of each set of ten tens.









Name this third man the "Hundreds-man." Have a child place one finger on the Hundreds-man to record the set of ten tens. When he does this, have him remove the ten fingers from the Tens-man. Have a child write the numeral 100 on the chalkboard. Tell the children that this is a numeral for the number one hundred. Remind them that they can think of it as ten tens or as one hundred, zero tens, and zero ones.

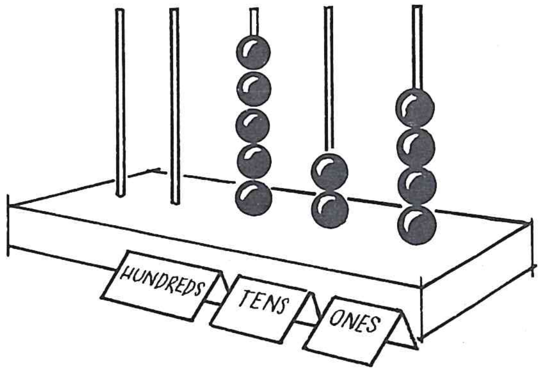
Ask a child to show the number which is one more than one hundred. After he puts one finger on the Ones-man, discuss the number which the Countingmen show. Have a child write a numeral for this number on the chalkboard and read it aloud: 101.

Now have a child place a finger on the Tens-man and tell what number is shown by the three Countingmen. Have a child write a numeral for this number on the chalkboard. Have the class tell how many ones, how many tens, and how many hundreds are in 111. Point to each digit and have its value discussed.

Show several other numbers using the Countingmen: 504, 439, 760, 389. Discuss each of these numbers, having the children write their corresponding numerals on the chalkboard. Have other children write the numerals for the numbers before and after each given number

▶ Emphasize that with only ten basic numerals we can represent all the whole numbers greater than nine. Choose three teams of ten players each. Distribute to each team a set of large cards bearing the numerals 0 through 9. Designate one team the "hundreds" team, another team the "tens" team, and the third team, the "ones" team. Write *hundreds, tens,* and *ones* on the chalkboard. Call out a numeral, for example, five hundred thirty-six. The member of the hundreds team who holds the digit 5, the member of the tens team who holds the digit 3, and the member of the ones team who holds the digit 6 must come quickly forward and stand beneath the proper words to form the numeral 536. Call different numerals until each child in each team has had a turn.

▶ Use the bead frame to strengthen the children's understanding of three-digit numerals. Label the rods of the bead frame hundreds, tens, and ones. Have the children use colored beads to represent the



number five hundred twenty-four on the frame. They should place five beads on the hundreds rod, two beads on the tens rod, and four beads on the ones rod. Ask a child to write the place-value numeral for this number on the chalkboard. Repeat the activity with other numbers from one hundred through nine hundred ninety-nine.

▶ Play a matching game. Write a column of numerals on the chalkboard and to the right of this column write a column of phrases which describe the place-value meaning of these numerals.

436 four hundreds, six tens, three ones
346 six hundreds, three tens, four ones
634 three hundreds, four tens, six ones
463 four hundreds, three tens, six ones

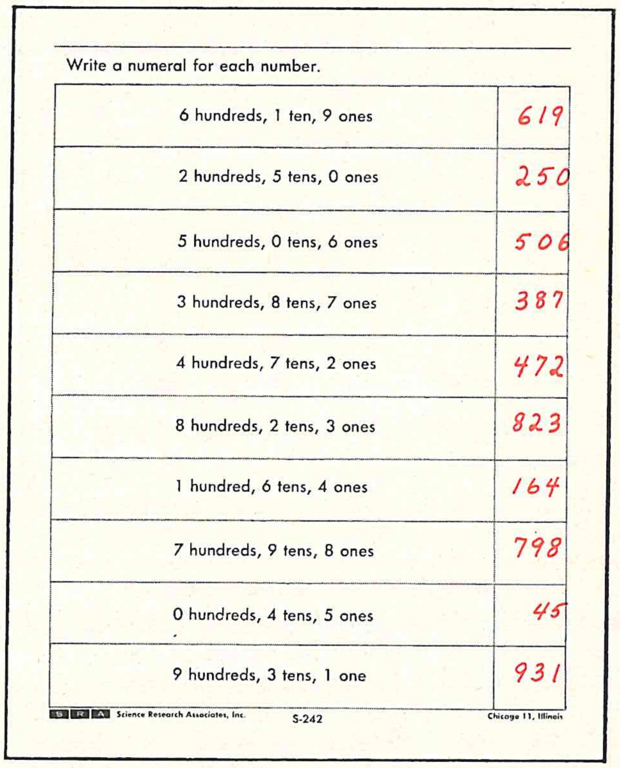
Call on a child to read the first numeral in the first column. Then have him search in the second column for the meaning of this numeral. Have him draw a line from the numeral to the phrase which describes it. Continue in a similar fashion to have the other numerals matched with the phrases.

▶ Use work page S-241 to give the children experience in writing three-digit numerals. Discuss the picture at the top of the page. Remind them that the number ten tens, or one hundred, may be represented by the numeral 100. Direct them to write a numeral for the number shown by the Countingmen in each row.

On work page S-242 direct the children to write a numeral for each of the numbers described. Work the first problem with the children and then encourage them to complete the page independently.

▶ Work page S-243 provides the children with an opportunity to demonstrate their understanding of three-digit numerals. Direct the children to read each numeral and to draw the corresponding number of fingers on each Countingman.

Direct the children to fill the blanks on work page S-244 to show the place-value meaning of each of these numerals.



Write a numeral for each number.

6 hundreds, 1 ten, 9 ones	619
2 hundreds, 5 tens, 0 ones	250
5 hundreds, 0 tens, 6 ones	506
3 hundreds, 8 tens, 7 ones	387
4 hundreds, 7 tens, 2 ones	472
8 hundreds, 2 tens, 3 ones	823
1 hundred, 6 tens, 4 ones	164
7 hundreds, 9 tens, 8 ones	798
0 hundreds, 4 tens, 5 ones	45
9 hundreds, 3 tens, 1 one	931

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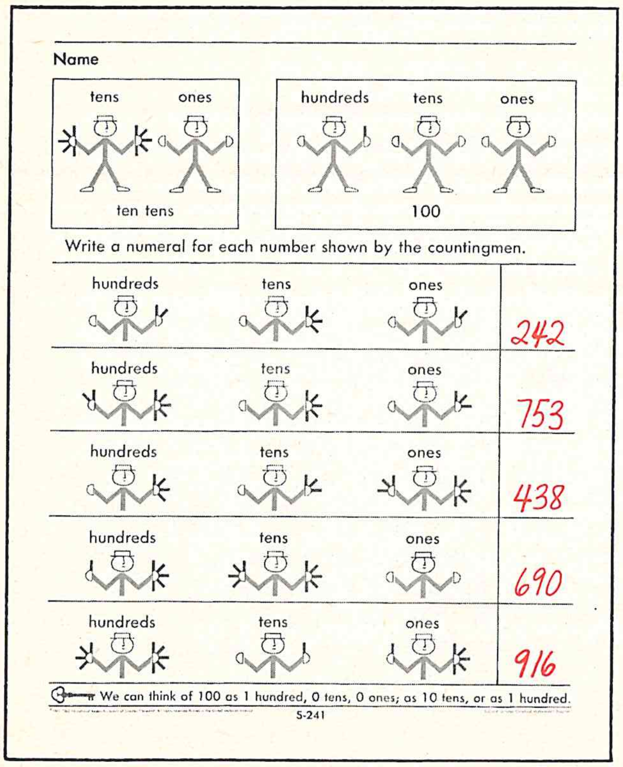


Name

Write a numeral for each number shown by the countingmen.

242

753

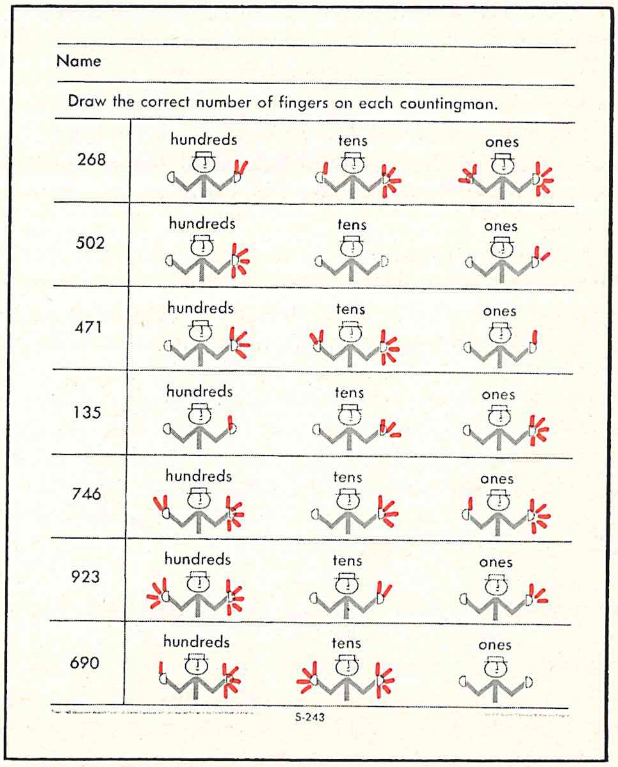
438

690

916

We can think of 100 as 1 hundred, 0 tens, 0 ones; as 10 tens, or as 1 hundred.

S-241



Name

Draw the correct number of fingers on each countingman.

268

502

471

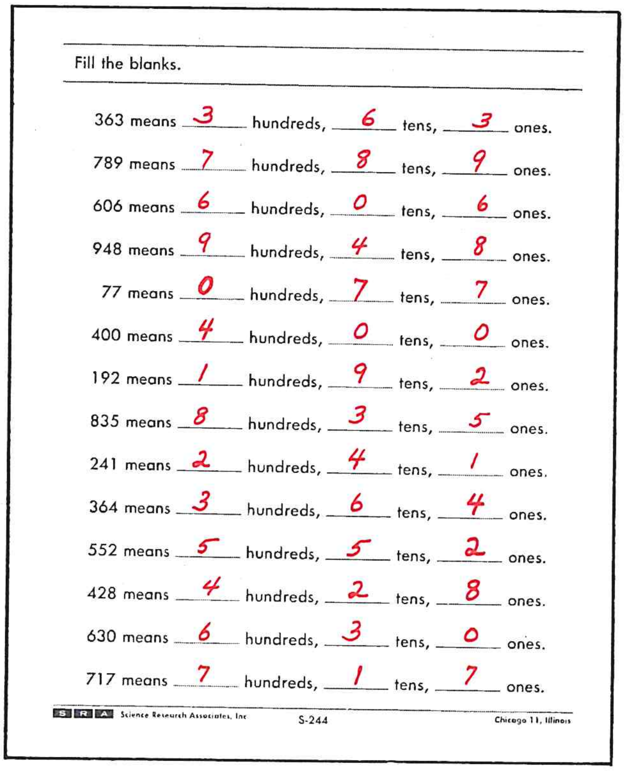
135

746

923

690

S-243



Fill the blanks.

363 means 3 hundreds, 6 tens, 3 ones.

789 means 7 hundreds, 8 tens, 9 ones.

606 means 6 hundreds, 0 tens, 6 ones.

948 means 9 hundreds, 4 tens, 8 ones.

77 means 0 hundreds, 7 tens, 7 ones.

400 means 4 hundreds, 0 tens, 0 ones.

192 means 1 hundreds, 9 tens, 2 ones.

835 means 8 hundreds, 3 tens, 5 ones.

241 means 2 hundreds, 4 tens, 1 ones.

364 means 3 hundreds, 6 tens, 4 ones.

552 means 5 hundreds, 5 tens, 2 ones.

428 means 4 hundreds, 2 tens, 8 ones.

630 means 6 hundreds, 3 tens, 0 ones.

717 means 7 hundreds, 1 tens, 7 ones.

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Supplemental Experiences

▶ Have the children play a guessing game. Choose one child to stand before the group and ask a riddle. He may say, "I'm thinking of the number that is three hundreds, five tens, and zero ones. Who can tell my number?" He then calls on another child to give the correct answer and write the numeral for this number on the chalkboard. If the answer is correct, the child who answered becomes "it" and thinks of another number. Suggest that some children describe their numbers in different ways. For example, one child might say, "I am thinking of a number that is one more than three hundred and twenty-seven." Another might say, "I'm thinking of a number which is one less than five hundred and sixty-three."

▶ Have the children use their plastic numerals to play a place-value game. Direct them to lay out a 2, a 3, and a 4. Tell them they may arrange these in any way they wish to make three-digit numerals. After each child has built his numeral, have the numerals listed on the chalkboard to show all the possible combinations: 234, 243, 342, 324, 423, 432. Have the children take turns explaining the meaning of each digit in each of these three-digit numerals. Repeat the activity with other combinations such as 5, 6, and 7, or 8, 0, and 9.

▶ Review the order concept using three-digit numerals. It is important for the children to become acquainted with the relative magnitude of the numbers represented by these numerals. The ability to decide which of two numbers is greater closely relates to an understanding of place value.

Write several rows of numerals on the chalkboard. Direct the children to think of the numbers represented by these numerals and to rewrite them on their own paper from the least to the greatest.

160, 106, 601
233, 332, 323
300, 330, 303
760, 607, 700
867, 678, 786

Vary the activity by asking the children to rewrite groups of numerals to represent the order of the numbers from the greatest to the least.

326, 632, 236
61, 160, 610
530, 503, 350
927, 792, 297
801, 108, 180

Place Value

Work Pages S-245 Through S-248

Objectives

To strengthen the children's understanding of place-value concepts.

To stress the idea of different numerals for the same number.

To extend the children's understanding of expanded notation.

Fundamentals

The child is familiar with the idea of expanded-notation numerals for the numbers eleven through ninety-nine. Now this knowledge will be extended to the numbers through nine hundred ninety-nine. It is important that the child have a clear understanding of expanded notation so that his work with three-digit addition and subtraction will be meaningful to him.

As a readiness for expanded notation of three-digit numerals, it will be well to pause briefly and review the meaning of the digit in the hundreds place. The child must have a complete grasp of the idea that a numeral such as 800 represents eight hundreds, zero tens, and zero ones, or simply eight hundred. With this clearly in mind, he will have little difficulty in understanding that in the numeral 888, for example, the digit 8 in the hundreds place stands for eight hundreds, the digit 8 in the tens place stands for eight tens, or 80, and the digit 8 in the ones place stands for eight ones, or 8. The expanded form for this numeral is, therefore, 800 + 80 + 8.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Systems of Numeration.

Readiness for Understanding

Understanding of numbers through nine hundred ninety-nine.

Ability to recognize the standard place-value numerals 0 through 999.

Awareness that different numerals may represent the same number.

Understanding of place-value concepts.

Developmental Experiences

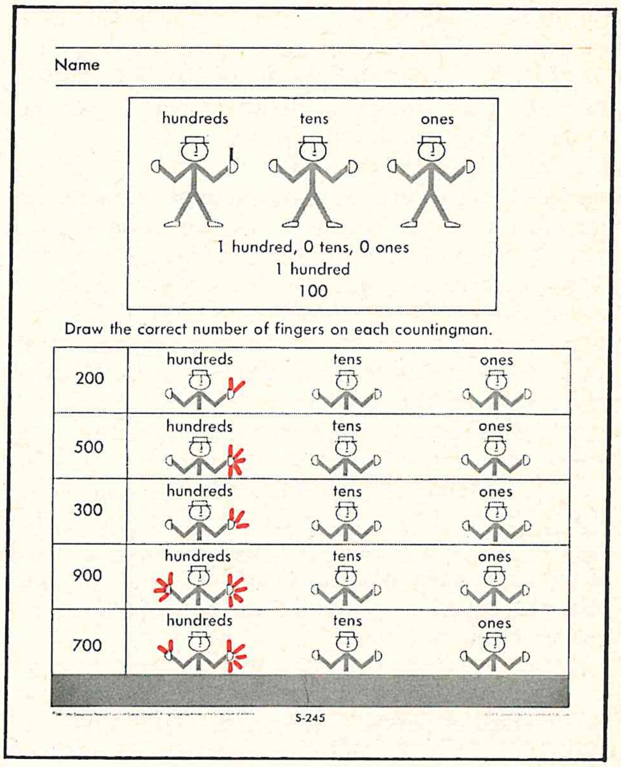
MATERIALS

sticks
2 pegboards
pegs

▶ Place a row of ten pegs in the pegboard. Have a child write the numeral that tells how many pegs there are. Place a second row of ten pegs in the pegboard. Call attention to the fact that there are now two sets of ten pegs, or twenty pegs, and have a child write the numeral for this number. Continue in this fashion to have ten pegs at a time placed in the pegboard and the corresponding numeral written each time. As the tenth row of pegs is placed in the pegboard, have the children tell how many sets of ten they see. Ask a child to give another name for ten tens. As the name "one hundred" is given, have the numeral 100 written on the chalkboard.

Have a child place a row of ten pegs in a second pegboard and tell how many pegs there are in each pegboard. When he answers that there is one set of one hundred pegs and one set of ten pegs, have the corresponding place-value numeral 110 written on the chalkboard. Now ask if some child can write a different numeral for this same number. Have the numeral 100 + 10 written on the chalkboard. Place one more peg in the second pegboard and ask if someone can write a numeral for this number. Have both 111 and the expanded form 100 + 10 + 1 written on the chalkboard. Proceed in a similar fashion to have other numbers of pegs displayed and the corresponding place-value and expanded-form numerals written.

▶ Use sticks to strengthen the children's understanding of the place-value meaning of three-digit numerals and to develop an understanding of the related expanded notations. Display eight sets of ten sticks each and three individual sticks. Ask a child to determine the number of sticks in this set and to write the place-value numeral and the expanded-form numeral for this number: 83 and 80 + 3. Repeat the activity with nine sets of ten sticks and nine single sticks. After the numerals 99 and 90 + 9 have been written, join one more stick to the nine single sticks. Bundle each set of ten sticks together and call the children's attention to the fact that now they see ten sets of ten. Bundle these ten sets of ten together and hold them up for all to see. Have a child tell how many are in this set. Ask for a volunteer to write the numeral for this number on the chalkboard: 100. Now lay out the set of one hundred sticks, three sets of ten sticks, and four single sticks. Call on a child to write the standard place-value numeral for the number of sticks in this set: 134. Ask another child to write the expanded numeral for this number: 100 + 30 + 4. Continue in the same way with different numbers of sticks.



Name

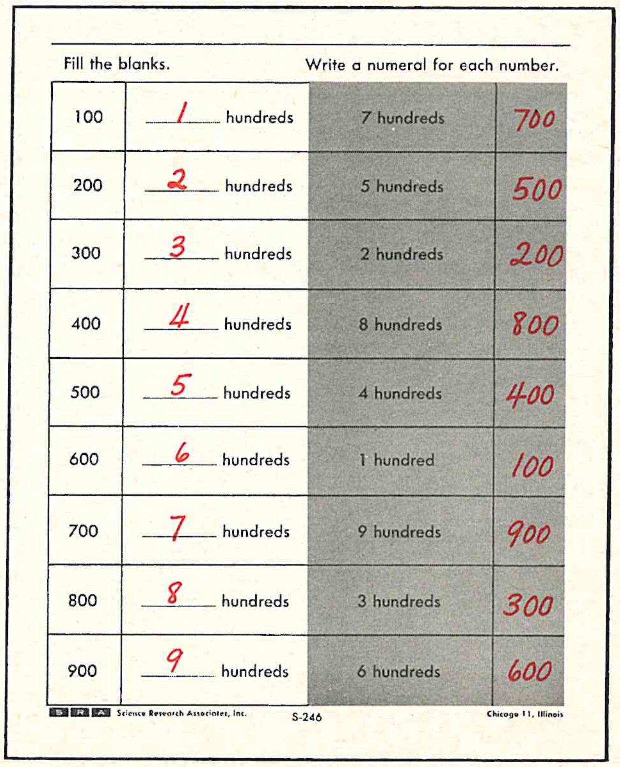
hundreds	tens	ones

1 hundred, 0 tens, 0 ones
1 hundred
100

Draw the correct number of fingers on each countingman.

200	hundreds	tens	ones
500	hundreds	tens	ones
300	hundreds	tens	ones
900	hundreds	tens	ones
700	hundreds	tens	ones

S-245



Fill the blanks. Write a numeral for each number.

100	1 hundreds	7 hundreds	700
200	2 hundreds	5 hundreds	500
300	3 hundreds	2 hundreds	200
400	4 hundreds	8 hundreds	800
500	5 hundreds	4 hundreds	400
600	6 hundreds	1 hundred	100
700	7 hundreds	9 hundreds	900
800	8 hundreds	3 hundreds	300
900	9 hundreds	6 hundreds	600

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▶ Use work page S-245 to review the meaning of the digit in the hundreds place. Discuss the picture at the top of the page, reminding the children to think of one hundred as one hundred, zero tens, and zero ones, or as simply one hundred. Then direct the children to read the numeral in each row and to draw the correct number of fingers on the Countingmen.

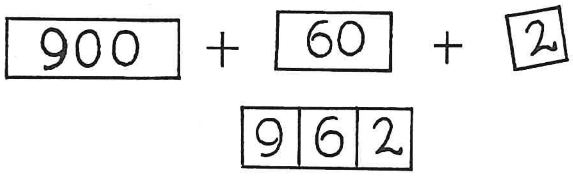
Direct the children to fill the blanks on the left half of work page S-246 to tell how many hundreds are indicated in each of the given numerals. Then direct them to read the phrase on the right half of the page and to write the standard place-value numeral for each number described.

▶ Use work page S-247 to give the children experience in reading expanded-notation numerals. Discuss the box at the top of the page, pointing out that the number three hundred fifty-four is represented here in four different ways. Now direct the children to the first row on the lower portion of the page. Tell them that they are to read the numeral in the first box and then to draw the correct number of fingers on each Countingman.

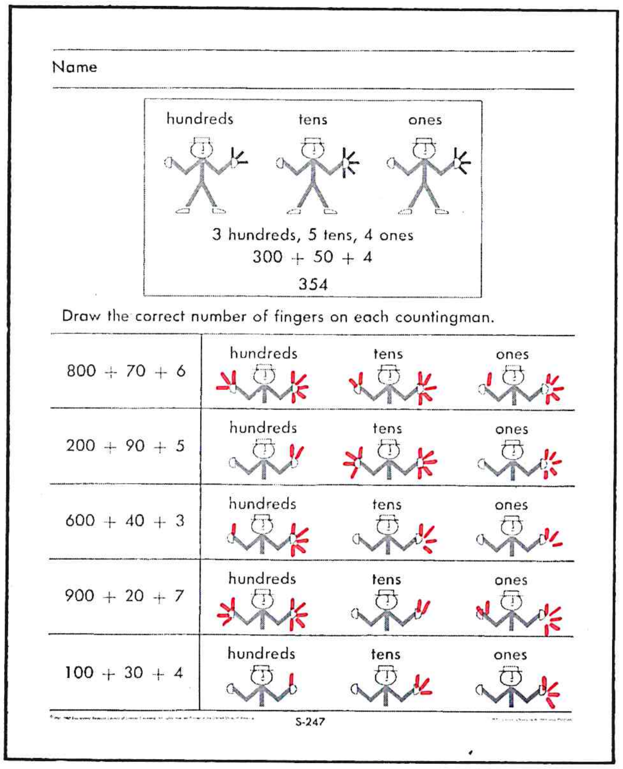
Use work page S-248 to provide experience with writing different numerals for the same number. Direct the children to complete each equation at the top of the page by writing the standard place-value numeral in each placeholder. On the lower portion of the page, have them read each numeral and complete the equation by writing the corresponding expanded form.

Supplemental Experiences

▶ On each of nine 4″ x 12″ cards, write a numeral 100, 200, 300, . . . 900; on each of nine 4″ x 8″ cards, write a numeral 10, 20, 30, . . . 90; on each of ten 4″ x 4″ cards, write a single digit 0 through 9. Distribute the cards to the children and say that you would like three children to show you a numeral for nine hundred sixty-two. Ask the children to think of the place-value meaning of the numeral 962 and to come forward if their cards can be used to construct such a numeral. The child with the 900, the child with the 60, and the child with the 2 should come forward and hold their cards near two plus signs which have been written on the chalkboard to show 900 + 60 + 2. Now have the 2 card held over the 0 of the 60 card to show the numeral 62. Have both these cards held over the two 0's of the 900 so that the numeral 962 is formed. Explain to the children that 962 is just a short way of showing that we mean 900 + 60 + 2. Once again display the numerals in expanded form. Continue to have the children show different numerals for different numbers.



▶ Continue to give the children experiences with comparing numbers. Encourage them to call on their understanding of place-value concepts in comparing numbers represented by three-digit numerals. Lead them to the realization that in comparing numbers represented by 632 and 326, a quick inspection of the digit in the hundreds place will lead them to conclude immediately that six hundred thirty-two is the greater number. In comparing 632 with 673 however, since the digits in the hundreds place are the same, they must then inspect the digits in the



Name

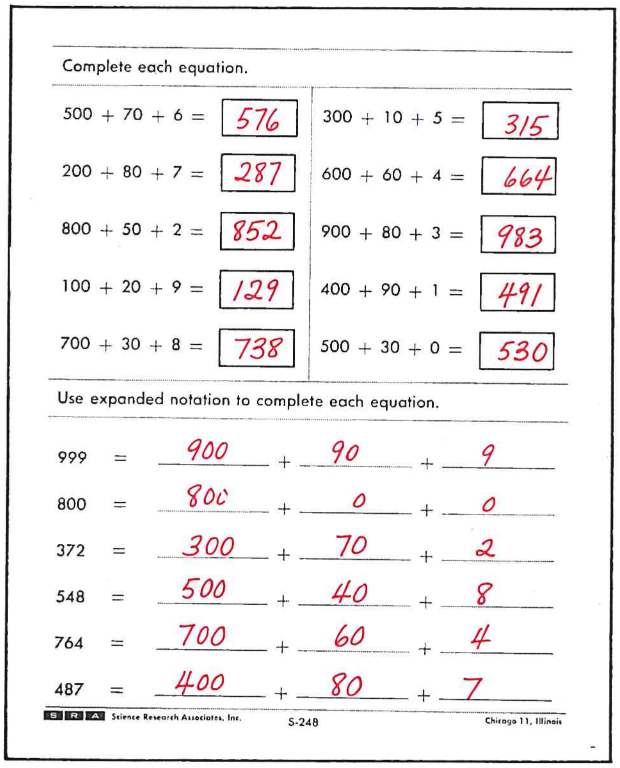
hundreds tens ones

3 hundreds, 5 tens, 4 ones
300 + 50 + 4
354

Draw the correct number of fingers on each countingman.

800 + 70 + 6	hundreds	tens	ones
200 + 90 + 5	hundreds	tens	ones
600 + 40 + 3	hundreds	tens	ones
900 + 20 + 7	hundreds	tens	ones
100 + 30 + 4	hundreds	tens	ones

S-247



Complete each equation.

500 + 70 + 6 = 576	300 + 10 + 5 = 315
200 + 80 + 7 = 287	600 + 60 + 4 = 664
800 + 50 + 2 = 852	900 + 80 + 3 = 983
100 + 20 + 9 = 129	400 + 90 + 1 = 491
700 + 30 + 8 = 738	500 + 30 + 0 = 530

Use expanded notation to complete each equation.

999 = 900 + 90 + 9
800 = 800 + 0 + 0
372 = 300 + 70 + 2
548 = 500 + 40 + 8
764 = 700 + 60 + 4
487 = 400 + 80 + 7

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tens place to decide which numeral represents the greater number. If both digits in the hundreds place and in the tens place are the same, as in 632 and 637, the digits in the ones place will determine which number is the greater.

▶ Write on the chalkboard several equations and inequalities in which the relation symbols are missing. Have the children write the correct relation symbol, $>$, $<$, or $=$ in each circle to make the statements true. In each case, ask them to explain their reasoning in selecting the answer.

754 ○ 547
672 ○ 600 + 70 + 2
900 + 0 + 3 ○ 900 + 30 + 0
821 ○ 800 + 20 + 1

Three-Digit Addition—Subtraction

NO CARRYING
NO BORROWING

Work Pages S-249 Through S-258

Objectives

To extend the development of addition and subtraction without carrying and borrowing.

To enable children to use their understanding of number concepts in work with three-digit addition and subtraction.

To emphasize the use of the commutative and associative properties.

To emphasize the inverse relation between addition and subtraction.

Fundamentals

By the very nature of its structure, arithmetic is sequential. Each concept grows out of previously developed ideas.

The child has acquired some skill in doing two-digit addition and subtraction with carrying and borrowing; he will now be given the opportunity to discover that known concepts apply to three-digit addition and subtraction without carrying and borrowing.

In moving to this new setting, three-digit additions and subtractions without carrying and borrowing will be examined first. The child will be given the opportunity to become accustomed to using the commutative and associative properties, combined in the process of scrambling, with numbers whose sums range in the hundreds.

$$\begin{aligned} 354 + 223 &= (300 + 50 + 4) + (200 + 20 + 3) \\ &= (300 + 200) + (50 + 20) + (4 + 3) \\ &= 500 + 70 + 7 \\ &= 577 \end{aligned}$$

The child will relate what he already knows to his new problems. He knows $4 + 3 = 7$. He knows $(10 + 4) + 3 = 17$. This will help him when he approaches problems such as $104 + 3$, $114 + 3$, or $994 + 3$.

The child has learned to join sets of ones, to join sets of tens, and then to put these together to get an answer. Now he will learn to join sets of ones, join sets of tens, join sets of hundreds, and then put these together to arrive at his answer. He will grow accustomed to using expanded notation in three-digit problems involving zero. He must learn that in problems such as the one illustrated, in expanded form, only one zero is needed in the tens position to represent the empty set—no tens—in the place-value interpretation of the numeral 104. There is one hundred, zero tens, four ones.

$$\begin{array}{r} 100 + 0 + 4 \\ +\qquad\quad 3 \\ \hline 100 + 0 + 7 \end{array} = 107$$

When working with numbers represented by one- and two-digit numerals, the child examined the operation of addition. Then he learned to undo his steps by working with the inverse operation of subtraction. In work with numbers represented by three-digit numerals, the same approach will be adopted. After the child is exposed to three-digit addition without carrying, three-digit subtraction without borrowing will fall into place naturally. The child will simply undo what was done in addition.

The child's horizon is widening. As the teacher introduces him to three-digit numerals, she will want to continue to nurture carefully his growing interests and ability with numbers. She will want to make him aware that the concepts he knows will help him to make new discoveries.

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Understanding of the concept of place value.

Understanding of the concepts of commutativity and associativity.

Understanding of the process of scrambling.

Ability to work with two-digit addition and subtraction.

Ability to work with expanded notation and vertical notation.

Developmental Experiences

MATERIALS

3 Countingmen

▶ Ask the children to tell the sum of $4 + 3$. Ask them to tell the sum of $14 + 3$, of $34 + 3$, of $84 + 3$. Review with them, either orally or in written form, the use of the associative property in these problems. Have children conclude that finding the sum of two single-digit addends did not involve the use of the property.

On the Countingmen, show the first addend in the problem 104 + 3 = □. Have children tell how many ones, how many tens, and how many hundreds must be added to one hundred four. Join the three ones to the four ones, and have children tell how many ones, how many tens, and how many hundreds are in the answer. Have them tell how they think the associative property can be used to show what happens in finding the sum of 104 + 3. Ask a child to write the way he thinks this problem would look as the associative property is used. Have a child write

$$\begin{aligned}(100 + 0 + 4) + 3 &= (100 + 0) + (4 + 3)\\ &= 100 + 7\\ &= 107\end{aligned}$$

the problem as he thinks it would look in expanded notation. Have the class tell the number of ones,

$$\begin{array}{rr} & 100 + 0 + 4\\ + & 3\\ \hline & 100 + 0 + 7 = 107\end{array}$$

tens, and hundreds in the first addend and in the answer. Ask why no tens appear in the answer even after the addition occurs. As a final step, have the problem written in vertical notation and read as it is being worked.

Repeat the activity several times; use such problems as 232 + 7 = □, 463 + 6 = □, 101 + 8 = □, and 400 + 5 = □.

On the Countingmen, show the first addend in the problem 104 + 30 = □. Adapt the procedure suggested above to this problem involving addition in both ones and tens places. Ask how the use of the commutative and associative properties, combined in the process of scrambling, help explain what happens in solving this addition problem. Some child might suggest that all of the ones should be grouped, all of the tens should be grouped, all of the hundreds should be grouped, and finally ones, tens, and hundreds should be put together for the answer.

$$\begin{gathered}(100 + 0 + 4) + (30 + 0) =\\ 100 + (0 + 30) + (4 + 0) =\\ 100 + 30 + 4\\ 134\end{gathered}$$

Have the problem written in expanded notation. Ask what the zeros in the ones and the tens place represent.

$$\begin{array}{rr} & 100 + \quad 0 + 4\\ + & (30 + 0)\\ \hline & 100 + \quad 30 + 4\end{array} = 134$$

Have the vertical-notation form of the problem used as a final step.

Repeat the activity with problems such as 700 + 17 = □, 3 + 205 = □, 242 + 30 = □, 110 + 50 = □, 52 + 306 = □, and 724 + 53 = □

Use this same approach in developing such sums as 600 + 200 = □, 500 + 101 = □, 206 + 380 = □, 304 + 175 = □ , and 725 + 263 = □.

▶ Use work pages S-249 and S-250 to provide practice in working three-digit addition problems. Discuss the problems at the top of the pages with the children. Have them write the needed numerals to complete the sample problems. Have them complete the remainder of each page on their own. Remind them to use care in lining up the numerals that represent ones, tens, and hundreds.

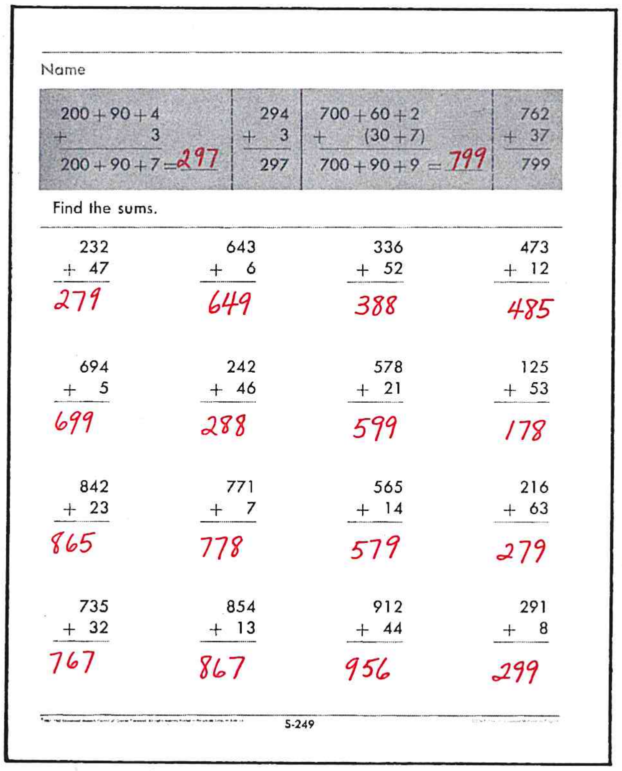
▶ Work pages S-251 and S-252 provide the opportunity to work with zero in various positions in three-digit addends. Discuss the two problems at the top of work page S-251 with the children. Have them write the needed numerals to complete each expanded notation. Direct them to find the sums for each of the remaining problems on these two pages.

▶ When children have an understanding of the concepts involved in additions with two three-digit addends without carrying, give them the opportunity to try to adapt their skill to column additions involving three to five two- and three-digit addends. Have them work several sample problems orally.

Place four or five problems on the chalkboard.

$$\begin{array}{rrr} 110 & 510 & 404\\ 422 & 306 & 21\\ 244 & 72 & 12\\ +\ 10 & +\ 11 & 500\\ \cline{1-2} & & +\ 61\\ \cline{3-3}\end{array}$$

Have one child add from top to bottom; have another child add from bottom to top; have a third child add in the way most convenient for him. Lead the children to discover that the commutative and associative properties of addition are still effective and permit a scrambling of addends in column addition as in all other additive settings encountered up to this point.



Name

$$\begin{array}{rr} & 200 + 90 + 4\\ + & 3\\ \hline & 200 + 90 + 7 = 297\end{array} \qquad \begin{array}{rr} & 294\\ + & 3\\ \hline & 297\end{array} \qquad \begin{array}{rr} & 700 + 60 + 2\\ + & (30 + 7)\\ \hline & 700 + 90 + 9 = 799\end{array} \qquad \begin{array}{rr} & 762\\ + & 37\\ \hline & 799\end{array}$$

Find the sums.

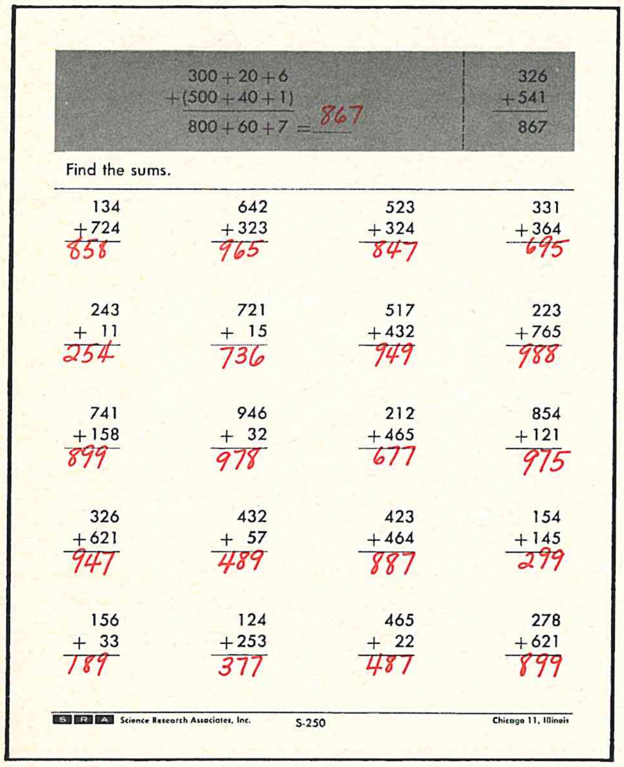
232 + 47 279	643 + 6 649	336 + 52 388	473 + 12 485
694 + 5 699	242 + 46 288	578 + 21 599	125 + 53 178
842 + 23 865	771 + 7 778	565 + 14 579	216 + 63 279
735 + 32 767	854 + 13 867	912 + 44 956	291 + 8 299

S-249

▶ Work pages S-253 and S-254 give children the opportunity to try out their growing skill in column additions. Direct them to find the sums.

▶ Demonstrate the problem 348 − 6 = ☐ by using the Countingmen. Tell a child to show the number three hundred forty-eight on the Countingmen. Ask another child to remove six ones: he should remove six fingers from the ones-man. Have someone tell what took place in terms of ones, tens, and hundreds. Ask a child to write the related expanded notation for the problem on the chalkboard.

$$\begin{array}{r} 300 + 40 + 8 \\ -\qquad\quad 6 \\ \hline 300 + 40 + 2 \end{array} = 342$$



$$\begin{array}{r} 300 + 20 + 6 \\ +(500 + 40 + 1) \\ \hline 800 + 60 + 7 \end{array} = 867 \qquad \begin{array}{r} 326 \\ +541 \\ \hline 867 \end{array}$$

Find the sums.

134 +724 858	642 +323 965	523 +324 847	331 +364 695
243 + 11 254	721 + 15 736	517 +432 949	223 +765 988
741 +158 899	946 + 32 978	212 +465 677	854 +121 975
326 +621 947	432 + 57 489	423 +464 887	154 +145 299
156 + 33 189	124 +253 377	465 + 22 487	278 +621 899

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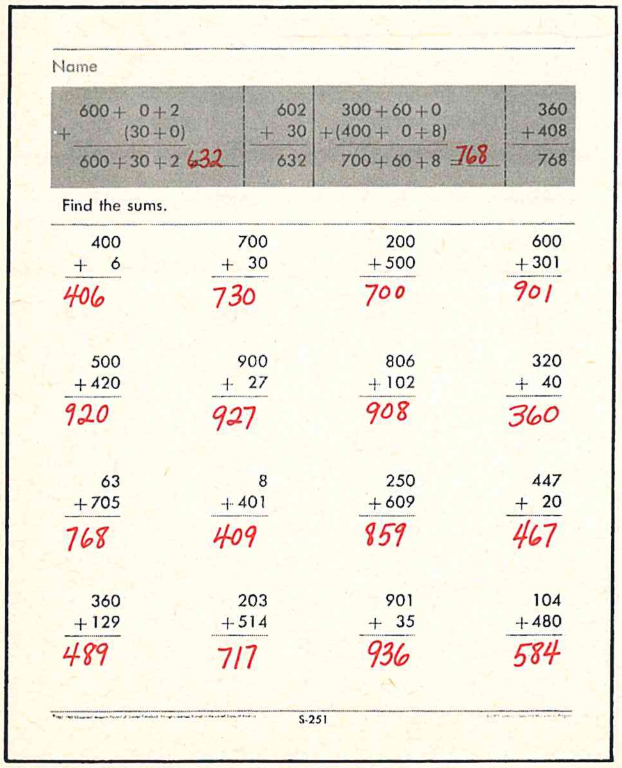


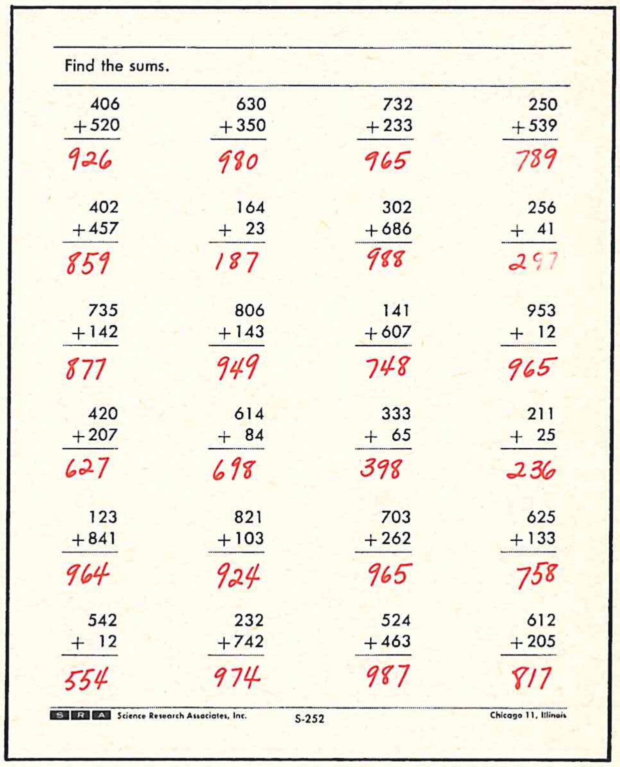
Name

$$\begin{array}{r} 600 + 0 + 2 \\ +\ (30 + 0) \\ \hline 600 + 30 + 2 \end{array}\ 632 \qquad \begin{array}{r} 602 \\ +\ 30 \\ \hline 632 \end{array} \qquad \begin{array}{r} 300 + 60 + 0 \\ +(400 + 0 + 8) \\ \hline 700 + 60 + 8 \end{array} = 768 \qquad \begin{array}{r} 360 \\ +408 \\ \hline 768 \end{array}$$

Find the sums.

400 + 6 406	700 + 30 730	200 +500 700	600 +301 901
500 +420 920	900 + 27 927	806 +102 908	320 + 40 360
63 +705 768	8 +401 409	250 +609 859	447 + 20 467
360 +129 489	203 +514 717	901 + 35 936	104 +480 584

S-251



Find the sums.

406 +520 926	630 +350 980	732 +233 965	250 +539 789
402 +457 859	164 + 23 187	302 +686 988	256 + 41 297
735 +142 877	806 +143 949	141 +607 748	953 + 12 965
420 +207 627	614 + 84 698	333 + 65 398	211 + 25 236
123 +841 964	821 +103 924	703 +262 965	625 +133 758
542 + 12 554	232 +742 974	524 +463 987	612 +205 817

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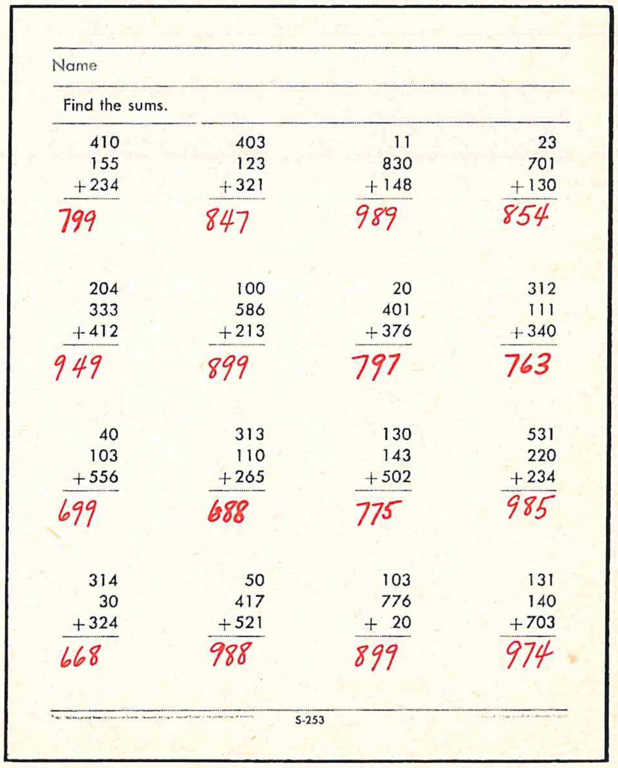


Name

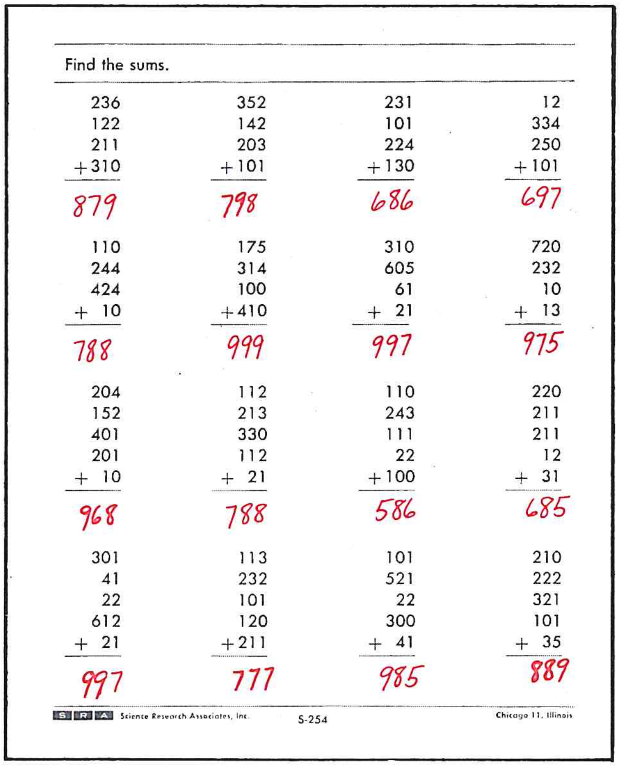
Find the sums.

410 155 +234 799	403 123 +321 847	11 830 +148 989	23 701 +130 854
204 333 +412 949	100 586 +213 899	20 401 +376 797	312 111 +340 763
40 103 +556 699	313 110 +265 688	130 143 +502 775	531 220 +234 985
314 30 +324 668	50 417 +521 988	103 776 + 20 899	131 140 +703 974

S-253

Have it read: "Eight minus six equals two; forty minus zero equals forty; three hundred minus zero equals three hundred; three hundred plus forty plus two equals three hundred forty-two." Help the children understand that it is not necessary to write the 0's in the tens place and the hundreds place when only ones are involved in a problem. We just think of them as being there in the subtraction.

Have a child write the vertical notation for the problem and have it read in terms of ones, tens, and hundreds.



Find the sums.

236 122 211 +310 879	352 142 203 +101 798	231 101 224 +130 686	12 334 250 +101 697
110 244 424 + 10 788	175 314 100 +410 999	310 605 61 + 21 997	720 232 10 + 13 975
204 152 401 201 + 10 968	112 213 330 112 + 21 788	110 243 111 22 +100 586	220 211 211 12 + 31 685
301 41 22 612 + 21 997	113 232 101 120 +211 777	101 521 22 300 + 41 985	210 222 321 101 + 35 889

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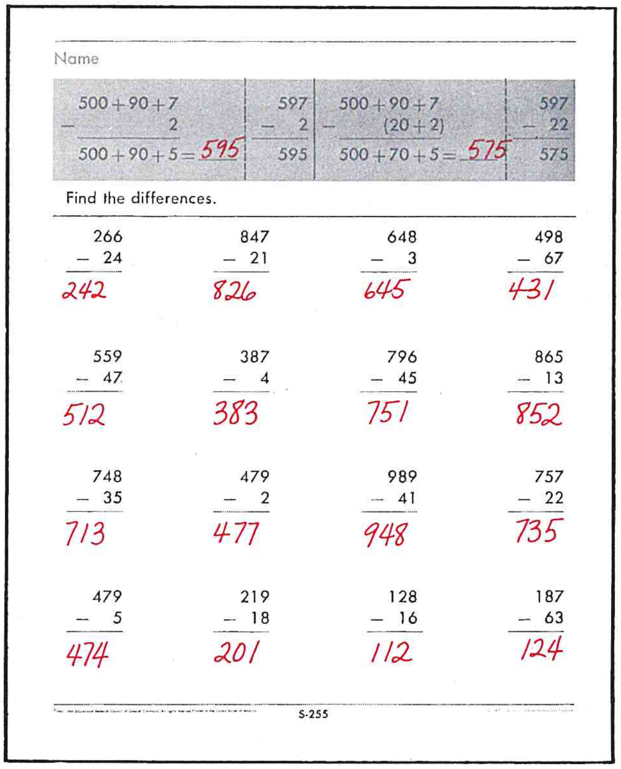


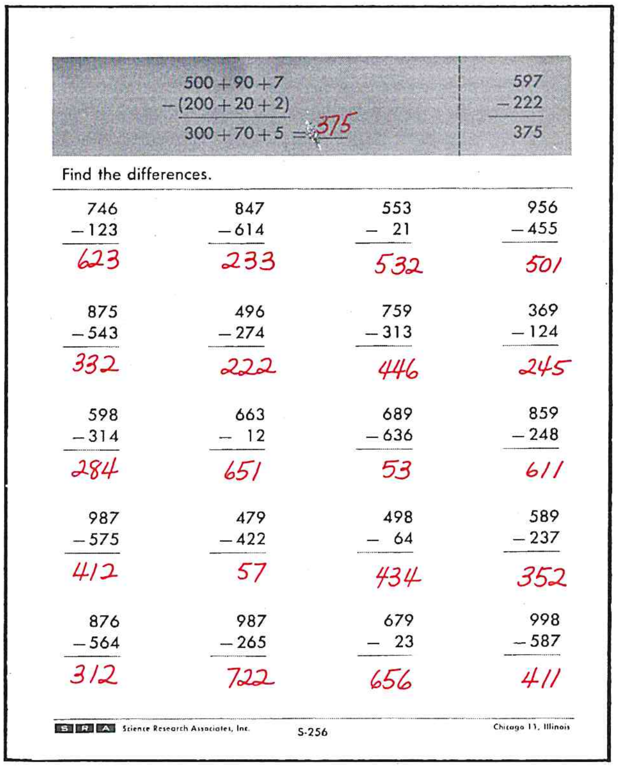
Name

500 + 90 + 7 − 2 500 + 90 + 5 = 595	597 − 2 595	500 + 90 + 7 − (20 + 2) 500 + 70 + 5 = 575	597 − 22 575

Find the differences.

266 − 24 242	847 − 21 826	648 − 3 645	498 − 67 431
559 − 47 512	387 − 4 383	796 − 45 751	865 − 13 852
748 − 35 713	479 − 2 477	989 − 41 948	757 − 22 735
479 − 5 474	219 − 18 201	128 − 16 112	187 − 63 124

S-255



500 + 90 + 7 − (200 + 20 + 2) 300 + 70 + 5 = 375	597 − 222 375

Find the differences.

746 − 123 623	847 − 614 233	553 − 21 532	956 − 455 501
875 − 543 332	496 − 274 222	759 − 313 446	369 − 124 245
598 − 314 284	663 − 12 651	689 − 636 53	859 − 248 611
987 − 575 412	479 − 422 57	498 − 64 434	589 − 237 352
876 − 564 312	987 − 265 722	679 − 23 656	998 − 587 411

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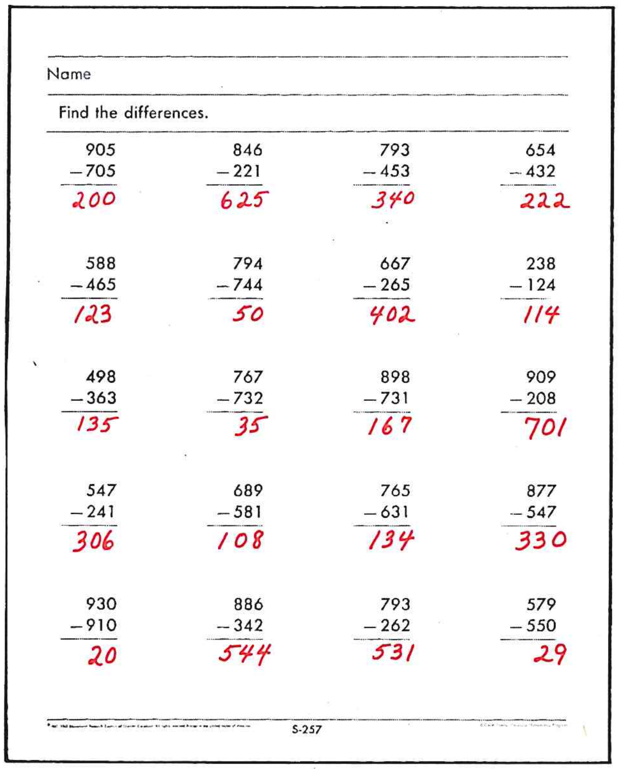


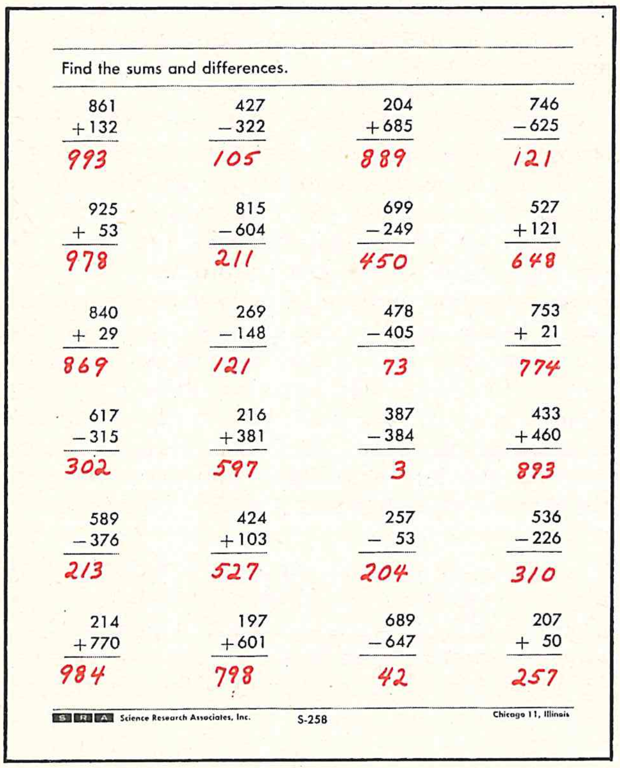
Name

Find the differences.

905 − 705 200	846 − 221 625	793 − 453 340	654 − 432 222
588 − 465 123	794 − 744 50	667 − 265 402	238 − 124 114
498 − 363 135	767 − 732 35	898 − 731 167	909 − 208 701
547 − 241 306	689 − 581 108	765 − 631 134	877 − 547 330
930 − 910 20	886 − 342 544	793 − 262 531	579 − 550 29

S-257

Continue in this way to develop other subtractions such as 308 − 2 = ☐, 348 − 26 = ☐, 544 − 40 = ☐, and 729 − 29 = ☐. Set up some addition situations and then "undo" them. Have related expanded notation and vertical notation accompany the action of the Countingmen in each problem.



Find the sums and differences.

861 +132 993	427 −322 105	204 +685 889	746 −625 121
925 + 53 978	815 −604 211	699 −249 450	527 +121 648
840 + 29 869	269 −148 121	478 −405 73	753 + 21 774
617 −315 302	216 +381 597	387 −384 3	433 +460 893
589 −376 213	424 +103 527	257 − 53 204	536 −226 310
214 +770 984	197 +601 798	689 −647 42	207 + 50 257

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▶ Use work pages S-255 and S-256 to provide practice in working three-digit subtraction problems. Discuss the problems at the top of the pages with the children. Have them write the needed numerals to complete the sample problems. They are to complete the remainder of each page on their own. Remind them to use care in lining up the numerals that represent ones, tens, and hundreds.

▶ Use work page S-257 to give children the opportunity to try out their growing skill in solving three-digit subtraction problems without borrowing. Direct them to find the differences.

Work page S-258 provides an opportunity to review three-digit addition and subtraction without carrying and borrowing. Have children find the sums or differences.

Supplemental Experiences

▶ The teacher may wish to present some children with a variation in writing and working three-digit addition and subtraction problems without carrying and borrowing.

(a)

$$\begin{array}{r} 400 + 30 + 2 \\ +(300 + 20 + 7) \\ \hline 700 + 50 + 9 = 759 \end{array}$$

(b)

$$\begin{array}{r} 432 \\ +\,327 \\ \hline 9 \\ 50 \\ 700 \\ \hline 759 \end{array}$$

(c)

$$\begin{array}{r} 432 \\ +\,327 \\ \hline 759 \end{array}$$

Relate the different forms which can be used in working problems of this type. Discuss the fact that the numbers involved remain the same—only the symbolism and the approach vary.

(a)

$$\begin{array}{r} 400 + 30 + 7 \\ -(300 + 10 + 6) \\ \hline 100 + 20 + 1 = 121 \end{array}$$

(b)

$$\begin{array}{rrrr} 437 & 437 & & \\ -316 & -6 & & \\ \hline & 431 & 431 & \\ & & -10 & \\ & & 421 & 421 \\ & & & -300 \\ & & & 121 \end{array}$$

(c)

$$\begin{array}{r} 437 \\ -316 \\ \hline 121 \end{array}$$

▶ The teacher may wish to challenge some children by presenting oral chain problems involving hundreds, tens, and ones. Say: "One hundred plus three hundred," pause, "plus two hundred." Call upon someone to give the answer. Present another problem: "Two hundred plus four hundred," pause, "plus seventy," pause, "plus twenty." Call upon someone to give the answer. Present a third problem: "Six hundred plus two hundred," pause, "plus twenty," pause, "plus forty," pause, "plus two," pause, "plus three."

Continue in this way with other oral problems. Include subtractions. Say: "Seven hundred fifty minus one hundred," pause, "minus two hundred," pause, "minus ten"; or, "one hundred twenty-five minus one hundred," pause, "minus twenty," pause, "minus five."

▶ Place a series of comparisons on the chalkboard. Call upon children to complete each statement by placing the correct symbol, >, <, or =, in the circle. Have the class determine the accuracy of the symbols chosen.

124 + 5 ○ 125 + 4
307 + 90 ○ 380 + 17
222 − 111 ○ 111 + 222
309 ○ 903
415 + 123 ○ 115 + 423
105 + 710 ○ 705 + 110
372 − 100 ○ 272 + 100

Discuss the thinking involved in determining the relation between pairs of numbers. Have children decide if addition and/or subtraction is necessary in each case.

Three-Digit Addition

CARRYING FROM ONES PLACE TO TENS PLACE

Work Pages S-259 Through S-264

Objectives

To extend the development of addition with carrying.

To strengthen the understanding of the concept of carrying.

To help the children develop skill in addition with carrying.

Fundamentals

The child has worked with three-digit addition without carrying and is familiar with the place-value meaning of three-digit numerals. He is able to apply the commutative and associative properties. Now he will work with three-digit addition involving carrying. Justification for the steps taken in solving addition problems such as 429 + 257 = ☐ will once more rest in the application of the process of scrambling.

$$\begin{array}{lllll}
(400 + 20 + 9) & + (200 + 50 + 7) = & & & \\
(400 + 200) & + & (20 + 50) & + (9 + 7) = & \\
\quad 600 & + & \quad 70 & + (10 + 6) = & \\
\quad 600 & + & \quad 80 & + 6 = & \\
 & & & \quad 686 &
\end{array}$$

Carrying from the ones place to the tens place in three-digit addition involves no major conceptual difference from previous work with carrying. The only variation is that after one ten is carried to the tens place and the tens are added, the hundreds must be added.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Ability to work with number combinations through eighteen.

Understanding of the concept of place value.

Ability to work with three-digit addition without carrying.

Understanding of the concept of carrying.

Understanding of the commutative and associative properties of addition.

Ability to use different numerals for the same number.

Ability to work with expanded notation and vertical notation.

Developmental Experiences

MATERIALS

pocket chart
strips of colored paper
rubber bands
3 Countingmen

▶ Place the problem 348 + 6 = ☐ on the chalkboard. Have a child write the expanded notation for this problem next to the equation. Have someone place in the pocket chart the bundles of one hundred strips, bundles of ten strips, and single strips of paper needed to represent the first addend. (If the ten bundles of ten strips are too cumbersome, use one different-colored strip, clearly labeled 100, to represent ten tens.

Have another child place in the pocket chart the strips of paper needed to represent the second addend. Ask someone else to join the sets of ones, the sets of tens, and the sets of hundreds. When he observes that he has fourteen ones, four tens, and three hundreds, ask him to record this in the expanded-notation problem. Now have him bundle ten ones into one set of ten and tell how many ones, how many tens, and how many hundreds he has all together (four ones, five tens, and three hundreds). Help him write the next step in the expanded-notation problem. As a final step, have a child record the standard place-value numeral to represent the number three hundred plus fifty plus four.

Continue in this way having the children work with sets and expanded notation in similar addition problems. Next have them adapt this procedure for use with problems such as 156 + 35 = ☐, 204 + 87 = ☐, 305 + 65 = ☐, and 129 + 258 = ☐.

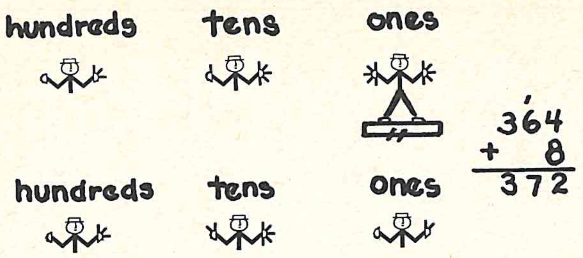
$$\begin{array}{crcrcrl}
 & & & \text{(a)} & & & \\
 & 100 & + & 50 & + & 6 & \\
+ & & & (30 & + & 5) & \\ \cline{4-6}
 & 100 & + & 80 & + & 11 & = \\
 & 100 & + & 90 & + & 1 & = 191
\end{array}
\qquad
\begin{array}{crcrcrl}
 & & & \text{(b)} & & & \\
 & 200 & + & 0 & + & 4 & \\
+ & & & (80 & + & 7) & \\ \cline{4-6}
 & 200 & + & 80 & + & 11 & = \\
 & 200 & + & 90 & + & 1 & = 291
\end{array}$$

$$\begin{array}{crcrcrl}
 & & & \text{(c)} & & & \\
 & 300 & + & 0 & + & 5 & \\
+ & & & (60 & + & 5) & \\ \cline{2-6}
 & 300 & + & 60 & + & 10 & = \\
 & 300 & + & 70 & + & 0 & = 370 \\ \cline{2-6}
\end{array}
\qquad
\begin{array}{crcrcrl}
 & & & \text{(d)} & & & \\
 & 100 & + & 20 & + & 9 & \\
+ & (200 & + & 50 & + & 8) & \\ \cline{2-6}
 & 300 & + & 70 & + & 17 & = \\
 & 300 & + & 80 & + & 7 & = 387 \\ \cline{2-6}
\end{array}$$

▶ Use the Countingmen to demonstrate three-digit addition with carrying. Write the problem 364 + 8 = ☐ in vertical notation on the chalkboard.

Show the addend three hundred sixty-four on the Countingmen. Ask how many ones, tens, and hundreds are represented. Have the children explain what the problem requires: eight is to be added to three hundred sixty-four. Have them tell how many ones, tens, and hundreds must be added.

Begin by placing six of the eight fingers on the Ones-man. The class will notice that this Countingman has all ten fingers before the full set of eight has been joined with the set of four. They should say that the ten ones must be changed to one ten. Hold the set of one ten up so that it is visible to the class,



Record the digit for the number of hundreds on the Countingman. Have the class summarize the steps in carrying as they refer to the vertical notation on the chalkboard: Four plus eight equals twelve; record 2 ones, carry one ten over to the tens place. Ten plus sixty equals seventy; record 7 in the tens place to represent seventy. Three hundred plus zero equals three hundred; record 3 in the hundreds place to represent three hundred. Three hundred plus seventy plus two equals three hundred seventy-two. The sum of three hundred sixty-four and eight is three hundred seventy-two.

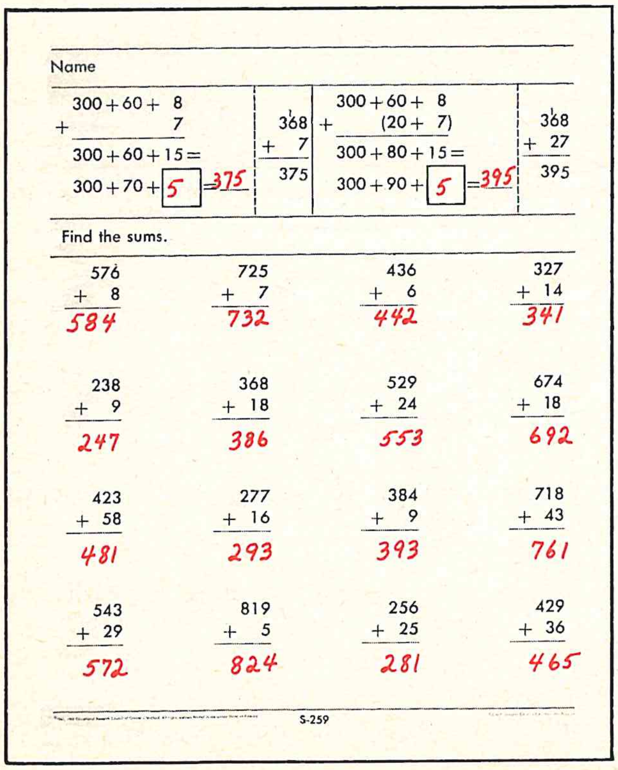
Continue using the same procedure. With similar problems, go through each step carefully on the Countingmen and have the children write the algorism on the chalkboard.

Next have children adapt this procedure to problems such as 439 + 54 = □, 508 + 77 = □, 129 + 61 = □, 367 + 215 = □, 472 + 118 = □, and 603 + 207 = □.

▶ Use work pages S-259 through S-264 as a follow-up to the activities with sets and the Countingmen. As each sheet is presented, work through the problems at the top of the front page with the children. Have them write the appropriate numerals as the discussion proceeds. Then have them complete both sides of the sheet on their own.

Work pages S-259 and S-260 provide practice in adding a number represented by a one-digit or a two-digit numeral to a number represented by a three-digit numeral.

Work pages S-261 and S-262 provide practice with a pair of three-digit addends.


Name

300 + 60 + 8
+ 7
300 + 60 + 15 =
300 + 70 + $\boxed{5}$ = 375

$3\overset{1}{6}8$
+ 7
375

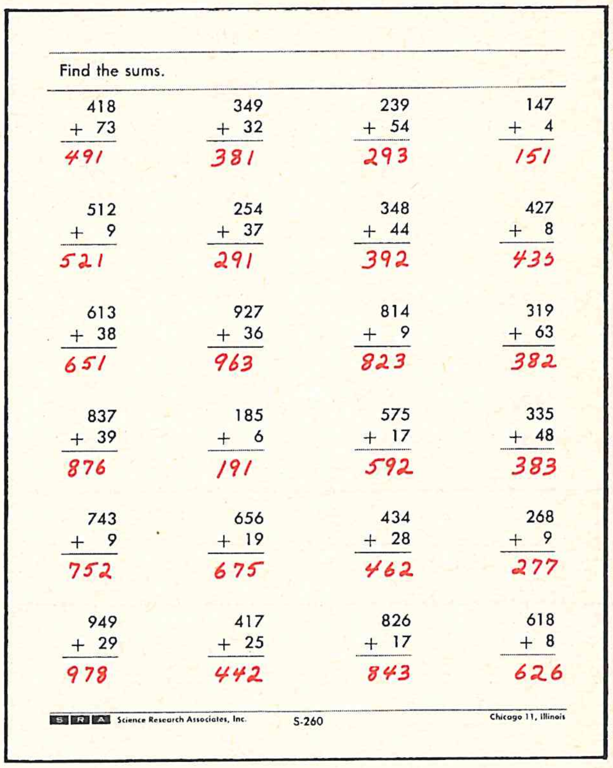
300 + 60 + 8
+ (20 + 7)
300 + 80 + 15 =
300 + 90 + $\boxed{5}$ = 395

$3\overset{1}{6}8$
+ 27
395

Find the sums.

576 + 8 584	725 + 7 732	436 + 6 442	327 + 14 341
238 + 9 247	368 + 18 386	529 + 24 553	674 + 18 692
423 + 58 481	277 + 16 293	384 + 9 393	718 + 43 761
543 + 29 572	819 + 5 824	256 + 25 281	429 + 36 465

S-259


Find the sums.

418 + 73 491	349 + 32 381	239 + 54 293	147 + 4 151
512 + 9 521	254 + 37 291	348 + 44 392	427 + 8 435
613 + 38 651	927 + 36 963	814 + 9 823	319 + 63 382
837 + 39 876	185 + 6 191	575 + 17 592	335 + 48 383
743 + 9 752	656 + 19 675	434 + 28 462	268 + 9 277
949 + 29 978	417 + 25 442	826 + 17 843	618 + 8 626

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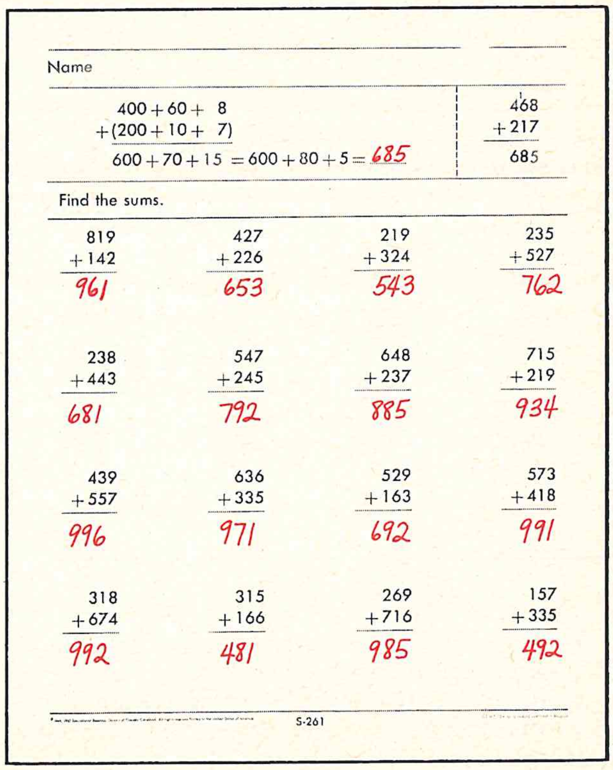

Name

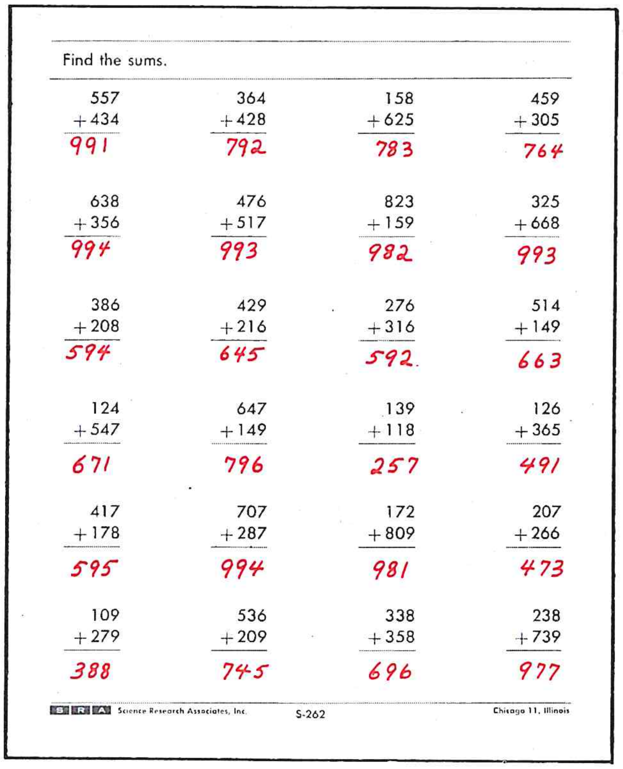
400 + 60 + 8
+ (200 + 10 + 7)
600 + 70 + 15 = 600 + 80 + 5 = 685

$4\overset{1}{6}8$
+ 217
685

Find the sums.

819 + 142 961	427 + 226 653	219 + 324 543	235 + 527 762
238 + 443 681	547 + 245 792	648 + 237 885	715 + 219 934
439 + 557 996	636 + 335 971	529 + 163 692	573 + 418 991
318 + 674 992	315 + 166 481	269 + 716 985	157 + 335 492

S-261



Find the sums.

557 +434 991	364 +428 792	158 +625 783	459 +305 764
638 +356 994	476 +517 993	823 +159 982	325 +668 993
386 +208 594	429 +216 645	276 +316 592	514 +149 663
124 +547 671	647 +149 796	139 +118 257	126 +365 491
417 +178 595	707 +287 994	172 +809 981	207 +266 473
109 +279 388	536 +209 745	338 +358 696	238 +739 977

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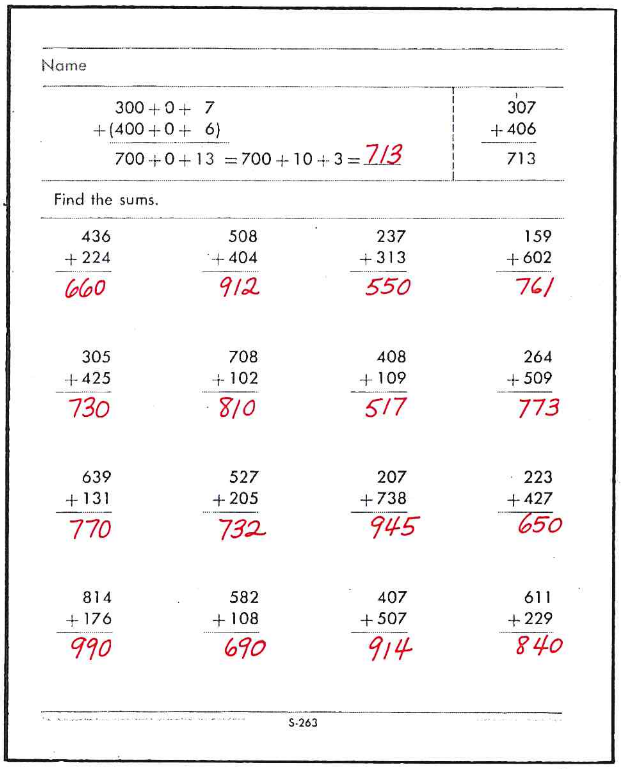


Name

$$\begin{array}{r} 300+0+7 \\ +(400+0+6) \\ 700+0+13 = 700+10+3 = \underline{713} \end{array} \qquad \begin{array}{r} 3\overset{1}{0}7 \\ +406 \\ \hline 713 \end{array}$$

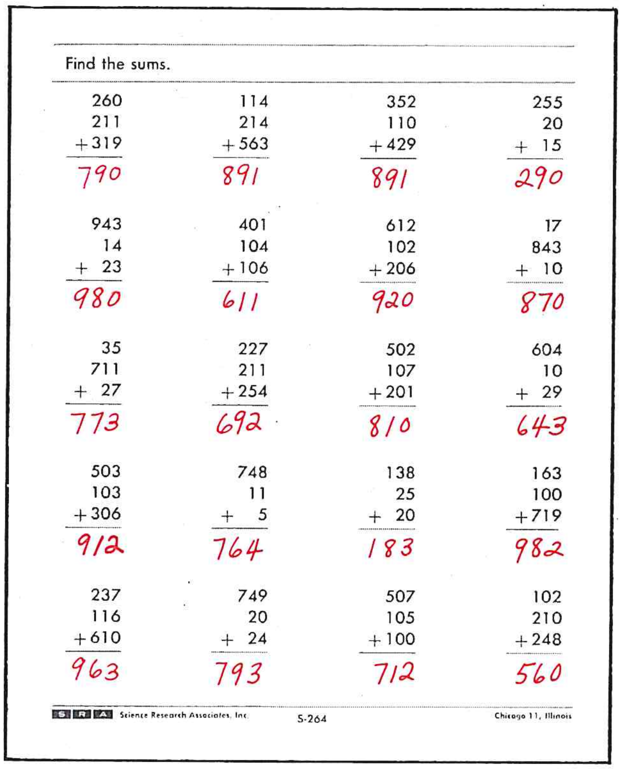
Find the sums.

436 +224 660	508 +404 912	237 +313 550	159 +602 761
305 +425 730	708 +102 810	408 +109 517	264 +509 773
639 +131 770	527 +205 732	207 +738 945	223 +427 650
814 +176 990	582 +108 690	407 +507 914	611 +229 840

S-263

Work page S-263 provides practice with a pair of three-digit addends, with 0's included.

Work page S-264 provides practice with three-digit column addition, with carrying from the ones to the tens place.



Find the sums.

260 211 +319 790	114 214 +563 891	352 110 +429 891	255 20 + 15 290
943 14 + 23 980	401 104 +106 611	612 102 +206 920	17 843 + 10 870
35 711 + 27 773	227 211 +254 692	502 107 +201 810	604 10 + 29 643
503 103 +306 912	748 11 + 5 764	138 25 + 20 183	163 100 +719 982
237 116 +610 963	749 20 + 24 793	507 105 +100 712	102 210 +248 560

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Supplemental Experiences

▶ Present children with a variation in writing and working three-digit addition problems with carrying.

$$\begin{array}{r} 200+40+\ 5 \\ +\ (400+30+\ 8) \\ \hline 600+70+13\ = \\ 600+80+\ 3\ = \\ 683 \end{array} \qquad \begin{array}{r} 245 \\ +\ 438 \\ \hline 13 \\ 70 \\ +\ 600 \\ \hline 683 \end{array} \qquad \begin{array}{r} 2\overset{1}{4}5 \\ +\ 438 \\ \hline 683 \end{array}$$

Have children relate the different forms that can be used in addition computations.

▶ The development of number sequences may be made more complex as a challenge for some classes.

1, 2, 4, 7,____, 16, 22,____,____,

Pattern: The difference between successive terms increases by one each time.

3, 3, 3, 3, 3, 3,____, 3, 3,____, 3.

Pattern: Addition (or subtraction) of 0 occurs each time.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34,____.

Pattern: After the first two terms, add each term to its predecessor to get the next term.

Place a series of number sequences on the chalkboard. Have children take turns, supplying the missing terms.

Three-Digit Subtraction — Addition

BORROWING AND CARRYING

Work Pages S-265 Through S-270

Objectives

To extend the development of subtraction with borrowing.

To strengthen the children's understanding of the concept of borrowing in subtraction.

To help the children develop skill in subtraction with borrowing and addition with carrying.

Fundamentals

The child should be encouraged to apply his understanding of the inverse relation between addition and subtraction as he works with three-digit subtraction problems involving borrowing from the tens place. He should learn to use the appropriate symbolism as ideas take shape and become meaningful to him.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

An understanding of the concept of place value.

Ability to work with three-digit subtraction without carrying.

Mastery of number combinations through eighteen.

Ability to use different numerals for the same number.

Ability to work with expanded notation and vertical notation.

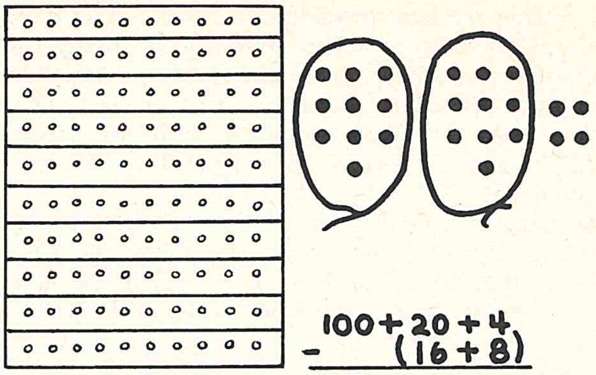
Awareness of the inverse relation between addition and subtraction.

Developmental Experiences

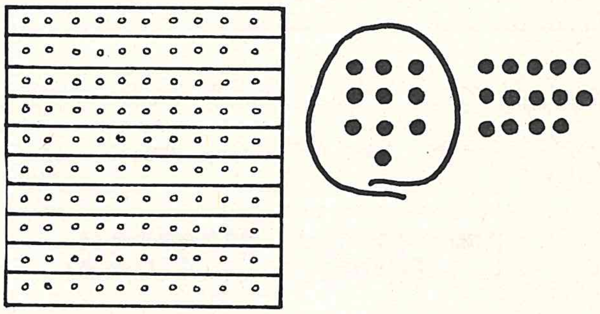
MATERIALS

flannel board
felt disks
yarn
tagboard card
3 Countingmen

▶ Write the expanded notation for the problem 124 − 18 = ☐ on the chalkboard. Place pictures of sets to represent the first number in this subtraction on the flannel board. Use a tagboard card to picture the set of one hundred. Have the children explain what they must do first in this problem—subtract eight from four. Ask them whether eight disks can be removed from the four loose disks. They will immediately discover that this is impossible, that one of the sets of ten must be thought of as ten ones. Remove the yarn from around one set of ten; say that one hundred plus twenty plus four can be thought of as one hundred plus ten plus fourteen.

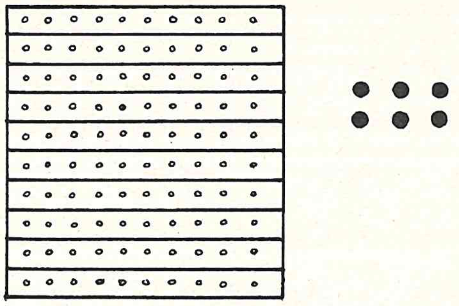


$$\begin{array}{rr} & 100+20+4 \\ - & (16+8) \\ \hline \end{array}$$



$$\begin{array}{rr} & 100+20+4 \\ - & (10+8) \\ \hline \end{array} = \begin{array}{rr} & 100+10+14 \\ - & (10+8) \\ \hline \end{array}$$

It is now possible to remove eight from fourteen. Help a child write on the chalkboard the expanded notation related to the action. Have the subtraction of ones completed in the expanded notation and have the children decide whether any more subtraction operations must be performed to solve the problem. Now return to the sets; have one set of ten removed from the set of ten. Ask how many sets of hundreds, tens, and ones remain. Have the expanded notation completed and the standard place-value numeral for one hundred plus zero plus six written as the final step.

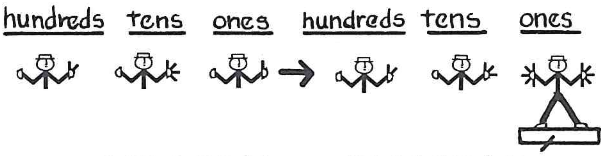


$$\begin{array}{rr} & 100+10+14 \\ - & (10+8) \\ \hline & 100+0+6=106 \end{array}$$

Follow the same procedure to develop other three-digit subtraction problems involving borrowing from the tens place. Help the children write the expanded notation related to the action with the sets. Have them work out other subtraction problems, such as 131 − 8 = ☐, 131 − 28 = ☐, 140 − 24 = ☐, and 480 − 279 = ☐.

▶ Use the Countingmen to demonstrate three-digit subtractions with borrowing from the tens place. Write the problem 241 − 18 = ☐ in expanded notation and in algorism form on the chalkboard.

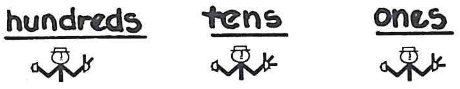
Show the first addend in the problem on the Countingmen. Begin to remove eight from one. The class will tell you that this is impossible; you must get one ten from the Tens-man and change it back to ten ones. Move the one from the hand to the base of the Ones-man. Bring one ten back from the Tens-man and put ten ones on the Ones-man's hands. Ask the class to describe the action: two hundreds, four tens, one one are now shown as two hundreds, three tens, eleven ones.



$$\begin{array}{rcl} 200+40+1 & = & 200+30+11 \\ -\ (10+8) & = & -\ (10+8) \end{array}$$

Refer to the computational form for the problem. Cross out the digit representing four tens, or forty, and write above it the digit 3 to represent three tens, or thirty. Place a small numeral 1 to the left of the digit 1 to represent eleven.

Remove a set of eight from the set of eleven now on the Ones-man. Take away the one on the base first and then seven more fingers. Subtract eight from eleven in the vertical notation and write the numeral 3 in the ones place below the line.

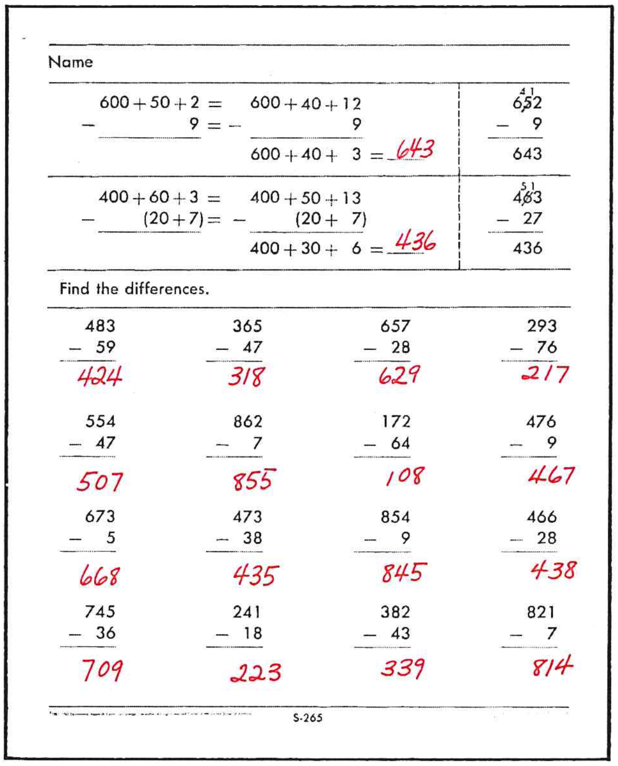


$$\begin{array}{rcl} 200+40+1 & = & 200+30+11 \\ \underline{-\ (10+8)} & & \underline{-\ (10+8)} = \underline{223} \\ & & 200+20+3 \end{array}$$

Have the class decide whether the problem is complete; they will tell you that you must remove one ten from the Tens-man. Complete the subtraction of ten in the algorism. The class will conclude that no hundreds need to be subtracted in this problem. Complete the problem and have the difference read.

Summarize the steps in working subtraction problems with borrowing. Relate the symbolism in the algorism to the symbolism in the expanded notation.

Continue in this way, developing other subtractions of this kind. Have the children take turns writing the algorism related to the action on the Countingmen. Include such problems as 682 − 7



Name

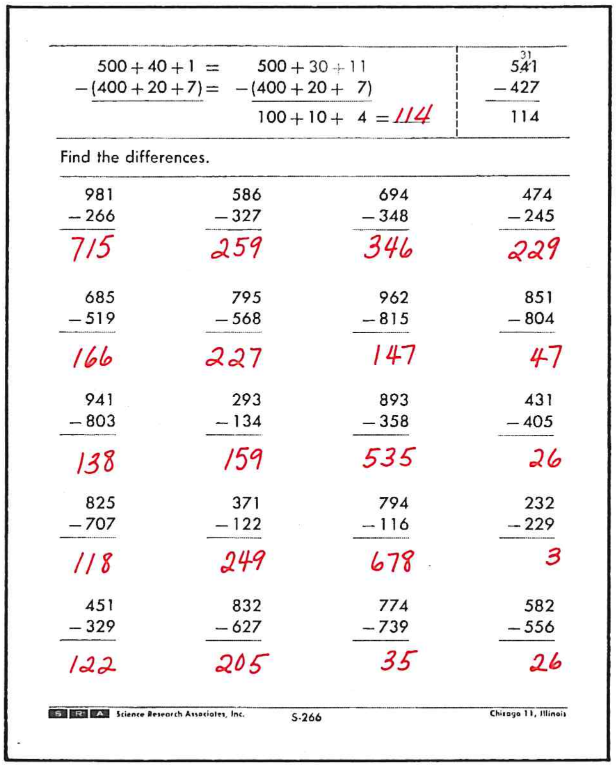
$$\begin{array}{rcl} 600+50+2 & = & 600+40+12 \\ -\ \underline{9} & = & -\ \underline{9} \\ & & 600+40+\ 3 = \underline{643} \end{array} \qquad \begin{array}{r} 6\overset{4}{\cancel{5}}{}^{1}2 \\ -\ \ 9 \\ \hline 643 \end{array}$$

$$\begin{array}{rcl} 400+60+3 & = & 400+50+13 \\ -\ \underline{(20+7)} & = & -\ \underline{(20+\ 7)} \\ & & 400+30+\ 6 = \underline{436} \end{array} \qquad \begin{array}{r} 4\overset{5}{\cancel{6}}{}^{1}3 \\ -\ 27 \\ \hline 436 \end{array}$$

Find the differences.

483 − 59 424	365 − 47 318	657 − 28 629	293 − 76 217
554 − 47 507	862 − 7 855	172 − 64 108	476 − 9 467
673 − 5 668	473 − 38 435	854 − 9 845	466 − 28 438
745 − 36 709	241 − 18 223	382 − 43 339	821 − 7 814

S-265



$$\begin{array}{rcl} 500+40+1 & = & 500+30+11 \\ -\underline{(400+20+7)} & = & -\underline{(400+20+\ 7)} \\ & & 100+10+\ 4 = \underline{114} \end{array} \qquad \begin{array}{r} 5\overset{3}{\cancel{4}}{}^{1}1 \\ -427 \\ \hline 114 \end{array}$$

Find the differences.

981 −266 715	586 −327 259	694 −348 346	474 −245 229
685 −519 166	795 −568 227	962 −815 147	851 −804 47
941 −803 138	293 −134 159	893 −358 535	431 −405 26
825 −707 118	371 −122 249	794 −116 678	232 −229 3
451 −329 122	832 −627 205	774 −739 35	582 −556 26

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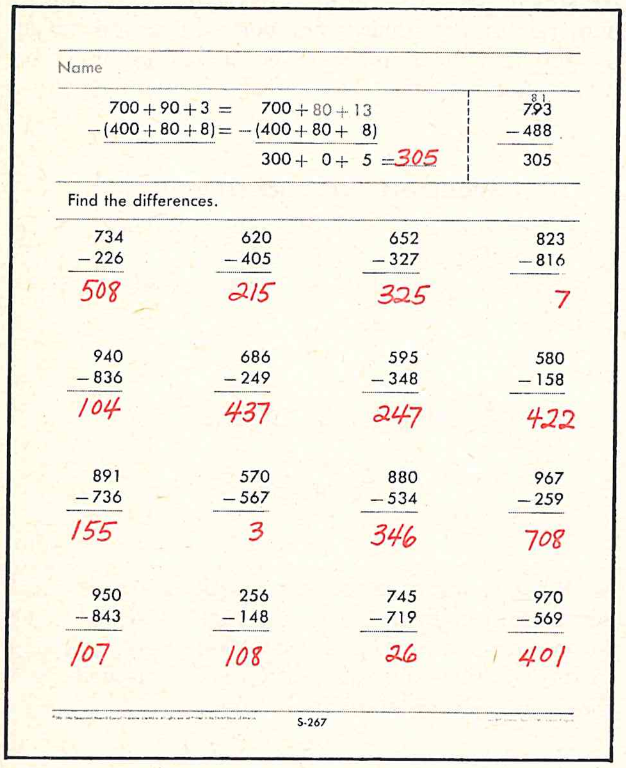
= □, 457 − 39 = □, 891 − 622 = □, and 323 − 214 = □.

▶ Use work pages S-265 and S-266 for practice in subtracting numbers represented by one-, two-, and three-digit numerals from a number represented by a three-digit numeral involving borrowing from the tens place. Work the subtraction problems at the top of each page with the class. Have them write the needed numerals to complete each example. Tell them to finish the remainder of each page by finding the differences.

▶ Use work pages S-267 and S-268 for practice in solving three-digit subtraction problems involving borrowing, with zero included.



Name

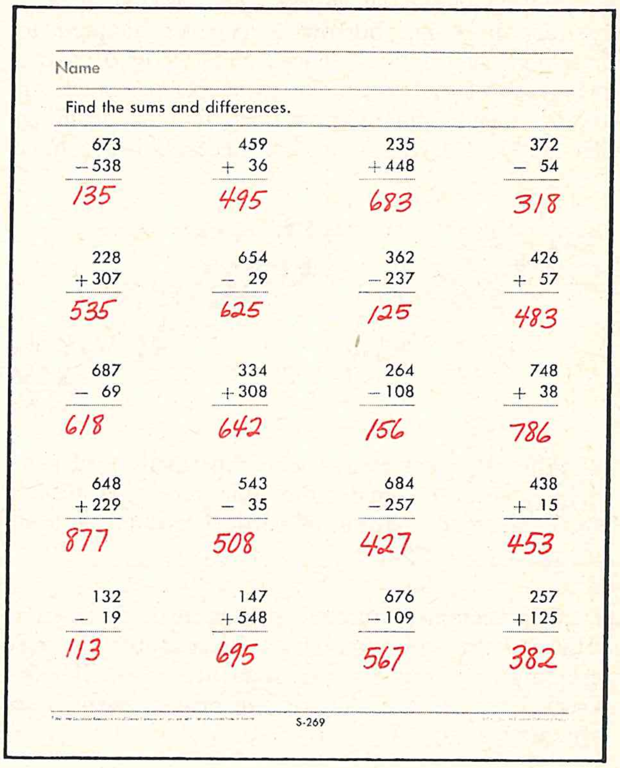
$$\begin{array}{rcl} 700+90+3 & = & 700+80+13 \\ -(400+80+8) & = & -(400+80+\ \ 8) \\ & & 300+\ 0+\ 5 = 305 \end{array}$$

$$\begin{array}{r} \overset{8\,1}{7\cancel{9}3} \\ -488 \\ \hline 305 \end{array}$$

Find the differences.

734 − 226 508	620 − 405 215	652 − 327 325	823 − 816 7
940 − 836 104	686 − 249 437	595 − 348 247	580 − 158 422
891 − 736 155	570 − 567 3	880 − 534 346	967 − 259 708
950 − 843 107	256 − 148 108	745 − 719 26	970 − 569 401

S-267

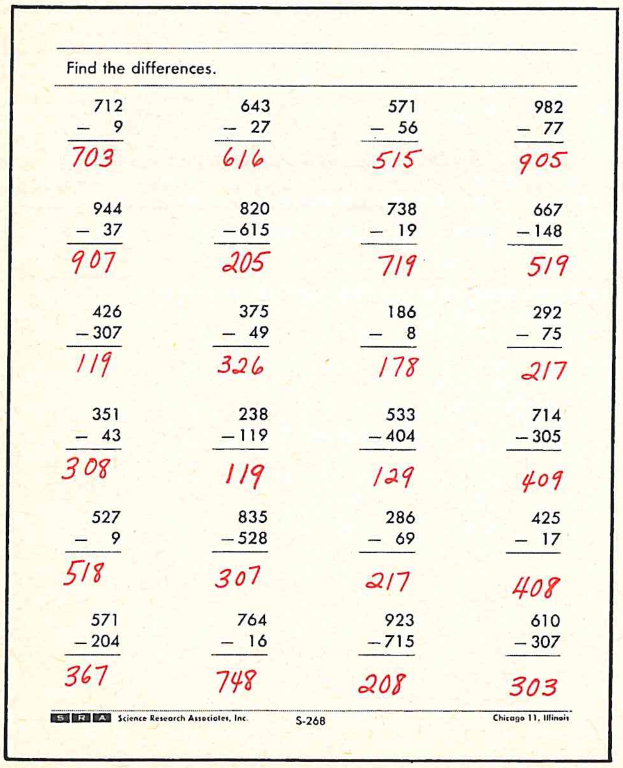


Name

Find the sums and differences.

673 − 538 135	459 + 36 495	235 + 448 683	372 − 54 318
228 + 307 535	654 − 29 625	362 − 237 125	426 + 57 483
687 − 69 618	334 + 308 642	264 − 108 156	748 + 38 786
648 + 229 877	543 − 35 508	684 − 257 427	438 + 15 453
132 − 19 113	147 + 548 695	676 − 109 567	257 + 125 382

S-269



Find the differences.

712 − 9 703	643 − 27 616	571 − 56 515	982 − 77 905
944 − 37 907	820 − 615 205	738 − 19 719	667 − 148 519
426 − 307 119	375 − 49 326	186 − 8 178	292 − 75 217
351 − 43 308	238 − 119 119	533 − 404 129	714 − 305 409
527 − 9 518	835 − 528 307	286 − 69 217	425 − 17 408
571 − 204 367	764 − 16 748	923 − 715 208	610 − 307 303

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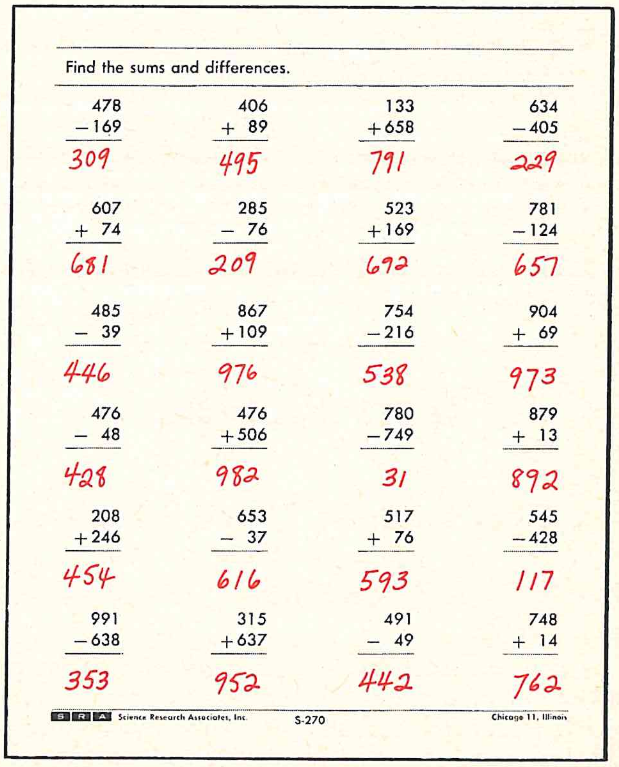

Find the sums and differences.

478 − 169 309	406 + 89 495	133 + 658 791	634 − 405 229
607 + 74 681	285 − 76 209	523 + 169 692	781 − 124 657
485 − 39 446	867 + 109 976	754 − 216 538	904 + 69 973
476 − 48 428	476 + 506 982	780 − 749 31	879 + 13 892
208 + 246 454	653 − 37 616	517 + 76 593	545 − 428 117
991 − 638 353	315 + 637 952	491 − 49 442	748 + 14 762

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▶ Work pages S-269 and S-270 provide an opportunity for the children to test their growing skill in solving three-digit addition and subtraction problems involving carrying and borrowing, with zero included.

Supplemental Experiences

▶ A variation in the writing and working of subtractions involving borrowing can be adapted to three-digit subtractions. Those children who have a firm grasp of the subtraction concepts presented up to this time in the program may find challenge in this method of subtracting in pieces. Always have the difference that results from this method of subtracting checked against the difference that results when expanded notation or vertical notation is used for the same problem.

$$\begin{array}{r} 232 \\ -\ 18 \\ \hline \end{array} \rightarrow \begin{array}{r} 232 \\ -\ 8 \\ \hline 224 \end{array} \rightarrow \begin{array}{r} 224 \\ -\ 10 \\ \hline 214 \end{array} \qquad \begin{array}{r} 457 \\ -239 \\ \hline \end{array} \rightarrow \begin{array}{r} 457 \\ -\ 9 \\ \hline 448 \end{array} \rightarrow \begin{array}{r} 448 \\ -\ 30 \\ \hline 418 \end{array} \rightarrow \begin{array}{r} 418 \\ 200 \\ \hline 218 \end{array}$$

▶ More complex number sequences may involve both addition and subtraction. Place numerals representing a series of number sequences on the chalkboard. Have children supply the missing terms and state the pattern.

1, 3, 2, 4, 3, 5, 4, ____, ____.
Pattern: add 2, subtract 1.

1, 4, 2, 5, 3, 6, 4, 7, ____, ____, ____.
Pattern: add 3, subtract 2.

After the pattern has been discovered, conduct a round-robin activity. Have each child in turn name a number to continue the pattern.

Children may wish to invent number sequences of their own and give the pattern for each.

Story Problems

Work Pages S-271 and S-272

Objectives

To extend the basic concepts involved in solving story problems.

To strengthen the children's ability to apply their understanding of three-digit addition and subtraction.

Fundamentals

The story problems on these work pages offer additional practice with three-digit addition and subtraction. The child should be kept aware that careful analysis of story problems is essential. He should also be reminded of the necessity of deciding which number is to be subtracted in subtraction problems. He should learn that in the subtraction algorism the numeral for the greater number always appears at the top and the number to be subtracted must be represented below it.

Readiness for Understanding

Ability to analyze and solve story problems involving three-digit addition and subtraction.

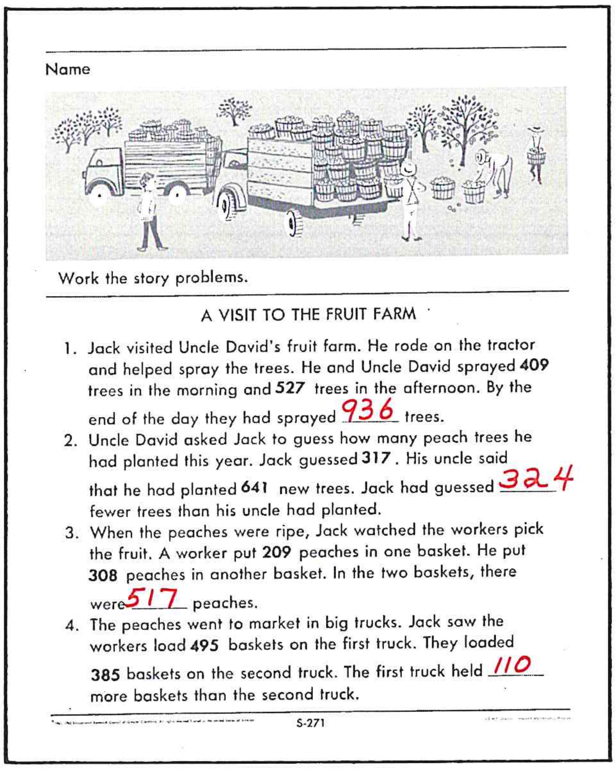
Ability to compare two numbers and decide which is greater.

Developmental Experiences

MATERIALS

3 Countingmen

▶ Review addition of three-digit addends with carrying from the ones to the tens place. Use the Countingmen and expanded notation to give meaning to the addition algorism. Remind the children that they can add two numbers in any order.



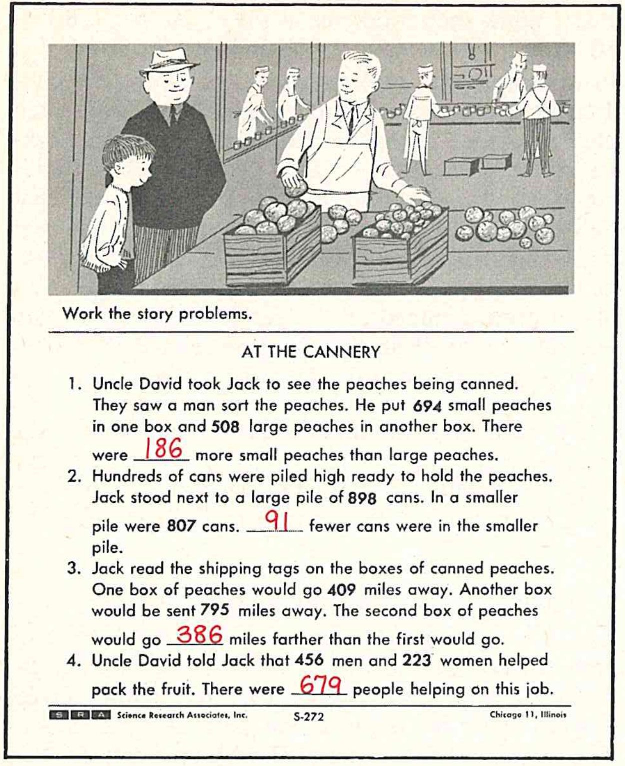
Name

Work the story problems.

A VISIT TO THE FRUIT FARM

1. Jack visited Uncle David's fruit farm. He rode on the tractor and helped spray the trees. He and Uncle David sprayed **409** trees in the morning and **527** trees in the afternoon. By the end of the day they had sprayed 936 trees.
2. Uncle David asked Jack to guess how many peach trees he had planted this year. Jack guessed **317**. His uncle said that he had planted **641** new trees. Jack had guessed 324 fewer trees than his uncle had planted.
3. When the peaches were ripe, Jack watched the workers pick the fruit. A worker put **209** peaches in one basket. He put **308** peaches in another basket. In the two baskets, there were 517 peaches.
4. The peaches went to market in big trucks. Jack saw the workers load **495** baskets on the first truck. They loaded **385** baskets on the second truck. The first truck held 110 more baskets than the second truck.

S-271



Work the story problems.

AT THE CANNERY

1. Uncle David took Jack to see the peaches being canned. They saw a man sort the peaches. He put **694** small peaches in one box and **508** large peaches in another box. There were 186 more small peaches than large peaches.
2. Hundreds of cans were piled high ready to hold the peaches. Jack stood next to a large pile of **898** cans. In a smaller pile were **807** cans. 91 fewer cans were in the smaller pile.
3. Jack read the shipping tags on the boxes of canned peaches. One box of peaches would go **409** miles away. Another box would be sent **795** miles away. The second box of peaches would go 386 miles farther than the first would go.
4. Uncle David told Jack that **456** men and **223** women helped pack the fruit. There were 679 people helping on this job.

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▶ Review three-digit subtraction with borrowing from the tens place. Use the Countingmen and expanded notation to explain why the subtraction algorism works. Point out that the numeral for the greater number must be written on top in the algorism and the numeral for the lesser number on the bottom. Help the children recall that order does make a difference in subtraction.

▶ A discussion of the pictures of the orchard and cannery on work pages S-271 and S-272 will lead into the story problems. Remind children to think through each problem, to decide on the operation, and to judge whether or not their answer is reasonable. Have them check both addition and subtraction problems.

Have children complete the pages on their own but give individual help when needed. Remind them to read each problem carefully.

Supplemental Experiences

▶ Display a state mileage map and have the children compute distances of imaginary vacation trips.

▶ Encourage children to bring in newspaper clippings containing references to numbers, such as number of people, money, mileage, or sports scores. Have them read the clippings and discuss the numbers involved. Some of them may be used as a basis for story problems.

Three-Digit Addition

CARRYING TO THE HUNDREDS PLACE

Work Pages S-273 Through S-280

Objectives

To extend the development of addition with carrying.

To strengthen the children's understanding of the concept of carrying in addition.

To help the children develop skill in addition with carrying.

Fundamentals

In carrying to the hundreds place, the child will be extending his knowledge of addition. He must apply his understanding of place value and of the associative and commutative properties.

Consider the solution to the problem $123 + 295 = \square$. Mathematically the following steps are involved:

$$(100 + 20 + 3) + (200 + 90 + 5) = \square$$
$$(100 + 200) + (20 + 90) + (3 + 5) = \square$$

combined use of
associative and
commutative properties

$$300 + 110 + 8 = \square$$
$$300 + (100 + 10) + 8 = \square$$
$$(300 + 100) + 10 + 8 = \square$$
$$400 + 10 + 8 = \square$$
$$418 = \square$$

The child will approach the computational rule through work with sets and the related expanded notation:

$$\begin{array}{r} 100 + 20 + 3 \\ (200 + 90 + 5) \\ \hline 300 + 110 + 8 \\ 400 + 10 + 8 = 418 \end{array}$$

After he thoroughly understands the concepts involved, he will be introduced to the algorism.

$$\begin{array}{r} {\scriptstyle 1} \\ 123 \\ 295 \\ \hline 418 \end{array}$$

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

An understanding of the concept of place value.

An understanding of the concept of carrying.

An understanding of the concepts of commutativity and associativity.

Ability to work with three-digit addition without carrying.

Ability to work with number combinations through eighteen.

Ability to use different numerals for the same number.

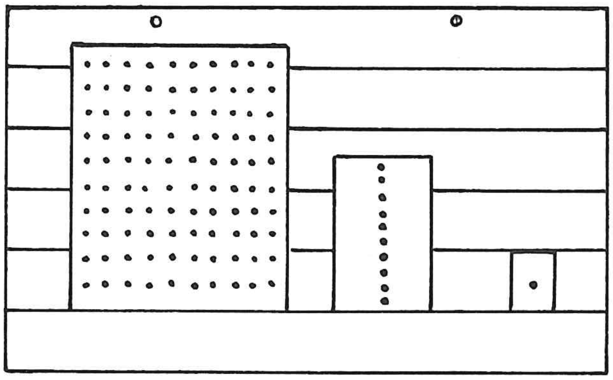
Ability to work with expanded notation and vertical notation.

Developmental Experiences

MATERIALS

- pocket chart
- tagboard cards and strips
- numeral cards 0 through 9
- 3 Countingmen

▶ Review the meaning of place value. Place tagboard cards in the pocket chart to illustrate the number one hundred eleven. Place three cards, each with the digit 1, below the set-cards. Discuss the fact that the same digit is used in each position but that its meaning depends on the place-value definition of the position. Emphasize that each digit represents one set: one set of one, one set of ten, or one set of one hundred.

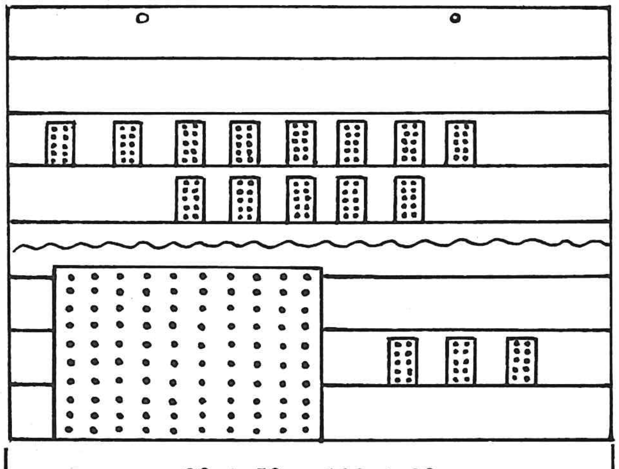


1 1 1

$(100 + 10 + 1)$

Continue in this way with other illustrations of the place-value definition of numbers.

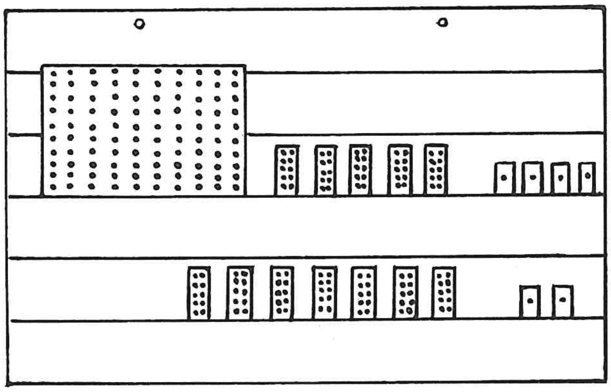
▶ Have children use pictures of sets in the pocket chart. Work such problems as $90 + 20 = \square$, $80 + 50 = \square$, $70 + 60 = \square$, and $30 + 80 = \square$. In working such problems as $9 + 2 = \square$, the children have learned to think of nine plus two as ten plus one, going up to ten and then beyond. In working problems like $90 + 20 = \square$, guide their thinking along the same lines. Help them discover that they must go to one hundred first and then beyond: $90 + 20$ will be thought of as $100 + 10$. Ten sets of ten become one set of one hundred. Eleven sets of ten are regrouped as one set of one hundred and one set of ten. Write the equation related to the action, on the chalkboard, and have it read.



$80 + 50 = 100 + 30$

Be sure that the children can readily follow the thinking steps in problems of this kind and can write equations related to the action with sets before you have them work with carrying in problems such as $191 + 23 = \square$.

Use the pocket chart and pictures of sets to demonstrate carrying from the tens to the hundreds position. Place the problem $154 + 72 = \square$ on the chalkboard. Have children place set pictures in the pocket chart to illustrate the first and second addends. Write the expanded-notation form for the problem, on the chalkboard, below the equation.

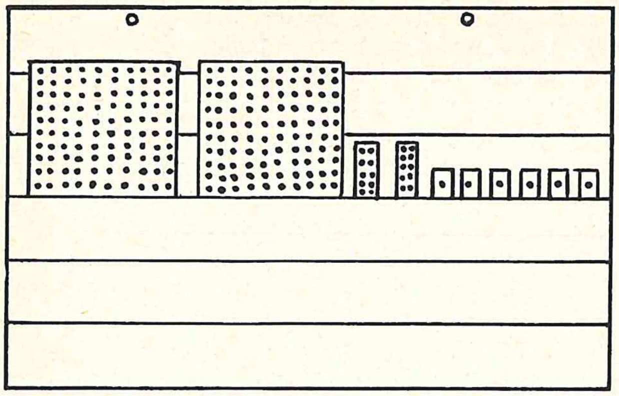


$154 + 72 = \square$

$$\begin{array}{r} 100 + 50 + 4 \\ + \quad (70 + 2) \\ \hline \end{array}$$

Have the ones grouped; write the numeral 6 in the expanded notation. Have the tens grouped; write the numeral 120 in the expanded notation to represent twelve tens. Have the hundreds grouped and write the numeral 100 in the expanded notation. Ask the children whether the twelve tens could be thought of in a different way: one hundred and two tens. Rewrite $100 + 120 + 6$ as $200 + 20 + 6$ in the expanded notation.

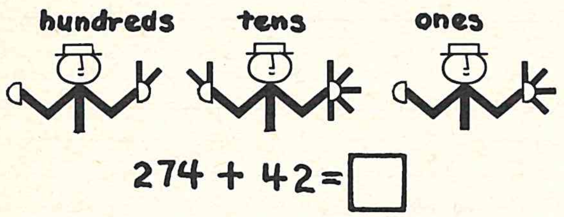
Have the ten tens removed from the chart and replaced with a set of one hundred. Ask how many hundreds, tens, and ones are now represented in the problem. Have someone write the standard place-value numeral for two hundred plus twenty plus six: 226.

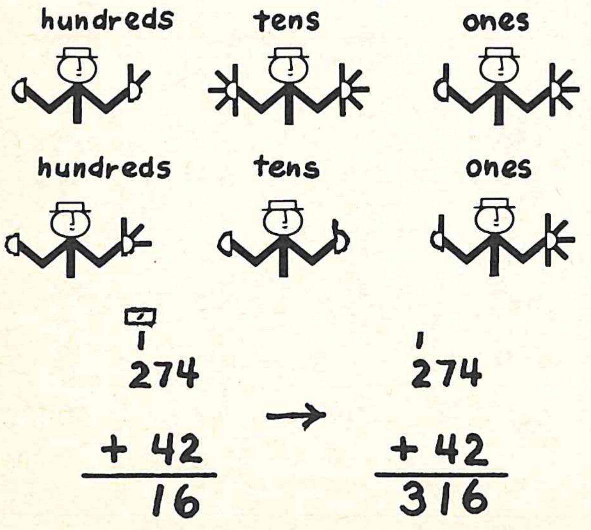


$$\begin{array}{rrrl} & 100 + & 50 + 4 & \\ + & & (70 + 2) & \\ \hline & 100 + & 120 + 6 & = \\ & 200 + & 20 + 6 & = 226 \end{array}$$

Continue in this way to have the children join sets and assist in the writing of related expanded notation for the ideas involved. Use such problems as 76 + 42 = □, 183 + 54 = □, and 265 + 380 = □.

▶ After the children have a satisfactory understanding of the concepts involved in carrying to the hundreds place and an ability to utilize expanded notation to symbolize ideas, use the Countingmen to demonstrate the algorism in such problems as 274 + 42 = □. The teacher can manipulate the Countingmen as she asks questions pertinent to the action. Have each action on the Countingmen recorded as it occurs. Have one child work the problem on the chalkboard using expanded notation, and have another record it in algorism form. Summarize by relating ideas implied in the two notations. The children should read both problems in terms of ones, tens, and hundreds, not in terms of single digits.



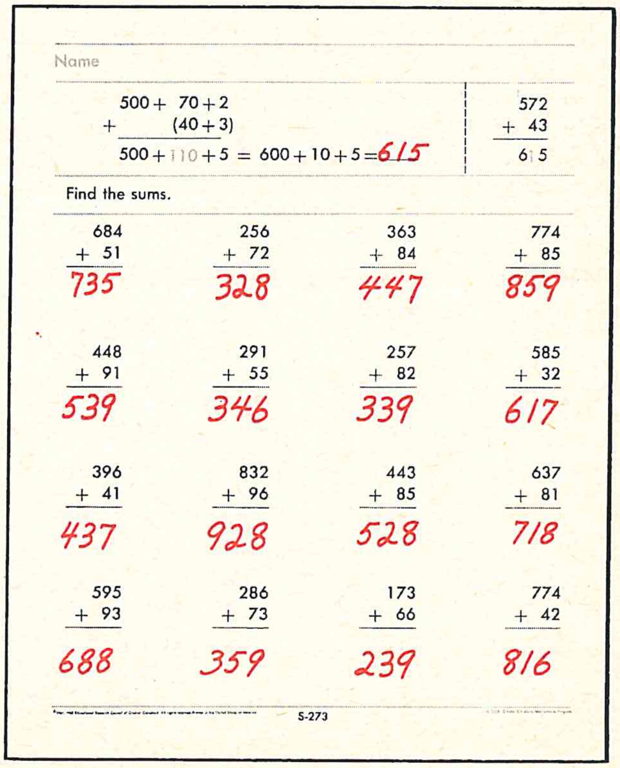


$$\begin{array}{rrrl} & 200 + & 70 + 4 & \\ + & & (40 + 2) & \\ \hline & 200 + & 110 + 6 & = \\ & 300 + & 10 + 6 & = \\ & & 316 & \end{array} \qquad \begin{array}{r} 274 \\ +\ 42 \\ \hline 16 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 274 \\ +\ 42 \\ \hline 316 \end{array}$$

The illustrated problem, for example, would be read: "Four plus two equals six. Seventy plus forty equals one hundred and ten; record one ten, carry one hundred. One hundred plus two hundred equals three hundred. Three hundred plus ten plus six equals three hundred sixteen." When the problem is read as "four plus two equals six, seven plus four equals eleven, and one plus two equals three," the place-value meaning is bypassed. In the initial stages of learning the basic meanings should be well established before any short cuts are taken.

Adapt the method described for using the Countingmen to work with such problems as 437 + 81 = □, 52 + 93 = □, and 346 + 560 = □.

▶ Use work pages S-273 and S-274 for practice in carrying from the tens to the hundreds place. Work the first problem at the top of work page S-273 with the class. Have them complete the front and back of this sheet on their own. Give help only if needed.



Name

$$\begin{array}{rrrl} & 500 + & 70 + 2 & \\ + & & (40 + 3) & \\ \hline & 500 + & 110 + 5 & = 600 + 10 + 5 = 615 \end{array} \qquad \begin{array}{r} 572 \\ +\ 43 \\ \hline 615 \end{array}$$

Find the sums.

$$\begin{array}{r} 684 \\ +\ 51 \\ \hline 735 \end{array} \qquad \begin{array}{r} 256 \\ +\ 72 \\ \hline 328 \end{array} \qquad \begin{array}{r} 363 \\ +\ 84 \\ \hline 447 \end{array} \qquad \begin{array}{r} 774 \\ +\ 85 \\ \hline 859 \end{array}$$

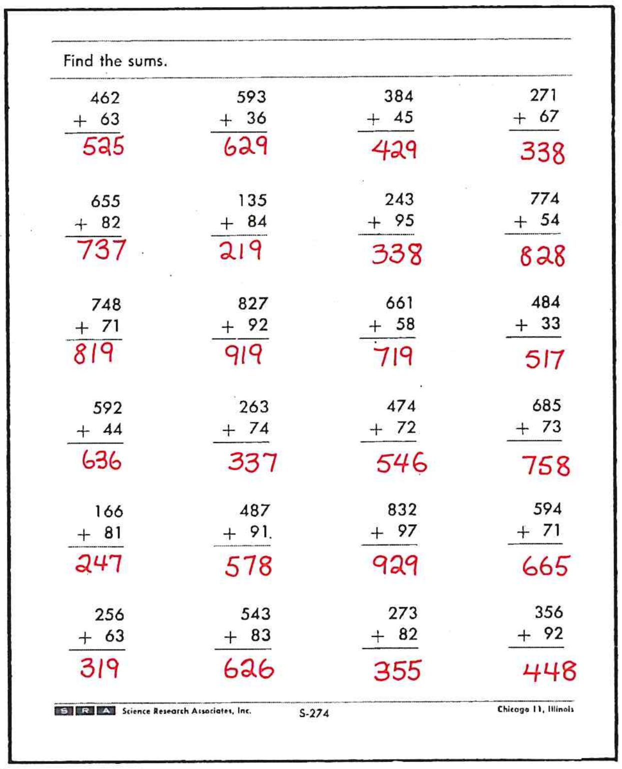
$$\begin{array}{r} 448 \\ +\ 91 \\ \hline 539 \end{array} \qquad \begin{array}{r} 291 \\ +\ 55 \\ \hline 346 \end{array} \qquad \begin{array}{r} 257 \\ +\ 82 \\ \hline 339 \end{array} \qquad \begin{array}{r} 585 \\ +\ 32 \\ \hline 617 \end{array}$$

$$\begin{array}{r} 396 \\ +\ 41 \\ \hline 437 \end{array} \qquad \begin{array}{r} 832 \\ +\ 96 \\ \hline 928 \end{array} \qquad \begin{array}{r} 443 \\ +\ 85 \\ \hline 528 \end{array} \qquad \begin{array}{r} 637 \\ +\ 81 \\ \hline 718 \end{array}$$

$$\begin{array}{r} 595 \\ +\ 93 \\ \hline 688 \end{array} \qquad \begin{array}{r} 286 \\ +\ 73 \\ \hline 359 \end{array} \qquad \begin{array}{r} 173 \\ +\ 66 \\ \hline 239 \end{array} \qquad \begin{array}{r} 774 \\ +\ 42 \\ \hline 816 \end{array}$$

S-273

▶ Work pages S-275 and S-276 provide practice in carrying to the hundreds place with one variation: there are zero hundreds to add to the one hundred being carried over from the tens place. After working the first problem at the top of work page S-275 with the class, have them complete the front and back of this sheet on their own.



Find the sums.

462 + 63 525	593 + 36 629	384 + 45 429	271 + 67 338
655 + 82 737	135 + 84 219	243 + 95 338	774 + 54 828
748 + 71 819	827 + 92 919	661 + 58 719	484 + 33 517
592 + 44 636	263 + 74 337	474 + 72 546	685 + 73 758
166 + 81 247	487 + 91 578	832 + 97 929	594 + 71 665
256 + 63 319	543 + 83 626	273 + 82 355	356 + 92 448

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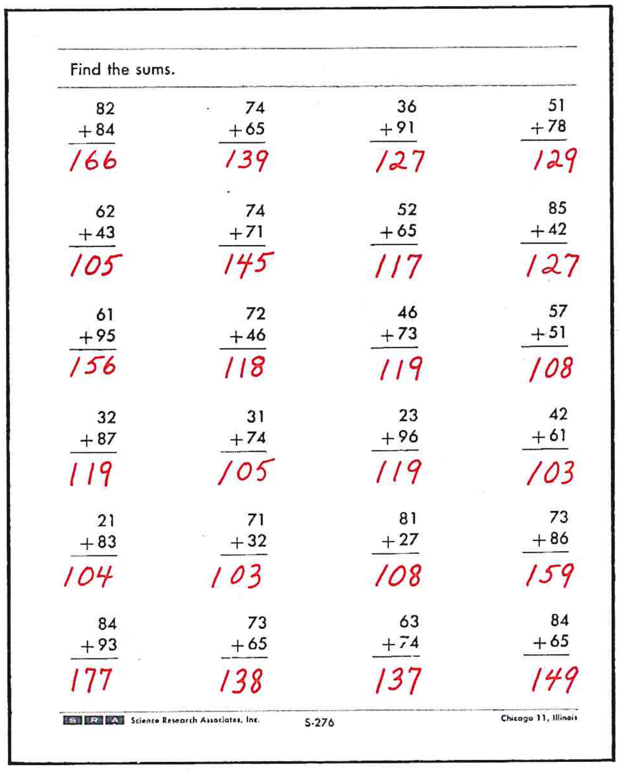

Find the sums.

82 +84 166	74 +65 139	36 +91 127	51 +78 129
62 +43 105	74 +71 145	52 +65 117	85 +42 127
61 +95 156	72 +46 118	46 +73 119	57 +51 108
32 +87 119	31 +74 105	23 +96 119	42 +61 103
21 +83 104	71 +32 103	81 +27 108	73 +86 159
84 +93 177	73 +65 138	63 +74 137	84 +65 149

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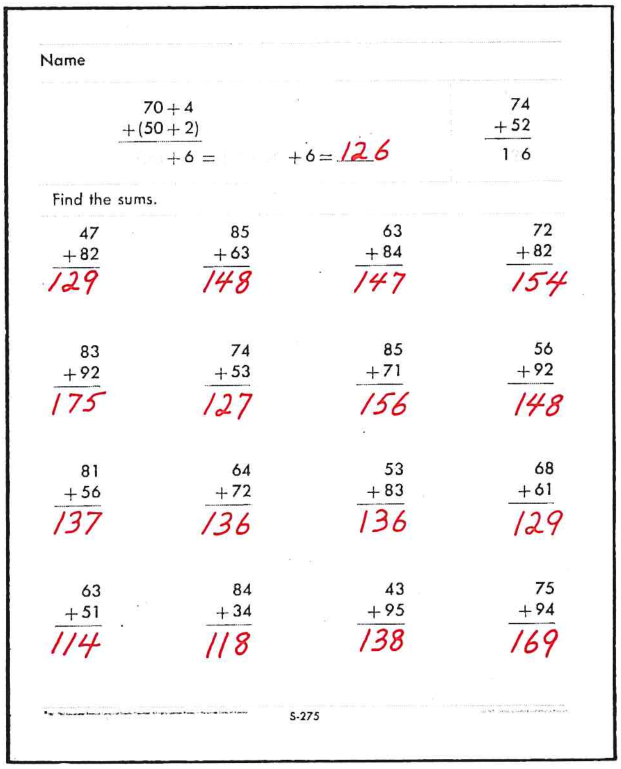


Name

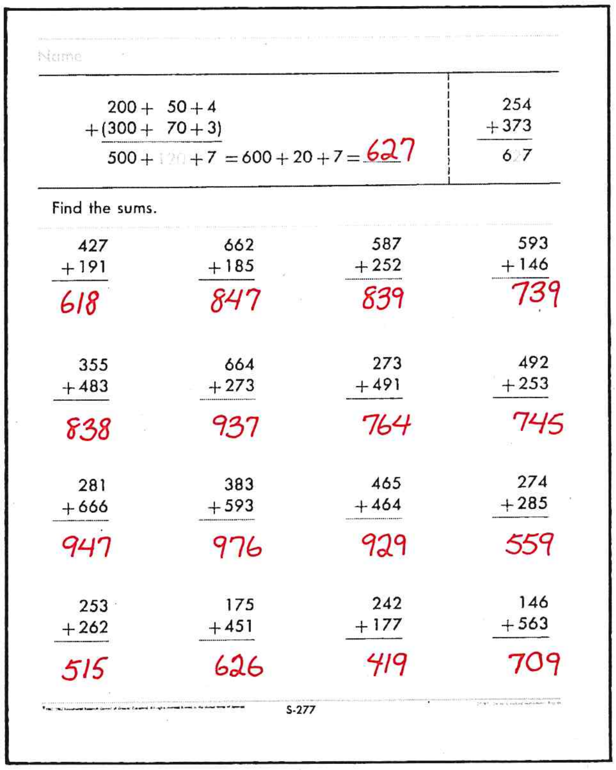
70 + 4
+ (50 + 2)
+ 6 = + 6 = 126

74
+ 52
126

Find the sums.

47 +82 129	85 +63 148	63 +84 147	72 +82 154
83 +92 175	74 +53 127	85 +71 156	56 +92 148
81 +56 137	64 +72 136	53 +83 136	68 +61 129
63 +51 114	84 +34 118	43 +95 138	75 +94 169

S-275



Name

200 + 50 + 4
+ (300 + 70 + 3)
500 + 120 + 7 = 600 + 20 + 7 = 627

254
+ 373
627

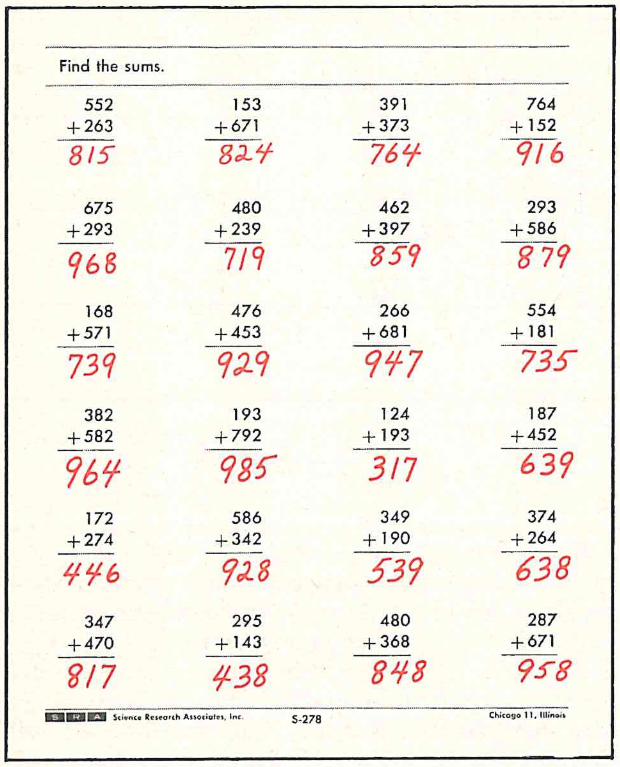
Find the sums.

427 +191 618	662 +185 847	587 +252 839	593 +146 739
355 +483 838	664 +273 937	273 +491 764	492 +253 745
281 +666 947	383 +593 976	465 +464 929	274 +285 559
253 +262 515	175 +451 626	242 +177 419	146 +563 709

S-277

▶ Work pages S-277 and S-278 provide practice with a pair of three-digit addends in addition problems involving carrying to the hundreds place, zero included. After working the first problem at the top of work page S-277 with the class, have them complete the front and back of this sheet on their own.

▶ Work page S-279 provides a review of the different phases of addition with carrying to the tens place. Have the children find the sums.



Find the sums.

552 +263 815	153 +671 824	391 +373 764	764 +152 916
675 +293 968	480 +239 719	462 +397 859	293 +586 879
168 +571 739	476 +453 929	266 +681 947	554 +181 735
382 +582 964	193 +792 985	124 +193 317	187 +452 639
172 +274 446	586 +342 928	349 +190 539	374 +264 638
347 +470 817	295 +143 438	480 +368 848	287 +671 958

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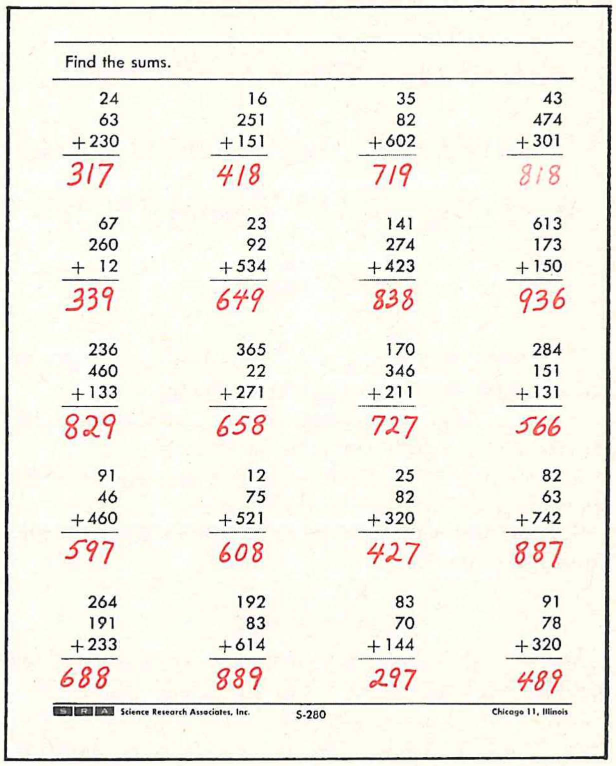

Find the sums.

24 63 +230 317	16 251 +151 418	35 82 +602 719	43 474 +301 818
67 260 + 12 339	23 92 +534 649	141 274 +423 838	613 173 +150 936
236 460 +133 829	365 22 +271 658	170 346 +211 727	284 151 +131 566
91 46 +460 597	12 75 +521 608	25 82 +320 427	82 63 +742 887
264 191 +233 688	192 83 +614 889	83 70 +144 297	91 78 +320 489

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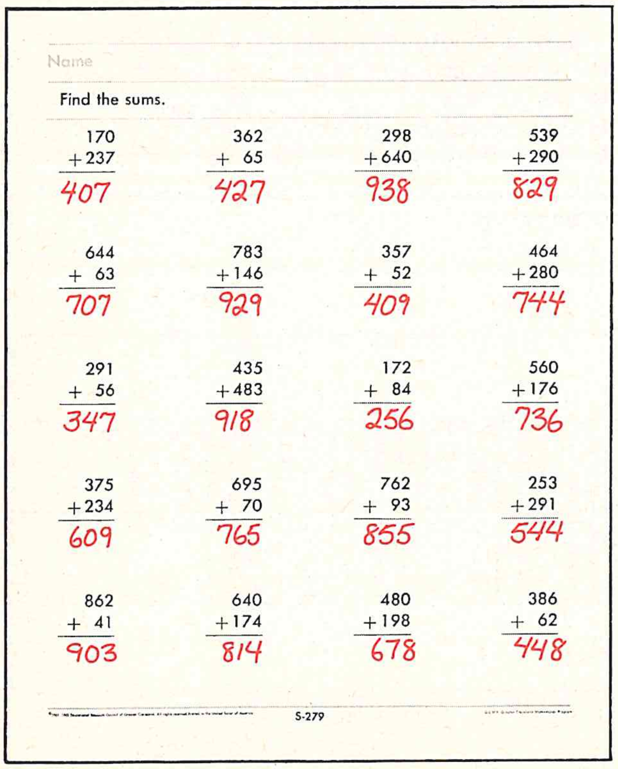


Name

Find the sums.

170 +237 407	362 + 65 427	298 +640 938	539 +290 829
644 + 63 707	783 +146 929	357 + 52 409	464 +280 744
291 + 56 347	435 +483 918	172 + 84 256	560 +176 736
375 +234 609	695 + 70 765	762 + 93 855	253 +291 544
862 + 41 903	640 +174 814	480 +198 678	386 + 62 448

S-279

Work page S-280 provides an opportunity for children to apply their growing skills in the setting of three-digit column addition. Have the children find the sums.

Supplemental Experiences

▶ Present the various ways of writing and working three-digit addition with carrying. Show the relations

$$\begin{array}{rrr}
500 + 90 + 6 & \\
+\ (200 + 70 + 3) & \\
\hline
700 + 160 + 9 = & \\
800 + 60 + 9 = & \\
869 &
\end{array}
\qquad
\begin{array}{r}
596 \\
+\ 273 \\
\hline
9 \\
160 \\
+\ 700 \\
\hline
869
\end{array}
\qquad
\begin{array}{r}
\overset{1}{5}96 \\
+\ 273 \\
\hline
869
\end{array}$$

between the different forms which can be used in working problems of this kind. Discuss the fact that the numbers involved remain the same, that only the symbolism and the approach vary.

▶ Present a series of placeholder equations on the chalkboard, and have children give the numeral for the number that completes each equation. Have

$$60 + 50 = 100 + \square$$
$$40 + 70 = \square + 10$$
$$\square + 30 = 100 + 20$$

them explain the thinking on which their answers are based.

▶ Have the children do mental computation in problems involving multiples of tens. Present orally such problems as 80 + 30 + 50 = □, 20 + 80 + 60 = □, and 50 + 40 + 70 = □.

Three-Digit Subtraction—Addition

BORROWING AND CARRYING

Work Pages S-281 Through S-294

Objectives

To strengthen and extend the understanding of the concept of borrowing and carrying.

To help children develop skill in subtraction with borrowing and addition with carrying.

To re-emphasize the inverse relation between the operations of addition and subtraction.

To evaluate the children's understanding of addition and subtraction.

Fundamentals

Work with subtraction will now be extended to include borrowing from the hundreds place. The teacher should encourage the child to apply his understanding of the inverse relation between addition and subtraction as he solves subtraction problems involving borrowing from the hundreds place.

Knowledge of the child's progress will be helpful in designing an effective learning environment for him. The frequent opportunity for check-ups is an intrinsic part of every well-planned program. Following presentation of the concept of borrowing from the hundreds place, the teacher will want to make a careful evaluation of the child's strengths and weaknesses and of his readiness to move on to the more complex concepts of addition and subtraction with carrying and borrowing in two positions.

See Key Topics in Mathematics for the Primary Teacher: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Understanding of the concept of place value.

Understanding of the concepts of borrowing and carrying.

Ability to work with number combinations through eighteen.

Awareness of the inverse relation between addition and subtraction.

Ability to use different numerals for a number.

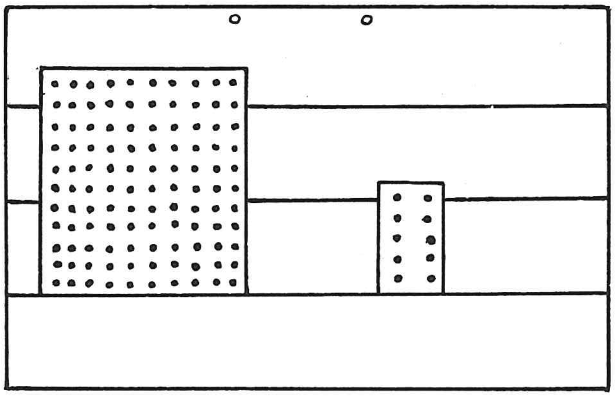
Ability to work with expanded notation and vertical notation.

Developmental Experiences

MATERIALS

pocket chart
tagboard cards and strips
3 Countingmen

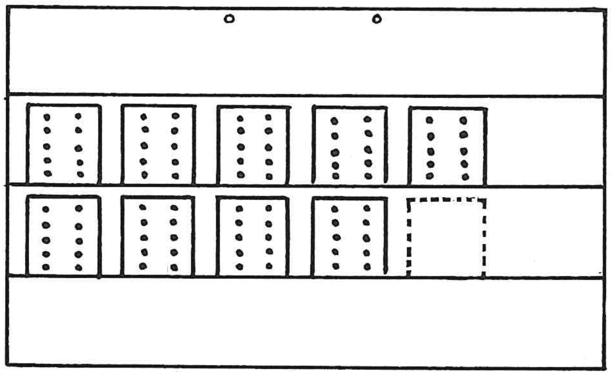
▶ Have children use pictures of sets in the pocket chart to work such problems as 110 − 20 = □,



$110-20=\square$

130 − 50 = □, 120 − 70 = □, and 110 − 80 = □. Relate what happens when working such problems as 11 − 2 = □ to problems such as 110 − 20 = □. Children learned to think of 11 − 2 as (11 − 1) −1, moving back to 10 first, and then completing the subtraction. In 110 − 20 = □, help them discover that they will move back to 100 first and then to the lesser multiples of ten; 110 − 20 will be thought of as (110 − 10) − 10.

Have a child remove one set of ten from a set of one hundred and a set of ten. Then have him replace the set of one hundred with ten sets of ten and remove the other ten which the problem states must be taken away: nine sets of ten or ninety will remain. On the chalkboard, write the equation related to the action.



$(110-10)-10=$
$100-10=90$

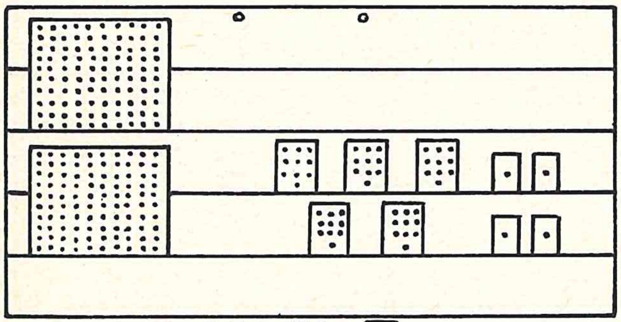
Continue in this way with other problems which involve moving from hundreds back to lesser multiples of ten.

▶ Write a series of equations on the chalkboard. Have children apply their understanding of the inverse concept in order to complete pairs of equations involving multiples of ten. When the equations are

$$\begin{array}{ll} 80 + 40 = \square & 70 + 60 = \square \\ \square - 40 = 80 & 130 - 60 = \square \\ & \\ 60 + \square = 140 & \square + 20 = 100 \\ 140 - \square = 60 & 100 - 20 = \square \end{array}$$

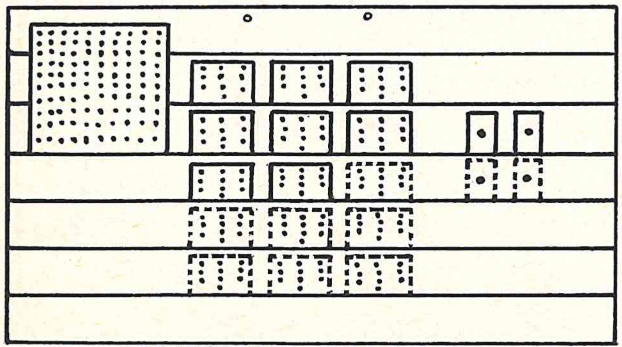
complete, have children read them in terms of tens. They may say: "Eight tens plus four tens equals twelve tens, or one hundred twenty." Reading the inverse equation, they may say: "Twelve tens minus four tens equals eight tens."

▶ Use the pocket chart and pictures of sets to demonstrate borrowing from hundreds to tens position. Place the problem 254 − 72 = □ on the chalkboard. In the pocket chart, have children place the needed set pictures to illustrate the first number in the problem.



254-72=□

Write the expanded notation for the problem, on the chalkboard, under the equation. Have children examine the problem and determine if the necessary number of ones can be subtracted. Have them determine if the necessary number of tens can be subtracted. At this point, they will discover a difficulty: there are not enough tens immediately available to perform the subtraction. Refer to the sets. Have a child replace one of the sets of one hundred with ten sets of ten. Rewrite 200 + 50 + 4 as 100 + 150 + 4.



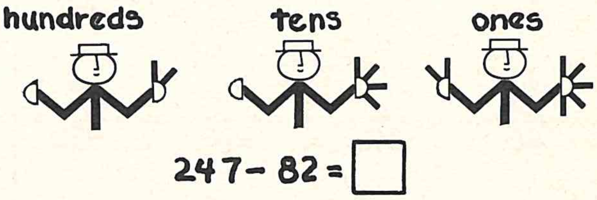
$$\begin{array}{rcl} 200+50+4 & = & 100+150+4 \\ -\quad(70+2) & = & -\quad(70+2) \\ \cline{1-1}\cline{3-3} & & 100+80+2 = 182 \end{array}$$

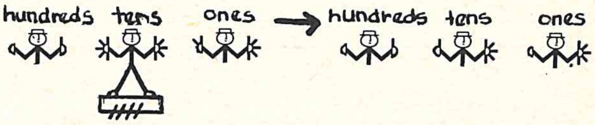
With fifteen tens now available, have the necessary removal of sets completed. Have two ones removed; write 2 to represent the difference in the expanded notation. Have the necessary number of tens removed; write 80 to represent the difference in the expanded notation. Have the necessary sets of hundreds removed; write 100 to represent the difference in the expanded notation.

Ask how many hundreds, tens, and ones are now in the problem. Have someone write the standard place-value numeral for the short way of saying one hundred plus eighty plus two: 182.

Continue in this way to have children make set removals and assist in writing the related expanded notations. Use such problems as 329 − 68 = □, 548 − 283 = □, and 409 − 186 = □.

▶ After children have a satisfactory understanding of the concepts involved in borrowing from the hundreds place and ability to utilize expanded notation, use the Countingmen to demonstrate the algorism in such problems as 247 − 82 = □. The teacher may manipulate the device as she asks questions pertinent to the action. Have each phase of the activity recorded as it occurs. On the chalkboard, have one child record the problem in expanded notation and another record it in vertical notation. Summarize by relating the ideas implied in the two notations. When children read the vertical notation, they should read it in terms of ones, tens, and hundreds.





$$\begin{array}{rcl} 200 + 40 + 7 & = & 100 + 140 + 7 \\ -\quad (80 + 2) & = & -\quad (80 + 2) \\ \cline{1-1}\cline{3-3} & & 100 + \ \ 60 + 5 = 165 \end{array}$$

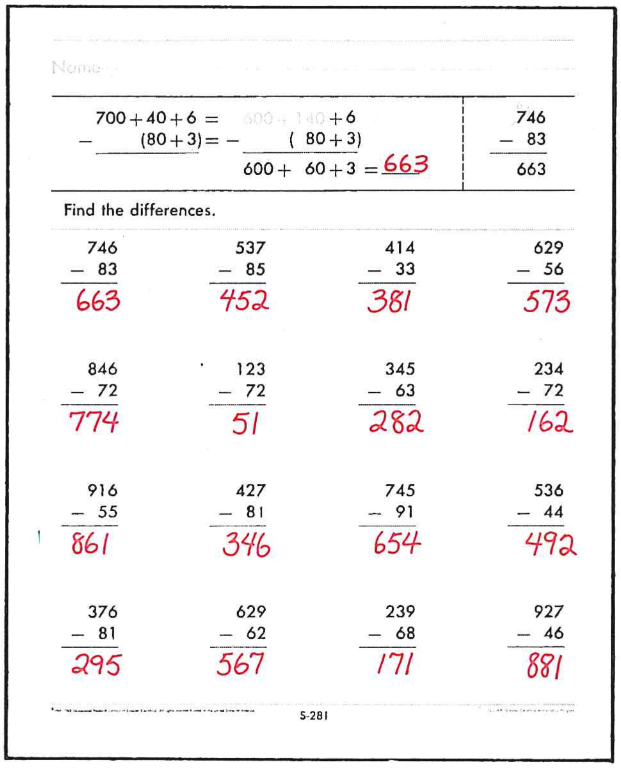
$$\begin{array}{r} \overset{1}{\cancel{2}}\,{}^{1}4\ 7 \\ -\quad 8\ 2 \\ \hline 1\ 6\ 5 \end{array}$$

Use the Countingmen to work other problems such as 267 − 85 = □, 579 − 188 = □, 916 − 896 = □, and 304 − 192 = □.

Review several addition problems with carrying to hundreds place involved. Have the algorism for each worked out on the chalkboard. Have children "undo" these problems and place the corresponding subtraction algorism on the board next to the addition algorism.

▶ Use work pages S-281 and S-282 to provide practice in borrowing from the hundreds place in subtraction. Numbers represented by two-digit numerals will be subtracted from numbers represented by three-digit numerals. Work the first problem on the top of work page S-281 with the class. Have them complete the front and back of this work sheet on their own. Give help only if needed.

▶ Work pages S-283 and S-284 provide practice in borrowing from the hundreds place with one variation from the preceding work pages: two three-digit numerals are involved. Work the first problem on the



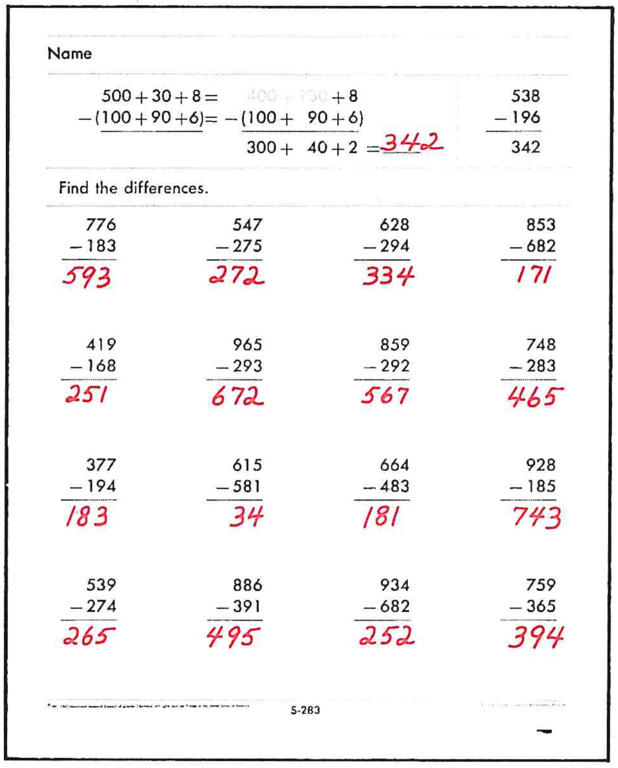
Name

700 + 40 + 6 = 600 + 140 + 6   746
− (80 + 3) = − (80 + 3)   − 83
600 + 60 + 3 = 663   663

Find the differences.

746 − 83 663	537 − 85 452	414 − 33 381	629 − 56 573
846 − 72 774	123 − 72 51	345 − 63 282	234 − 72 162
916 − 55 861	427 − 81 346	745 − 91 654	536 − 44 492
376 − 81 295	629 − 62 567	239 − 68 171	927 − 46 881

S-281



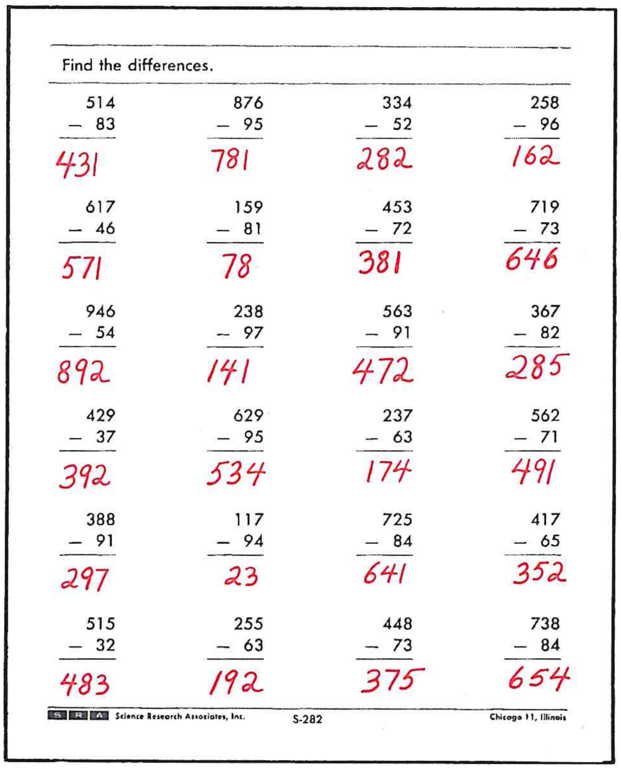
Name

500 + 30 + 8 = 400 + 130 + 8   538
− (100 + 90 + 6) = − (100 + 90 + 6)   − 196
300 + 40 + 2 = 342   342

Find the differences.

776 − 183 593	547 − 275 272	628 − 294 334	853 − 682 171
419 − 168 251	965 − 293 672	859 − 292 567	748 − 283 465
377 − 194 183	615 − 581 34	664 − 483 181	928 − 185 743
539 − 274 265	886 − 391 495	934 − 682 252	759 − 365 394

S-283



Find the differences.

514 − 83 431	876 − 95 781	334 − 52 282	258 − 96 162
617 − 46 571	159 − 81 78	453 − 72 381	719 − 73 646
946 − 54 892	238 − 97 141	563 − 91 472	367 − 82 285
429 − 37 392	629 − 95 534	237 − 63 174	562 − 71 491
388 − 91 297	117 − 94 23	725 − 84 641	417 − 65 352
515 − 32 483	255 − 63 192	448 − 73 375	738 − 84 654

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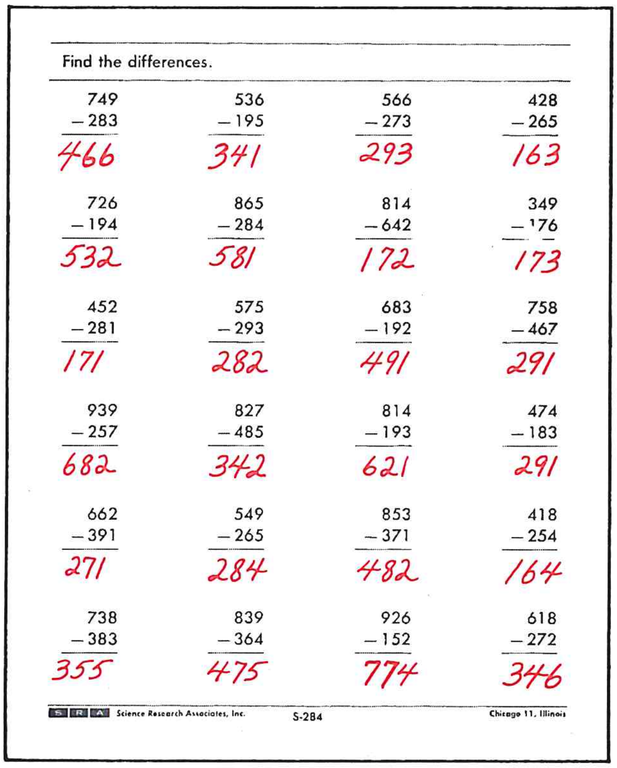

Find the differences.

749 − 283 466	536 − 195 341	566 − 273 293	428 − 265 163
726 − 194 532	865 − 284 581	814 − 642 172	349 − 176 173
452 − 281 171	575 − 293 282	683 − 192 491	758 − 467 291
939 − 257 682	827 − 485 342	814 − 193 621	474 − 183 291
662 − 391 271	549 − 265 284	853 − 371 482	418 − 254 164
738 − 383 355	839 − 364 475	926 − 152 774	618 − 272 346

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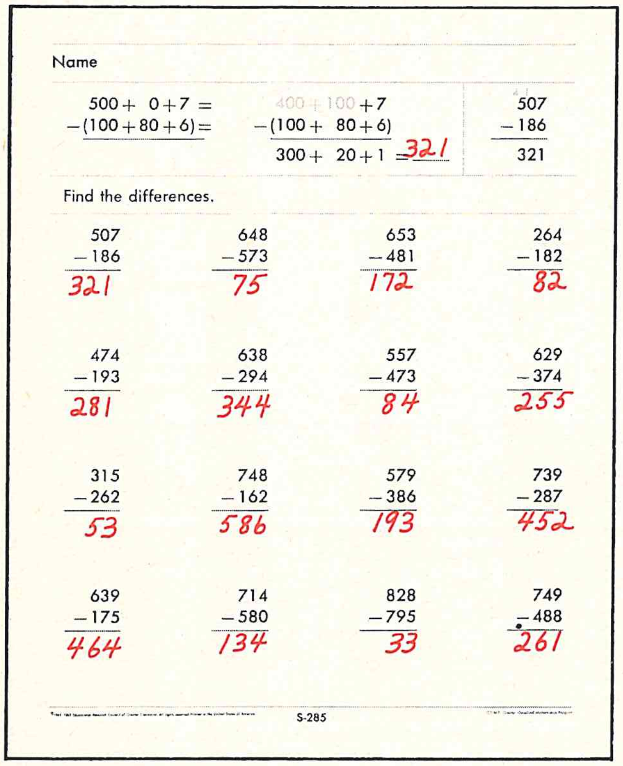
top of work page S-283 with the class. Have them complete the front and back of this sheet independently.

▶ Work pages S-285 and S-286 provide practice with two three-digit numerals in subtractions involving borrowing from the hundreds place; zero is included. After working the problem on the top of work page S-285 with the class, have them complete the front and back of this sheet on their own.

▶ Work pages S-287 and S-288 provide for a review of addition with carrying to the hundreds place and subtraction with borrowing from the hundreds place, zero included. Direct children to find the sums and differences.



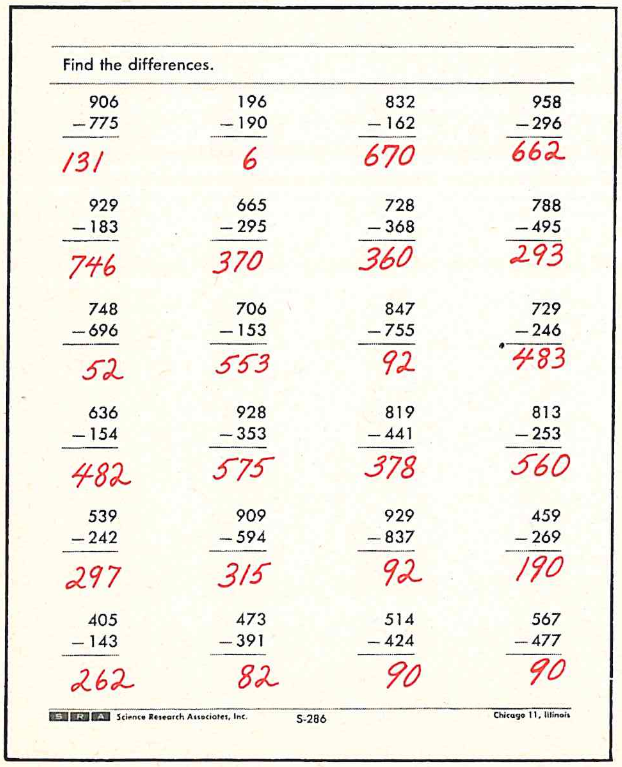
Name

500 + 0 + 7 = − (100 + 80 + 6) =	400 + 100 + 7 − (100 + 80 + 6) 300 + 20 + 1 = 321	507 − 186 321

Find the differences.

507 − 186 321	648 − 573 75	653 − 481 172	264 − 182 82
474 − 193 281	638 − 294 344	557 − 473 84	629 − 374 255
315 − 262 53	748 − 162 586	579 − 386 193	739 − 287 452
639 − 175 464	714 − 580 134	828 − 795 33	749 − 488 261

S-285



Find the differences.

906 − 775 131	196 − 190 6	832 − 162 670	958 − 296 662
929 − 183 746	665 − 295 370	728 − 368 360	788 − 495 293
748 − 696 52	706 − 153 553	847 − 755 92	729 − 246 483
636 − 154 482	928 − 353 575	819 − 441 378	813 − 253 560
539 − 242 297	909 − 594 315	929 − 837 92	459 − 269 190
405 − 143 262	473 − 391 82	514 − 424 90	567 − 477 90

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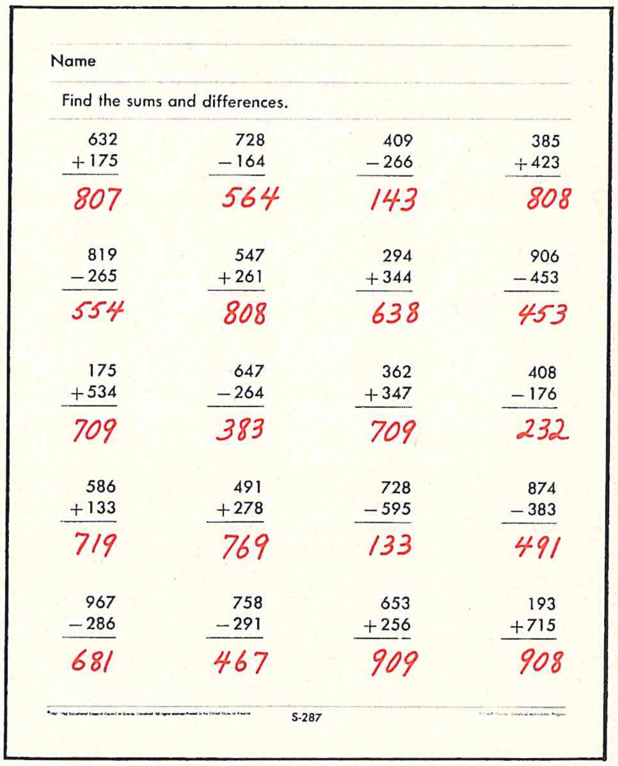
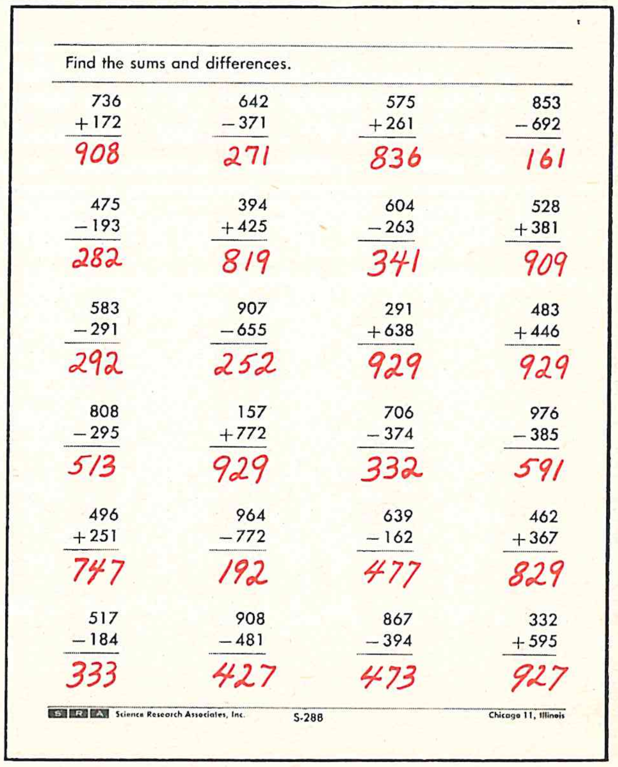


Name

Find the sums and differences.

632 + 175 807	728 − 164 564	409 − 266 143	385 + 423 808
819 − 265 554	547 + 261 808	294 + 344 638	906 − 453 453
175 + 534 709	647 − 264 383	362 + 347 709	408 − 176 232
586 + 133 719	491 + 278 769	728 − 595 133	874 − 383 491
967 − 286 681	758 − 291 467	653 + 256 909	193 + 715 908

S-287



Find the sums and differences.

736 + 172 908	642 − 371 271	575 + 261 836	853 − 692 161
475 − 193 282	394 + 425 819	604 − 263 341	528 + 381 909
583 − 291 292	907 − 655 252	291 + 638 929	483 + 446 929
808 − 295 513	157 + 772 929	706 − 374 332	976 − 385 591
496 + 251 747	964 − 772 192	639 − 162 477	462 + 367 829
517 − 184 333	908 − 481 427	867 − 394 473	332 + 595 927

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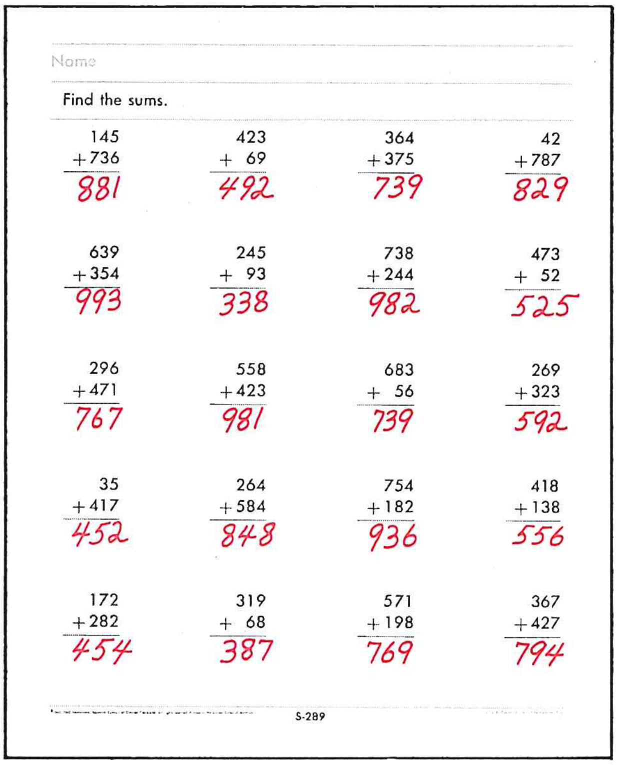
▶ Use work pages S-289 and S-290 to evaluate the children's ability to work with carrying to the tens place and carrying to the hundreds place. In any single problem, however, only one of these situations will be encountered. Problems contain two, three, or four addends. Direct children to find the sums.

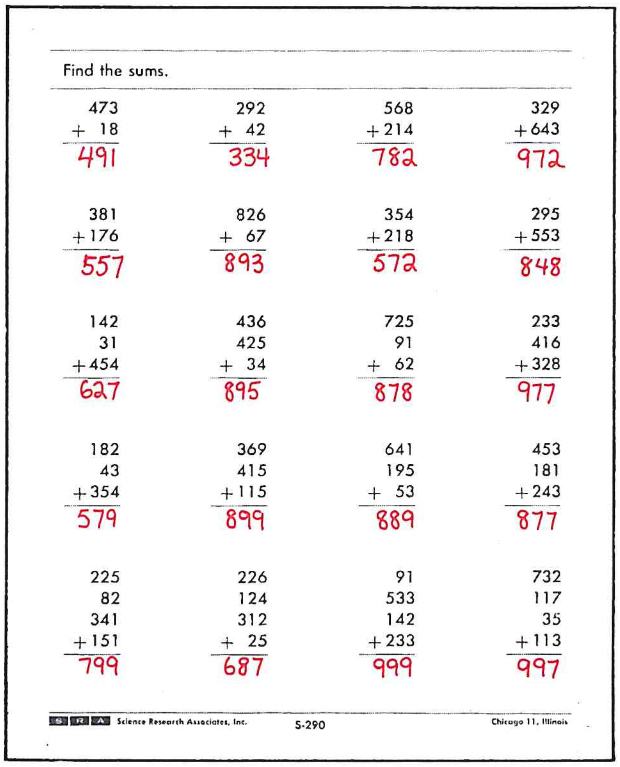


Name

Find the sums.

145 +736 881	423 + 69 492	364 +375 739	42 +787 829
639 +354 993	245 + 93 338	738 +244 982	473 + 52 525
296 +471 767	558 +423 981	683 + 56 739	269 +323 592
35 +417 452	264 +584 848	754 +182 936	418 +138 556
172 +282 454	319 + 68 387	571 +198 769	367 +427 794

S-289



Find the sums.

473 + 18 491	292 + 42 334	568 +214 782	329 +643 972
381 +176 557	826 + 67 893	354 +218 572	295 +553 848
142 31 +454 627	436 425 + 34 895	725 91 + 62 878	233 416 +328 977
182 43 +354 579	369 415 +115 899	641 195 + 53 889	453 181 +243 877
225 82 341 +151 799	226 124 312 + 25 687	91 533 142 +233 999	732 117 35 +113 997

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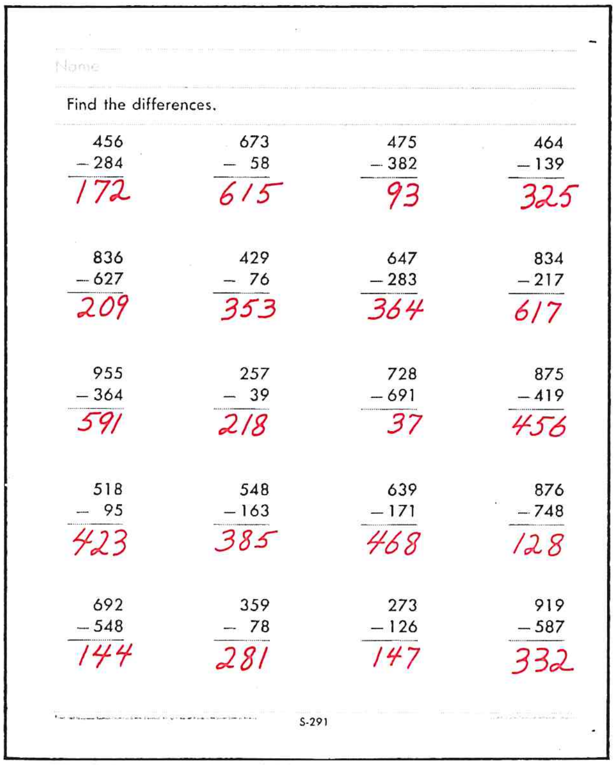
▶ Use work pages S-291 and S-292 to evaluate the children's ability to work with borrowing from the tens place and borrowing from the hundreds place. Only one of these situations is presented in each problem. Direct the children to find the differences.

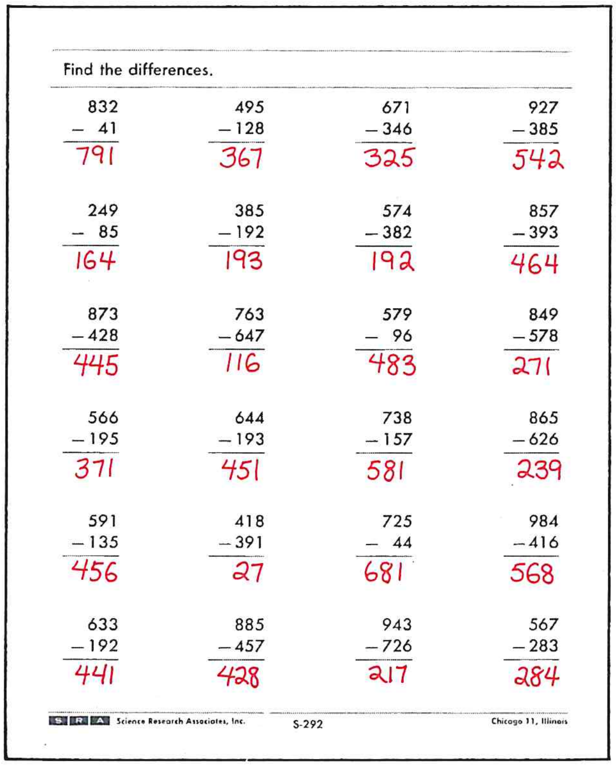


Name

Find the differences.

456 −284 172	673 − 58 615	475 −382 93	464 −139 325
836 −627 209	429 − 76 353	647 −283 364	834 −217 617
955 −364 591	257 − 39 218	728 −691 37	875 −419 456
518 − 95 423	548 −163 385	639 −171 468	876 −748 128
692 −548 144	359 − 78 281	273 −126 147	919 −587 332

S-291



Find the differences.

832 − 41 791	495 −128 367	671 −346 325	927 −385 542
249 − 85 164	385 −192 193	574 −382 192	857 −393 464
873 −428 445	763 −647 116	579 − 96 483	849 −578 271
566 −195 371	644 −193 451	738 −157 581	865 −626 239
591 −135 456	418 −391 27	725 − 44 681	984 −416 568
633 −192 441	885 −457 428	943 −726 217	567 −283 284

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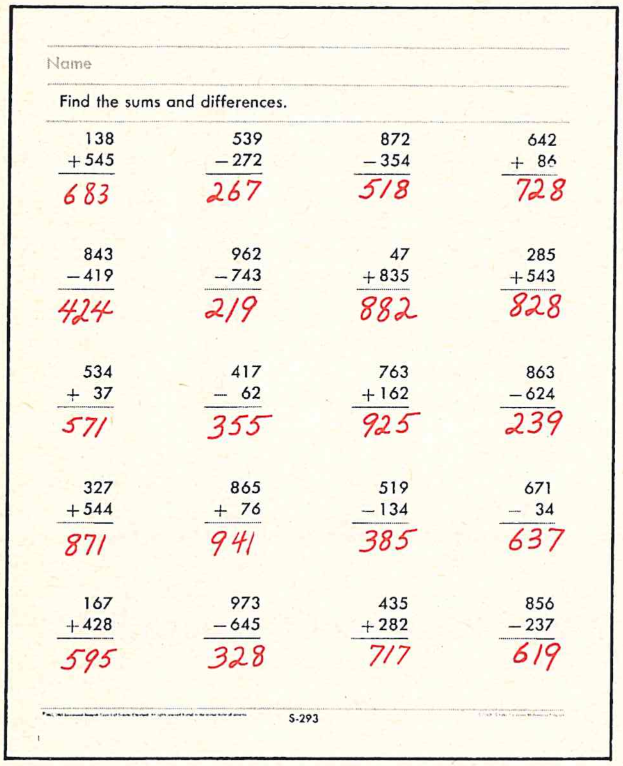
▶ Work pages S-293 and S-294 provide for a complete review of carrying and borrowing concepts developed up to this point. Use these two work pages to evaluate children's strengths and weaknesses in this area of number work. Direct children to find the sums and differences.

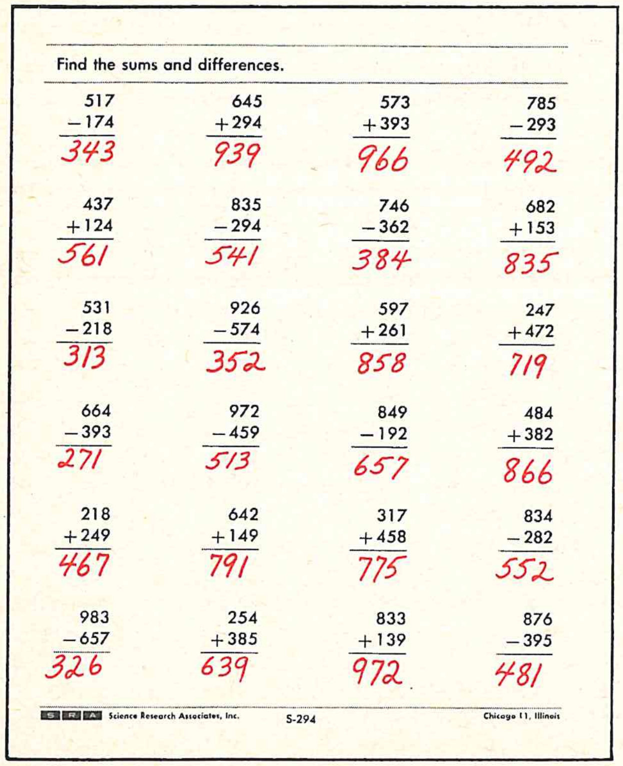


Name

Find the sums and differences.

138 +545 683	539 −272 267	872 −354 518	642 + 86 728
843 −419 424	962 −743 219	47 +835 882	285 +543 828
534 + 37 571	417 − 62 355	763 +162 925	863 −624 239
327 +544 871	865 + 76 941	519 −134 385	671 − 34 637
167 +428 595	973 −645 328	435 +282 717	856 −237 619

S-293



Find the sums and differences.

517 −174 343	645 +294 939	573 +393 966	785 −293 492
437 +124 561	835 −294 541	746 −362 384	682 +153 835
531 −218 313	926 −574 352	597 +261 858	247 +472 719
664 −393 271	972 −459 513	849 −192 657	484 +382 866
218 +249 467	642 +149 791	317 +458 775	834 −282 552
983 −657 326	254 +385 639	833 +139 972	876 −395 481

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Supplemental Experiences

▶ Present a series of placeholder equations on the chalkboard and have children complete each equation. Have them explain their answers.

$$90 + 50 = 180 - \square$$
$$230 - 40 = 110 + \square$$
$$40 + 80 = 120 + \square$$
$$360 - \square = 290 + 10$$
$$170 - 80 = 160 - \square$$
$$80 + \square = 90 + 60$$

▶ Have children do mental computation in problems involving multiples of ten. Orally, present such problems as "one hundred thirty minus thirty (pause) minus fifty; one hundred forty minus sixty (pause) minus twenty; and one hundred ten minus forty (pause) minus thirty."

Story Problems

Work Pages S-295 and S-296

Objective

To extend the basic concepts involved in solving story problems.

Fundamentals

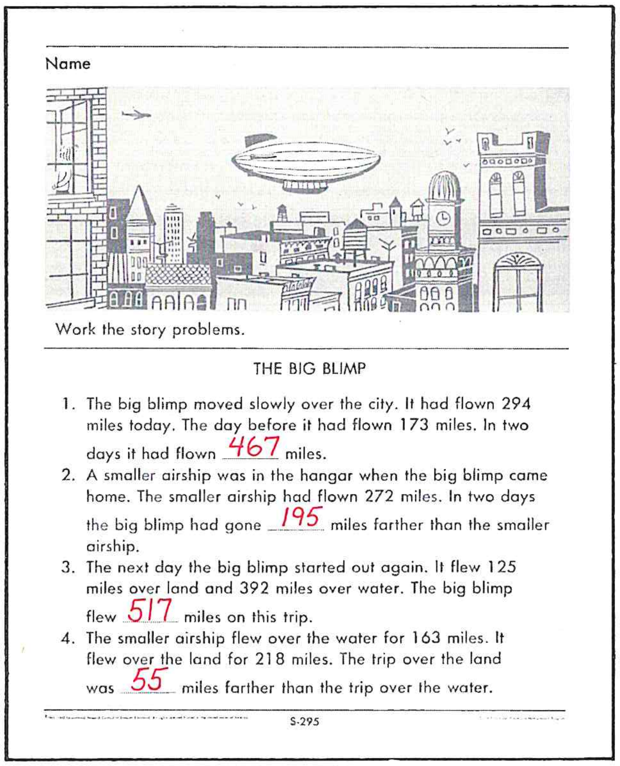
Much practice has been given with three-digit addition involving carrying and three-digit subtraction involving borrowing. Story problems should be used to help the child learn to apply his skills and understandings of these concepts to life situations. As he analyzes each problem situation the child should learn to extract from the story the facts which will help him find the solution. Then he must decide how to use these facts to arrive at the correct solution. Finally, he should judge whether or not he has found the best solution—whether or not his answer is reasonable.

Readiness for Understanding

The ability to analyze and solve story problems.
Clear understanding of the addition and subtraction algorisms.

Developmental Experiences

▶ Introduce the lesson with a discussion of blimps and jets. Ask the children to tell the difference in the speed and size of blimps and jets. Discuss the pictures on work pages S-295 and S-296 and have each problem read aloud.



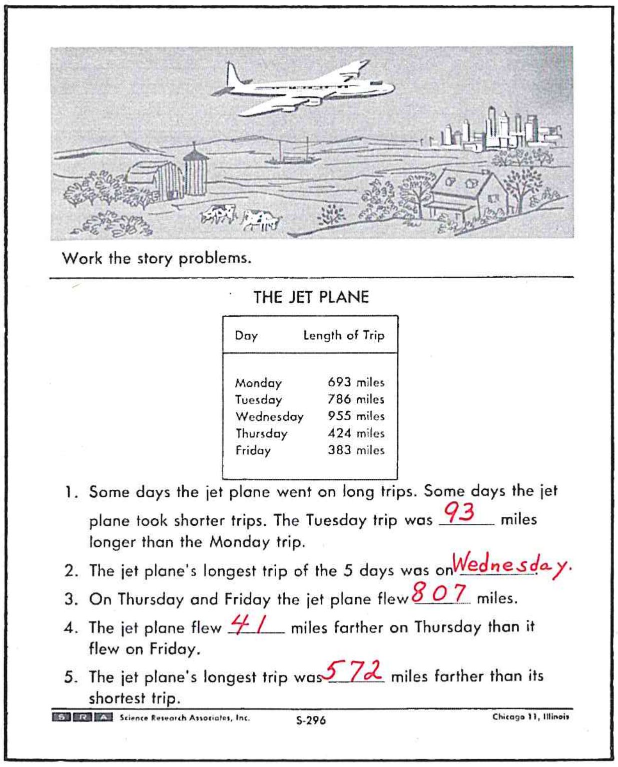
Name

Work the story problems.

THE BIG BLIMP

1. The big blimp moved slowly over the city. It had flown 294 miles today. The day before it had flown 173 miles. In two days it had flown 467 miles.
2. A smaller airship was in the hangar when the big blimp came home. The smaller airship had flown 272 miles. In two days the big blimp had gone 195 miles farther than the smaller airship.
3. The next day the big blimp started out again. It flew 125 miles over land and 392 miles over water. The big blimp flew 517 miles on this trip.
4. The smaller airship flew over the water for 163 miles. It flew over the land for 218 miles. The trip over the land was 55 miles farther than the trip over the water.

S-295



Work the story problems.

THE JET PLANE

Day	Length of Trip
Monday	693 miles
Tuesday	786 miles
Wednesday	955 miles
Thursday	424 miles
Friday	383 miles

1. Some days the jet plane went on long trips. Some days the jet plane took shorter trips. The Tuesday trip was 93 miles longer than the Monday trip.
2. The jet plane's longest trip of the 5 days was on Wednesday.
3. On Thursday and Friday the jet plane flew 807 miles.
4. The jet plane flew 41 miles farther on Thursday than it flew on Friday.
5. The jet plane's longest trip was 572 miles farther than its shortest trip.

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Have the children complete both pages independently; give individual help as needed. To check their answers, ask different children to work the problems at the chalkboard and explain their method of solution.

Supplemental Experiences

▶ Write addition and subtraction problems on slips of paper. Give one slip to each child. Have him make up a story problem using the given numbers and operation. He should then tell his story and ask another child to find the solution.

▶ As a variation, have each child write his story on a paper and illustrate it. Place one or two stories on the bulletin board under a sign which reads "Problem of the Day." Have answer slips and an answer box nearby. Each child must place his name and answer on an answer slip and deposit the slip in the box. At the end of the day the author checks to see how many children had the correct answer. He may explain his problem to those who did not get the correct answer.

Linear Measurement

Work Pages S-297 Through S-300

Objectives

To strengthen the concept of linear measurement.

To review the foot and inch as units of linear measure.

To develop an understanding of the relation between inch and foot as units of linear measure.

Fundamentals

The child has had many informal experiences with the concepts of length, distance, height, and width. In most instances he has had formal experiences in the first grade with the measurement of length and has some understanding of the foot and inch as units of linear measure. Now these concepts will be strengthened and the child will develop a further understanding of the relation between the inch and foot as units of measurement. He will recall that the choice of a unit of linear measure is arbitrary but that there is an advantage in having certain standard units of measure.

The concept of approximation is extremely important in the study of measurement, since one can never be assured of arriving at an exact measure. Stating that a boy is 5 feet tall means that he is closer to 5 feet than he is to 4 feet or to 6 feet. If the same boy is measured in terms of inches, a more precise measure can be found. Stating that he is 59 inches tall means that he is closer to 59 inches than to 60 inches or to 58 inches. These measures are approximations; they are given to the nearest unit and are not exact.

See Key Topics in Mathematics for the Primary Teacher: Measurement.

Readiness for Understanding

Some idea of the meaning of length.

Developmental Experiences

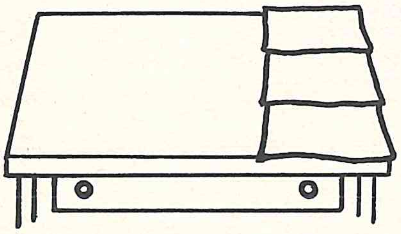
MATERIALS

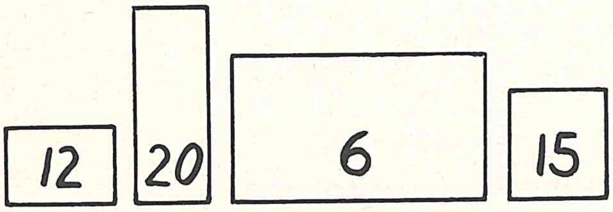
pads of paper
cardboard strips
foot rulers with
  inch marks

VOCABULARY

length
measurement
measure
inch
foot

▶ Review with the children the concept of linear *measurement.* Hold up a pad of paper and ask the children, "How many sheets do you think can be placed end-to-end across the front of my desk?" Following a short period of discussion and estimation, have a child try it. Stress the importance of careful end-to-end placement and counting of the sheets. The idea of approximation is involved in deciding which number is the better *measure.* When the sheets have been placed across the desk, the last sheet may not fit exactly at the edge of your desk. If more than half of the *length* of this sheet must be used, then the sheet should be included in the count. If less than half of the sheet is needed, it should not be included in the count. If just half the paper is needed, it may be counted or not as the children wish. Repeat this activity several times with different-size pads of paper. Record each measure in terms of the size of paper and help the children see that the number assigned to the length of the desk depends on the unit of measure which has been chosen.





Discuss with the children the choice of the unit of measure. Lead them to conclude that it would be wise to have a standard unit of measure for ease of communicating about length. Ask if any children know of some units of measure which are used by many people. Discuss the *foot* and *inch* as standard units of linear measure. Display strips of cardboard, one of which is one foot long and one of which is one inch long. Review with the children the relation between the inch and the foot as units of measure. Using the inch-long strip, mark off inch lengths on the foot-long cardboard strip. Have the children count to find the number of inch lengths on the foot-long strip. Help them see that twelve inches is not exactly the same as one foot, but that twelve inches measures the same amount of length as one foot. Write on the chalkboard: "12 inches $\overset{m}{=}$ 1 foot." Read this with the children: "Twelve inches measures the same amount of length as one foot." Point out that the equality sign cannot be used because the number twelve does not equal the number one.

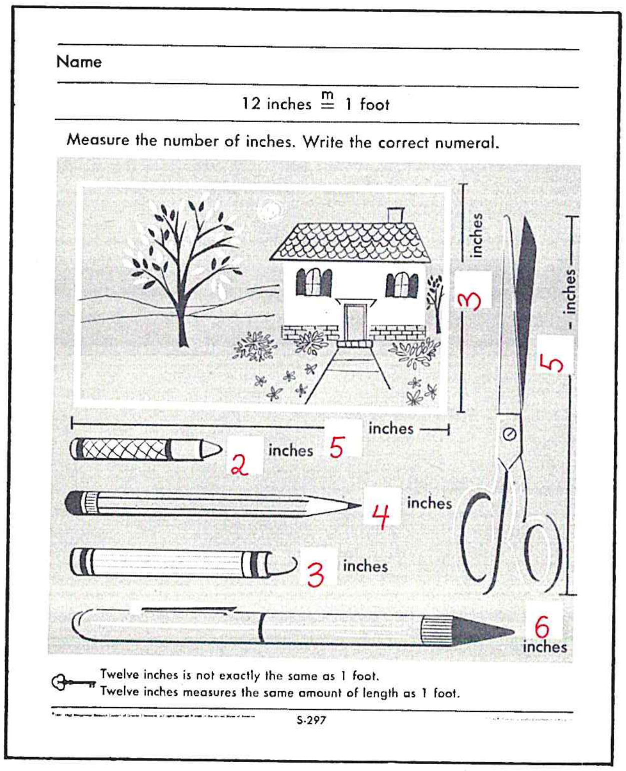
▶ Provide each child with a foot ruler which has been marked off in inches. Help them decide on the best method of using these instruments to measure various objects about the room. Help them conclude that it is easier to measure some objects in terms of inches and others in terms of feet. On some occasions, a measure may be given in terms of both feet and inches, as one foot, three inches.

▶ Help the children develop a feeling for comparative length. Ask, "Which is longer: eleven inches or one foot?" Have the children draw lines eleven inches long and one foot long on the chalkboard and again discuss which length is greater and which is lesser. Ask other questions: "Would it be better to buy two inches or two feet of ribbon for a hair bow?" "Why?" "Which is longer: two inches or two feet?" "Tommy had two sticks, one seven inches long and one six inches long. If he put them end-to-end, would they be longer or shorter than a foot ruler?" "If I have a piece of string one foot long, how many ways can I cut it so that each of the two pieces can be measured in whole inches?"

▶ Use work page S-297 to reinforce the idea of measuring to the nearest inch. Read with the children the statement at the top of the page. Help them see that twelve inches is not exactly the same as one foot but that twelve inches measures the same amount of length as one foot. Direct the children to measure the pictures and then to write the correct numerals in the boxes.

Work page S-298 provides practice in comparing lengths. Discuss the picture at the top of the page. Point out that the two shorter boards measure the same length as the one longer board. Read the first story problem with the children. Have them think of the answer and write the correct numeral in the blank. Note that a variety of correct answers are possible for problem 7.

▶ Work pages S-299 and S-300 provide further practice in measuring to the nearest inch. Have each child use his ruler to measure the pictures and record these measures in the blanks.



Name

$12 \text{ inches} \overset{m}{=} 1 \text{ foot}$

Measure the number of inches. Write the correct numeral.

3 inches

5 inches

5 inches

2 inches

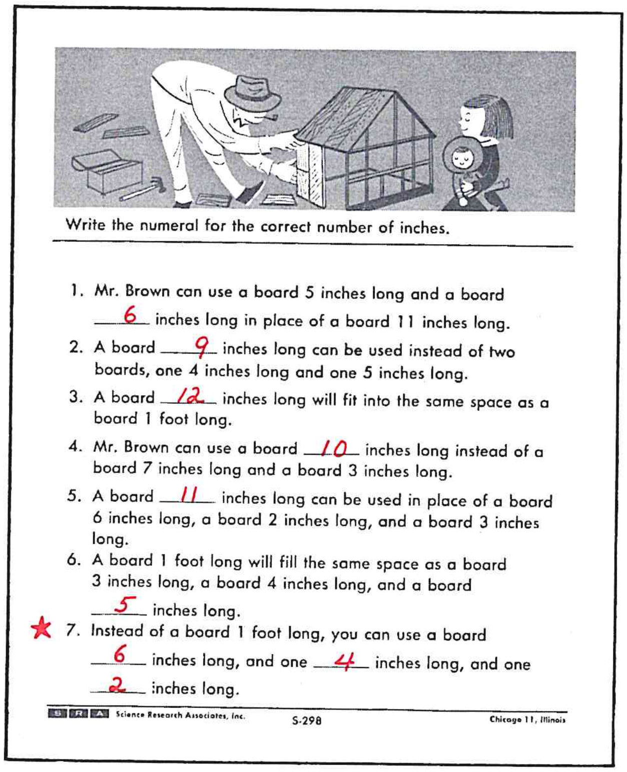
4 inches

3 inches

6 inches

Twelve inches is not exactly the same as 1 foot.
Twelve inches measures the same amount of length as 1 foot.

S-297



Write the numeral for the correct number of inches.

1. Mr. Brown can use a board 5 inches long and a board 6 inches long in place of a board 11 inches long.
2. A board 9 inches long can be used instead of two boards, one 4 inches long and one 5 inches long.
3. A board 12 inches long will fit into the same space as a board 1 foot long.
4. Mr. Brown can use a board 10 inches long instead of a board 7 inches long and a board 3 inches long.
5. A board 11 inches long can be used in place of a board 6 inches long, a board 2 inches long, and a board 3 inches long.
6. A board 1 foot long will fill the same space as a board 3 inches long, a board 4 inches long, and a board 5 inches long.

★ 7. Instead of a board 1 foot long, you can use a board 6 inches long, and one 4 inches long, and one 2 inches long.

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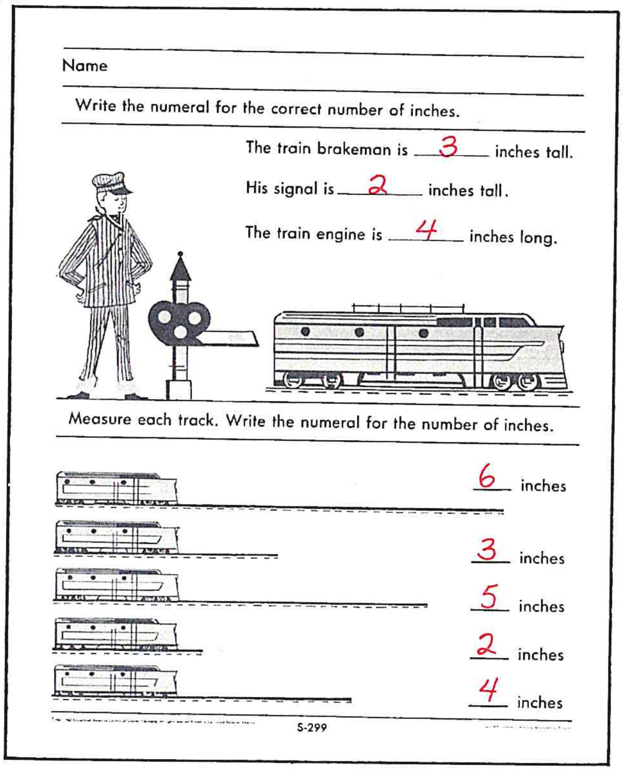
★ More than one correct answer is possible for each problem.



Name

Write the numeral for the correct number of inches.

The train brakeman is 3 inches tall.

His signal is 2 inches tall.

The train engine is 4 inches long.

Measure each track. Write the numeral for the number of inches.

6 inches

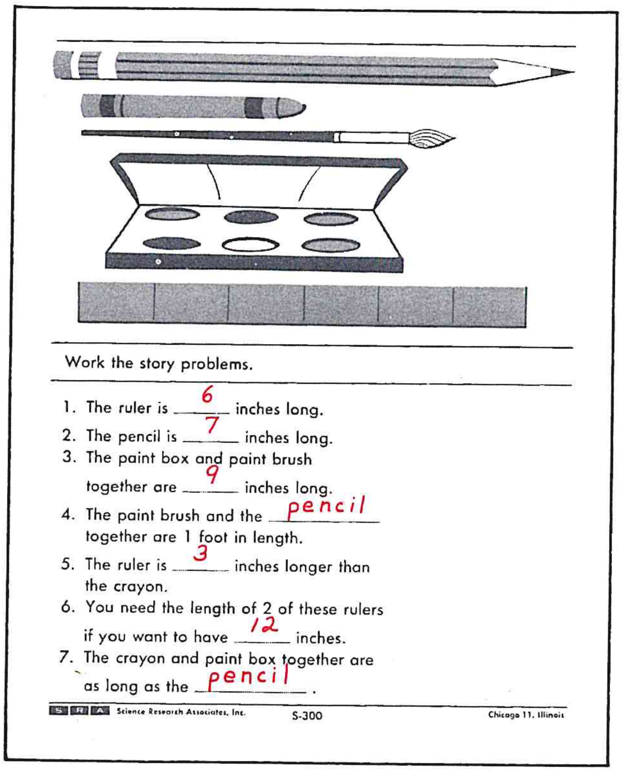
3 inches

5 inches

2 inches

4 inches

S-299



Work the story problems.

1. The ruler is 6 inches long.
2. The pencil is 7 inches long.
3. The paint box and paint brush together are 9 inches long.
4. The paint brush and the pencil together are 1 foot in length.
5. The ruler is 3 inches longer than the crayon.
6. You need the length of 2 of these rulers if you want to have 12 inches.
7. The crayon and paint box together are as long as the pencil.

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Supplemental Experiences

▶ Provide additional opportunities for the children to use their skills in measurement. A directed lesson in art class or a building project in a social science unit will give them a chance to measure to the nearest foot or inch.

▶ Post a linear measurement question on the bulletin board each day. Provide a box where the children can put their answers.

24 inches $\overset{m}{=}$ __________ feet
36 inches $\overset{m}{=}$ __________ feet
2 feet $\overset{m}{=}$ __________ inches

▶ Provide the children with opportunities to estimate and compare measures of length. Have them guess how long a book or stick is in inches and in feet. Then have them measure to see how closely they estimated. Ask a child to try to draw a line six inches long on the chalkboard. Have him measure his line to see if he was correct. Have other children try to draw lines of other given lengths.

Draw a series of vertical or horizontal lines on the chalkboard. Have the children discuss the lengths of these lines and estimate the number of inches or feet in each line. Have each estimate tested by measurement.

Have the children estimate the length of several pieces of string. Then have them measure each piece. Help them see that for some pieces of string, the inch is a better unit of measure; but for others it will be more natural to measure in terms of feet. On some occasions a measure may be given in terms of both feet and inches, as with an eighteen-inch string, which might be called one foot, six inches long.

▶ Draw a line four inches long on the chalkboard. Have a child measure the line and give its length in inches. Direct another child to draw a line three inches longer than the given line. Have still another child tell how long this line should be and measure it to see that the second child drew a line of the correct length. Continue this activity, having children draw lines which are longer or shorter than given lines. In such activities, point out that they can add or subtract numbers but that they cannot add or subtract the feet or inches.

Linear Measurement

Work Pages S-301 Through S-304

Objectives

To strengthen the concept of linear measurement.

To review the foot and the inch as units of linear measurement.

To review the relation between inch and foot as units of linear measurement.

To introduce the concept of the yard as a unit of linear measurement.

To develop an understanding of the relation between inch and yard and between foot and yard as units of linear measure.

Fundamentals

The child has had many informal experiences with concepts of length, distance, height, and width as measured in feet and inches. The children will recall the advantage of having certain standard units of measure. Now these concepts will be strengthened and the yard will be introduced as a unit of measure.

The child can then be guided to apply logic in developing an understanding of the relation between feet and yards by calling on his knowledge of inches and feet, and of inches and yards.

See Key Topics in Mathematics for the Primary Teacher: Measurement.

Readiness for Understanding

Understanding of the concept of linear measurement.

Understanding of feet and inches.

Understanding of the relation between inches and feet.

Developmental Experiences

MATERIALS	VOCABULARY
ribbon	yard
cardboard strip	
yardstick	
foot rulers	
tape measure	
cards	

▶ Help the children mark off inch lengths on a cardboard strip one *yard* long, using inch-long pieces of cardboard or a foot ruler with inch marks to do the measuring. Have the children count the spaces between marks. After they say that there are thirty-six spaces, tell them that another name for this length is yard. Write on the chalkboard, "36 inches $\overset{m}{=}$ 1 yard." Have the children read this statement. Beneath this statement, write "One foot $\overset{m}{=}$ ______ inches." Have some child fill in the blank with the correct numeral. Next ask how many inches are in two feet, and then how many are in three feet. Record the children's answers on the chalkboard, i.e., 2 feet $\overset{m}{=}$ 24 inches and 3 feet $\overset{m}{=}$ 36 inches. Lead the children to the conclusion that if they know these statements are true, they can answer the question, "How many feet measure the same length as one yard?"

▶ The yardstick should be introduced at this time. Have the children measure the yardstick with a foot ruler; they will see that 3 ft. $\overset{m}{=}$ 1 yd. They will see the inch marks on the yardstick and see that 36 in. $\overset{m}{=}$ 1 yd. Introduce the tape measure and compare it with the yardstick. Discuss the use of the tape measure and the yardstick in various practical situations, such as measuring the width of a room or finding the distance around a child's arm.

▶ Have the children measure three ribbons: two yards, three yards, and four yards in length. Have each of these measurements recorded in inches, feet, and yards.

▶ Measure the heights of children and record the measurements in terms of yards and inches as 1 yd. ____ in.

▶ Help the children develop a feeling for comparative length. Ask, "Which is longer: two feet or twenty-six inches?" Have the children explain their answers. To further develop this comparison, have the children draw a line two feet long and a line twenty-six inches long on the chalkboard. Again discuss which length is greater and which is less. Ask, "What is the difference between the two lengths?" This development of a feeling for comparative length can be continued with many such concrete examples. After the children have had sufficient practice with concrete problems, continue the comparisons orally without concrete examples. Children will realize that it is easier to compare lengths when the same unit of linear measure is used.

▶ Distribute cards on which are written various measurements. Ask the children oral riddles; the child with the correct answer on his card must stand and show his card to the group for approval. Say, "I am thinking of something which measures the same length as one foot." The child with the "12 inch" card must stand and say, "12 inches measures the same length as one foot." Continue with other riddles such as, "I am thinking of something which measures the same length as thirty-six inches." "I am thinking of a length which is two inches shorter than two feet."

22 inches

12 inches

1 yard

5 inches more than 1 foot

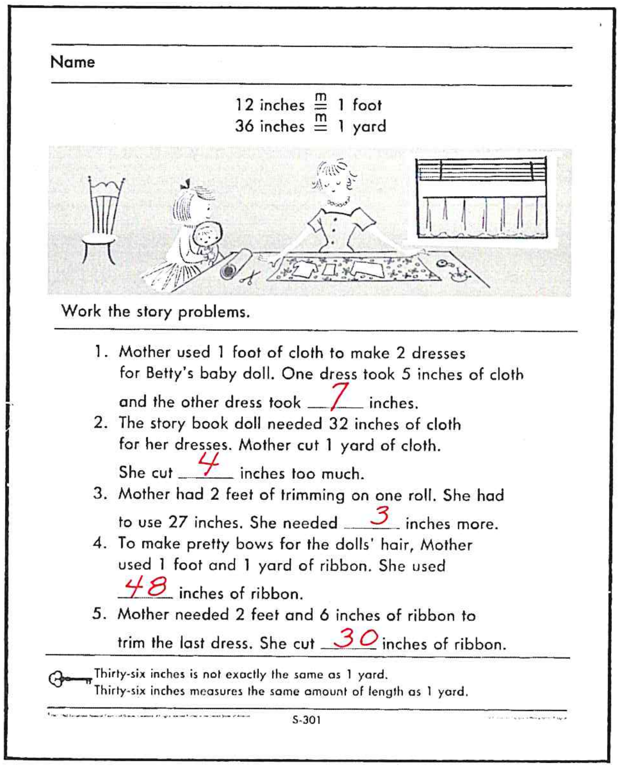
2 feet and 8 inches

3 feet

▶ Work page S-301 reinforces the idea of linear measurement (inches, feet, yards) by using story problems about realistic situations. Read the statements at the top of the page and discuss the picture. Tell the children that on this page they must fill in the blanks. Read the first story problem with the children. Have them think of the answer and write the correct numeral in the blank. Encourage the children to complete the page independently.

Work page S-302 provides further practice in the development of an understanding of comparative lengths. In the first column the children must mark the greater length. In the second column they are to mark the lesser length.

▶ On work page S-303 the children will have another opportunity to recall measurements they know.



Name ______________________

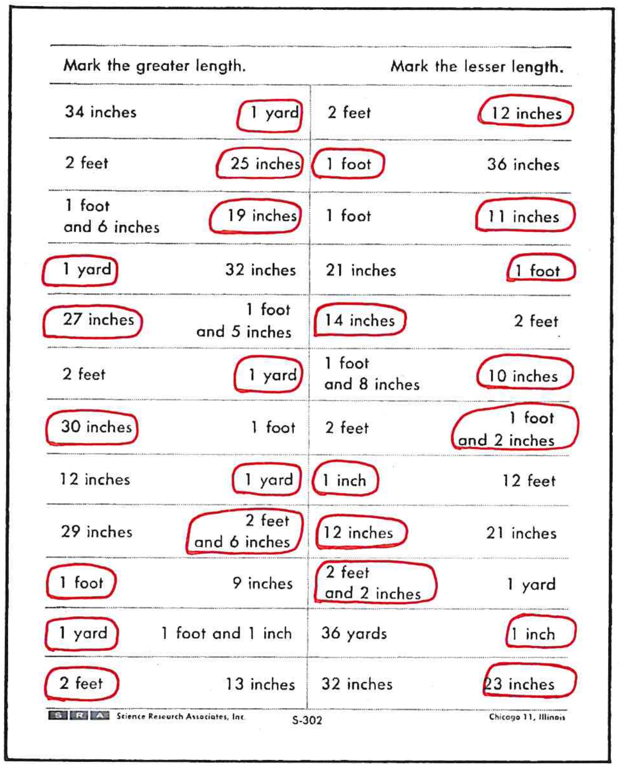
12 inches $\overset{m}{=}$ 1 foot
36 inches $\overset{m}{=}$ 1 yard

Work the story problems.

1. Mother used 1 foot of cloth to make 2 dresses for Betty's baby doll. One dress took 5 inches of cloth and the other dress took __7__ inches.
2. The story book doll needed 32 inches of cloth for her dresses. Mother cut 1 yard of cloth. She cut __4__ inches too much.
3. Mother had 2 feet of trimming on one roll. She had to use 27 inches. She needed __3__ inches more.
4. To make pretty bows for the dolls' hair, Mother used 1 foot and 1 yard of ribbon. She used __48__ inches of ribbon.
5. Mother needed 2 feet and 6 inches of ribbon to trim the last dress. She cut __30__ inches of ribbon.

Thirty-six inches is not exactly the same as 1 yard.
Thirty-six inches measures the same amount of length as 1 yard.

S-301



Mark the greater length.		Mark the lesser length.	
34 inches	[✓ 1 yard]	2 feet	[✓ 12 inches]
2 feet	[✓ 25 inches]	[✓ 1 foot]	36 inches
1 foot and 6 inches	[✓ 19 inches]	1 foot	[✓ 11 inches]
[✓ 1 yard]	32 inches	21 inches	[✓ 1 foot]
[✓ 27 inches]	1 foot and 5 inches	[✓ 14 inches]	2 feet
2 feet	[✓ 1 yard]	1 foot and 8 inches	[✓ 10 inches]
[✓ 30 inches]	1 foot	2 feet	[✓ 1 foot and 2 inches]
12 inches	[✓ 1 yard]	[✓ 1 inch]	12 feet
29 inches	[✓ 2 feet and 6 inches]	[✓ 12 inches]	21 inches
[✓ 1 foot]	9 inches	[✓ 2 feet and 2 inches]	1 yard
[✓ 1 yard]	1 foot and 1 inch	36 yards	[✓ 1 inch]
[✓ 2 feet]	13 inches	32 inches	[✓ 23 inches]

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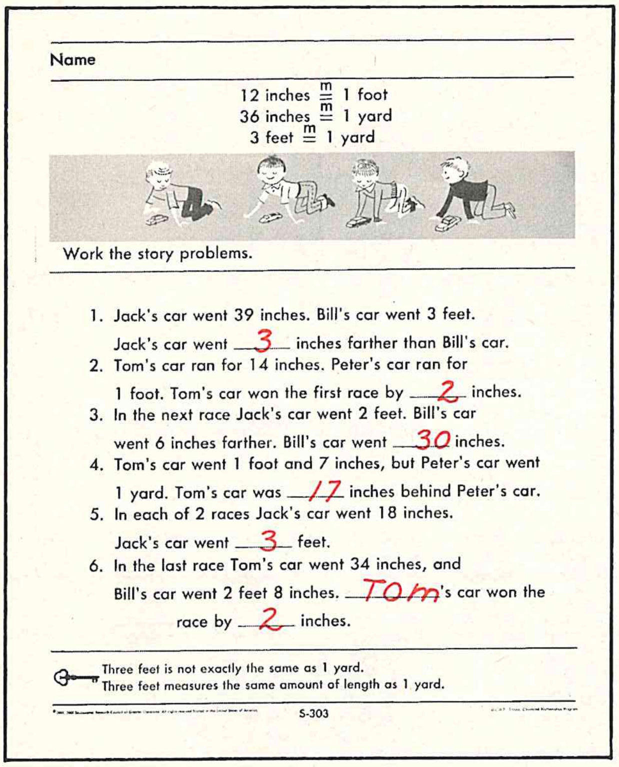
Read and discuss the statements and the picture at the top of the page. Direct the children to read each story problem very carefully, think what they must find out, and write the correct answers in the blanks.

Work page S-304 provides further practice in comparing different lengths. Recall the riddle game the class played with the cards. Direct the children's attention to the top of the page, and tell them to draw lines between cards which indicate the same amount of length. Do the first example with the children.

Have the children think of different measures for the same length and write the correct numeral in each blank at the bottom of the page.



Name

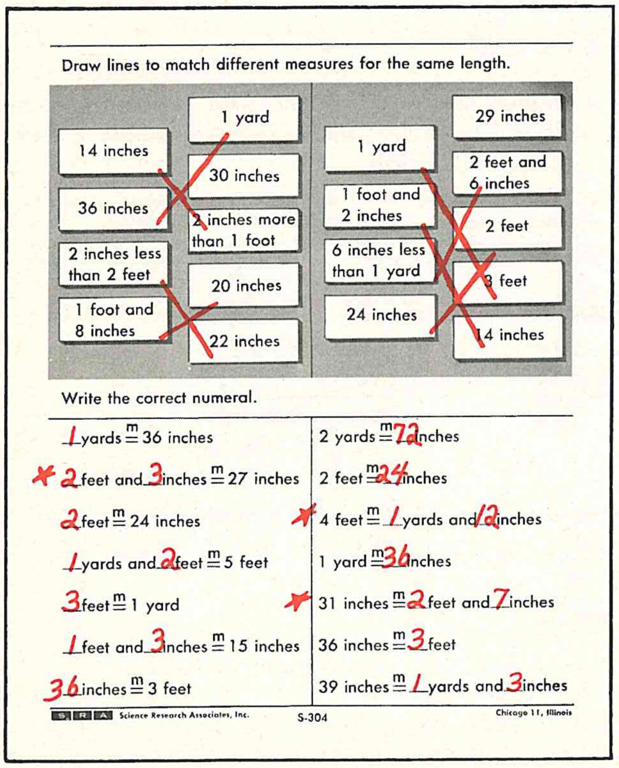
12 inches $\overset{m}{=}$ 1 foot
36 inches $\overset{m}{=}$ 1 yard
3 feet $\overset{m}{=}$ 1 yard

Work the story problems.

1. Jack's car went 39 inches. Bill's car went 3 feet. Jack's car went 3 inches farther than Bill's car.
2. Tom's car ran for 14 inches. Peter's car ran for 1 foot. Tom's car won the first race by 2 inches.
3. In the next race Jack's car went 2 feet. Bill's car went 6 inches farther. Bill's car went 30 inches.
4. Tom's car went 1 foot and 7 inches, but Peter's car went 1 yard. Tom's car was 17 inches behind Peter's car.
5. In each of 2 races Jack's car went 18 inches. Jack's car went 3 feet.
6. In the last race Tom's car went 34 inches, and Bill's car went 2 feet 8 inches. Tom's car won the race by 2 inches.

Three feet is not exactly the same as 1 yard.
Three feet measures the same amount of length as 1 yard.

S-303



Draw lines to match different measures for the same length.

14 inches
36 inches
2 inches less than 2 feet
1 foot and 8 inches
1 yard
30 inches
2 inches more than 1 foot
20 inches
22 inches

1 yard
1 foot and 2 inches
6 inches less than 1 yard
24 inches
29 inches
2 feet and 6 inches
2 feet
3 feet
14 inches

Write the correct numeral.

1 yards $\overset{m}{=}$ 36 inches	2 yards $\overset{m}{=}$ 72 inches
★ 2 feet and 3 inches $\overset{m}{=}$ 27 inches	2 feet $\overset{m}{=}$ 24 inches
2 feet $\overset{m}{=}$ 24 inches	★ 4 feet $\overset{m}{=}$ 1 yards and 12 inches
1 yards and 2 feet $\overset{m}{=}$ 5 feet	1 yard $\overset{m}{=}$ 36 inches
3 feet $\overset{m}{=}$ 1 yard	★ 31 inches $\overset{m}{=}$ 2 feet and 7 inches
1 feet and 3 inches $\overset{m}{=}$ 15 inches	36 inches $\overset{m}{=}$ 3 feet
36 inches $\overset{m}{=}$ 3 feet	39 inches $\overset{m}{=}$ 1 yards and 3 inches

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★ More than one correct answer is possible for each problem.

Supplemental Experiences

▶ Help the children make a giant measuring tape from adding machine tape. The yards can be marked in blue, the feet in red, and the inches in black. Have children use this tape to measure such things as the playground, hall, or classroom. Display the tape on the wall in the classroom for future reference. Whenever the children read of a giant whose feet are ten feet long or a rabbit that can jump seven feet into the air, refer them to the measuring tape.

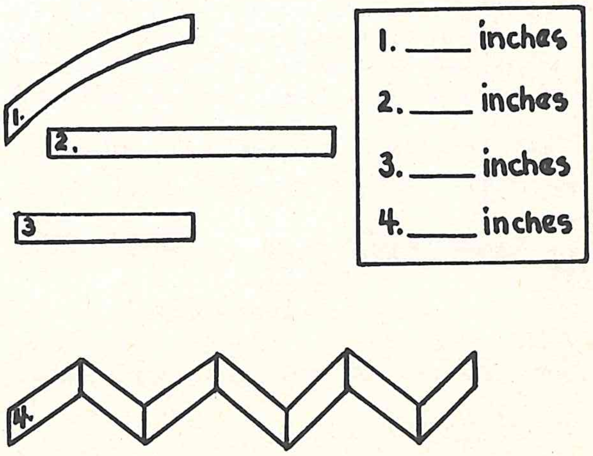
▶ Divide the children into two teams for a game of measuring. Write symbols for different lengths in a column on the chalkboard.

3 in.
15 in.
12 in.
4 in.

Have one child from each team in turn estimate and try to draw the desired length on the chalkboard. The team with the most lines nearer the stated length wins. This activity requires a great deal of board space; if this space is not available, have each child use newspaper and scissors to try to cut strips of the desired length.

During the measuring and judging of these measurements, the ½ inch measure can be introduced as enrichment.

▶ Provide each child with numbered strips of paper of varying lengths, rulers, and an answer sheet. Direct the children to measure the slips of paper and put the answers in the corresponding blanks on the answer sheet. The child who completes his measuring and recording accurately and in the least amount of time wins the game.



Addition

CARRYING IN TWO PLACES

Work Pages S-305 Through S-312

Objectives

To extend the development of addition to carrying in two places.

To strengthen understanding of the concept of carrying in addition.

To help children develop skill in addition with carrying.

Fundamentals

Addition that involves carrying in both the ones and the tens places is the final step in developing the concept of carrying at the second-grade level. If the child has achieved competence in addition involving carrying to the ones and the tens places, he will be ready to work with carrying in both places when they appear in one problem.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers.

Readiness for Understanding

Understanding of the concept of place value.

Understanding of the concept of carrying.

Understanding of the concepts of commutativity and associativity.

Ability to work with number combinations through eighteen.

Ability to use different numerals for numbers.

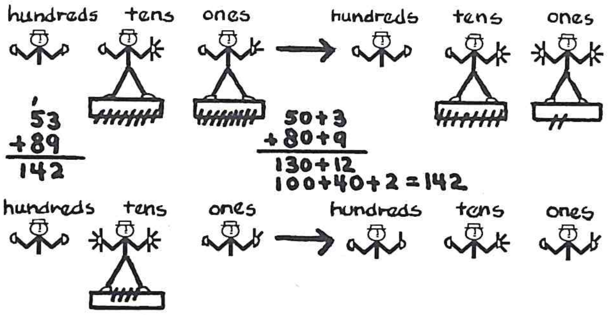
Ability to work with expanded notation and vertical notation.

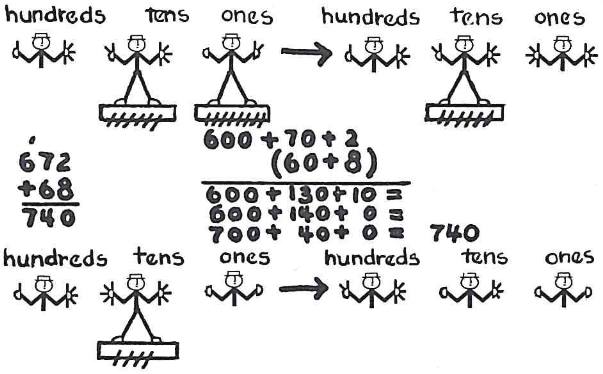
Developmental Experiences

MATERIALS

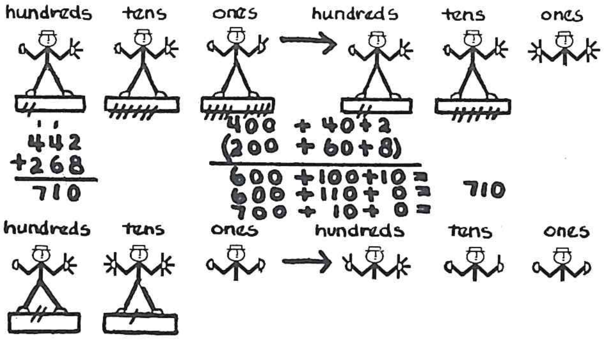
3 Countingmen

▶ Use the Countingmen to demonstrate problems that involve carrying in two places. Begin with addition problems such as 53 + 89 = ☐. The teacher may manipulate the Countingmen as she asks questions. Have each phase of the activity on the Countingmen recorded as it occurs. On the chalkboard, have one child record the ideas involved in expanded notation, and have another child record the ideas in vertical notation. Summarize by relating ideas implied in the two notations. When children read the vertical notation, they should read it in terms of ones, tens, and hundreds, not in terms of single digits.

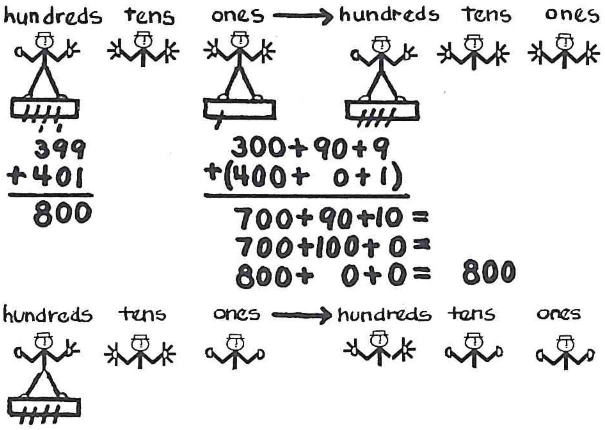




Adapt this approach to other problems such as 46 + 84 = ☐, 386 + 55 = ☐, and 672 + 68 = ☐.



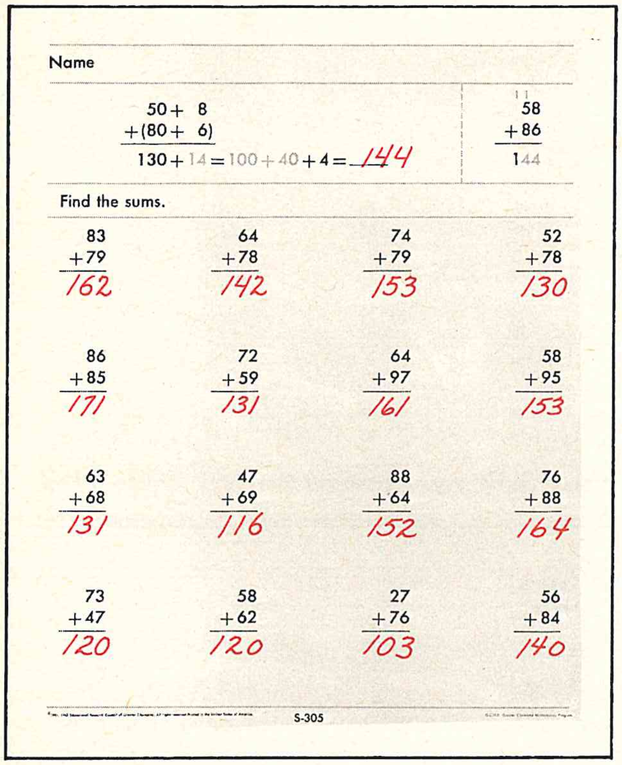
Continue the development with problems such as 289 + 374 = ☐, 565 + 275 = ☐, 356 + 449 = ☐, and 442 + 268 = ☐.



As a last step, demonstrate problems such as 399 + 401 = ☐.

▶ Use work pages S-305 and S-306 to provide practice in addition problems involving two-digit addends with carrying to tens and hundreds places. Work the first problem on the top of work page S-305 with the children. Have them complete the front and back of this sheet on their own. Give help only if needed.

▶ Use work pages S-307 and S-308 to provide practice with addition problems involving two- and three-digit addends, carrying to tens and hundreds places. Work the first problem on the top of each work page with the class. Have them complete the remainder of both pages on their own. Give help only if needed.



Name

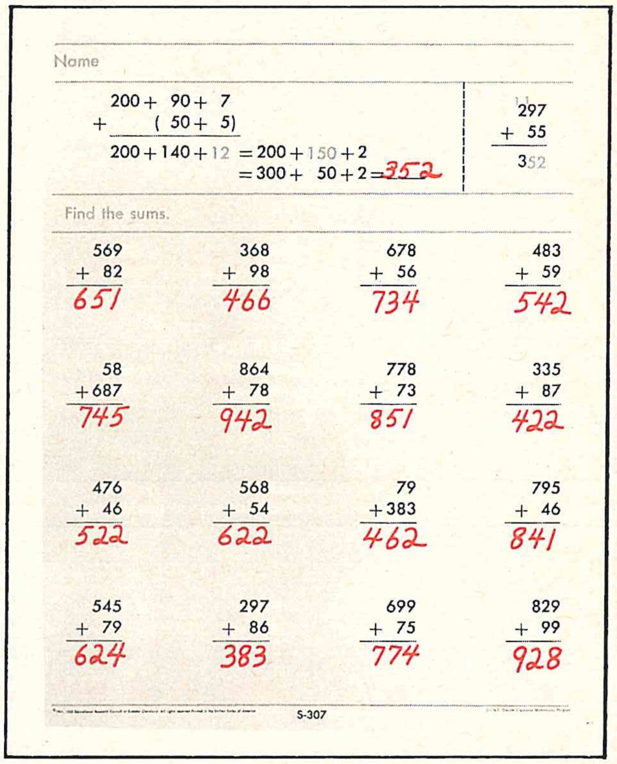
50 + 8
+(80 + 6)
130 + 14 = 100 + 40 + 4 = 144

11
58
+86
144

Find the sums.

83 +79 162	64 +78 142	74 +79 153	52 +78 130
86 +85 171	72 +59 131	64 +97 161	58 +95 153
63 +68 131	47 +69 116	88 +64 152	76 +88 164
73 +47 120	58 +62 120	27 +76 103	56 +84 140

S-305



Name

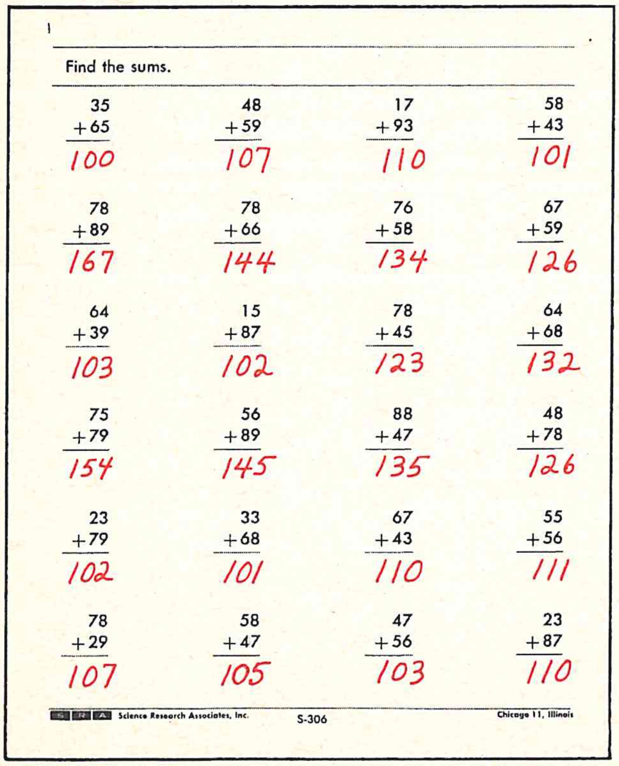
200 + 90 + 7
+ (50 + 5)
200 + 140 + 12 = 200 + 150 + 2
= 300 + 50 + 2 = 352

11
297
+ 55
352

Find the sums.

569 + 82 651	368 + 98 466	678 + 56 734	483 + 59 542
58 +687 745	864 + 78 942	778 + 73 851	335 + 87 422
476 + 46 522	568 + 54 622	79 +383 462	795 + 46 841
545 + 79 624	297 + 86 383	699 + 75 774	829 + 99 928

S-307



Find the sums.

35 +65 100	48 +59 107	17 +93 110	58 +43 101
78 +89 167	78 +66 144	76 +58 134	67 +59 126
64 +39 103	15 +87 102	78 +45 123	64 +68 132
75 +79 154	56 +89 145	88 +47 135	48 +78 126
23 +79 102	33 +68 101	67 +43 110	55 +56 111
78 +29 107	58 +47 105	47 +56 103	23 +87 110

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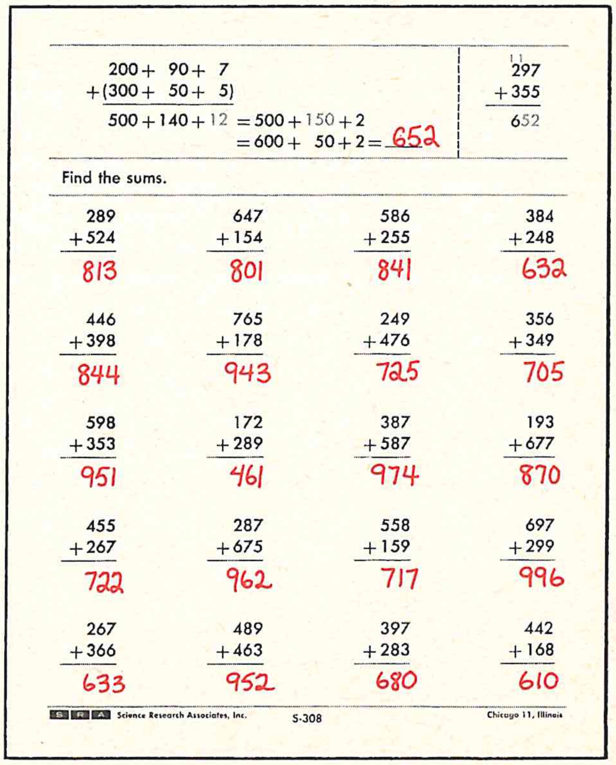

200 + 90 + 7
+(300 + 50 + 5)
500 + 140 + 12 = 500 + 150 + 2
= 600 + 50 + 2 = 652

11
297
+355
652

Find the sums.

289 +524 813	647 +154 801	586 +255 841	384 +248 632
446 +398 844	765 +178 943	249 +476 725	356 +349 705
598 +353 951	172 +289 461	387 +587 974	193 +677 870
455 +267 722	287 +675 962	558 +159 717	697 +299 996
267 +366 633	489 +463 952	397 +283 680	442 +168 610

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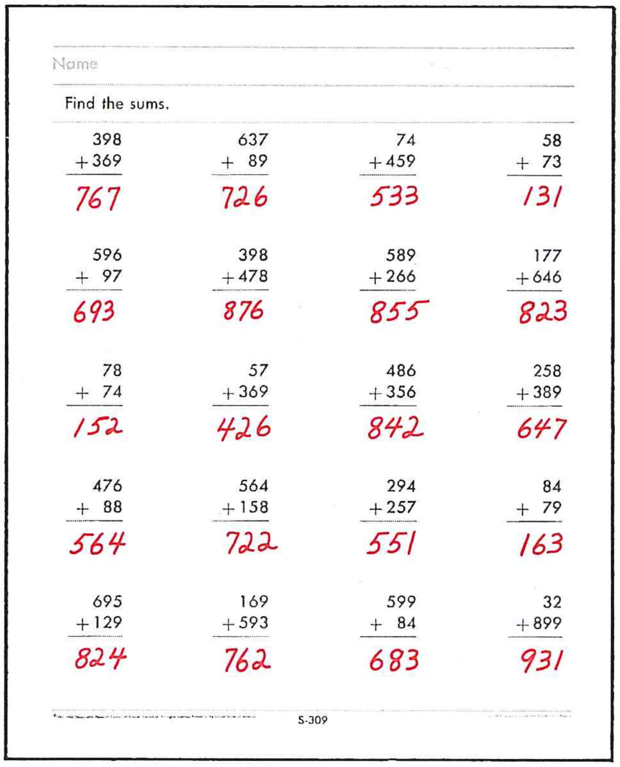
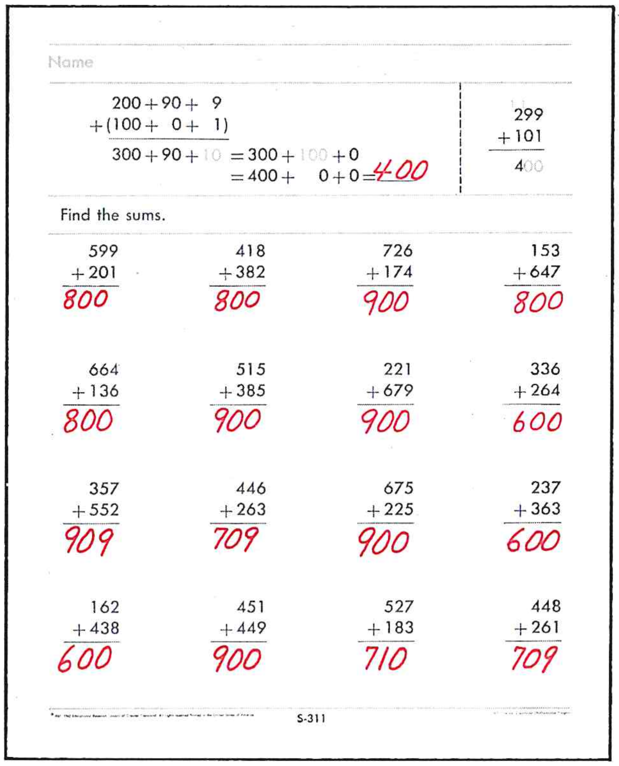


Name

Find the sums.

398 +369 767	637 + 89 726	74 +459 533	58 + 73 131
596 + 97 693	398 +478 876	589 +266 855	177 +646 823
78 + 74 152	57 +369 426	486 +356 842	258 +389 647
476 + 88 564	564 +158 722	294 +257 551	84 + 79 163
695 +129 824	169 +593 762	599 + 84 683	32 +899 931

S-309



Name

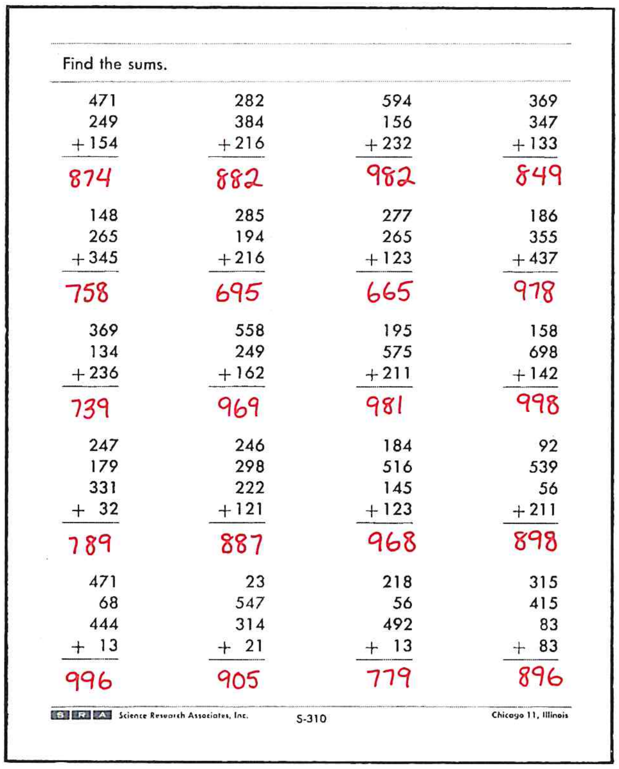
200 + 90 + 9
+(100 + 0 + 1)
300 + 90 + 10 = 300 + 100 + 0
= 400 + 0 + 0 = 400

11
299
+101
400

Find the sums.

599 +201 800	418 +382 800	726 +174 900	153 +647 800
664 +136 800	515 +385 900	221 +679 900	336 +264 600
357 +552 909	446 +263 709	675 +225 900	237 +363 600
162 +438 600	451 +449 900	527 +183 710	448 +261 709

S-311



Find the sums.

471 249 +154 874	282 384 +216 882	594 156 +232 982	369 347 +133 849
148 265 +345 758	285 194 +216 695	277 265 +123 665	186 355 +437 978
369 134 +236 739	558 249 +162 969	195 575 +211 981	158 698 +142 998
247 179 331 + 32 789	246 298 222 +121 887	184 516 145 +123 968	92 539 56 +211 898
471 68 444 + 13 996	23 547 314 + 21 905	218 56 492 + 13 779	315 415 83 + 83 896

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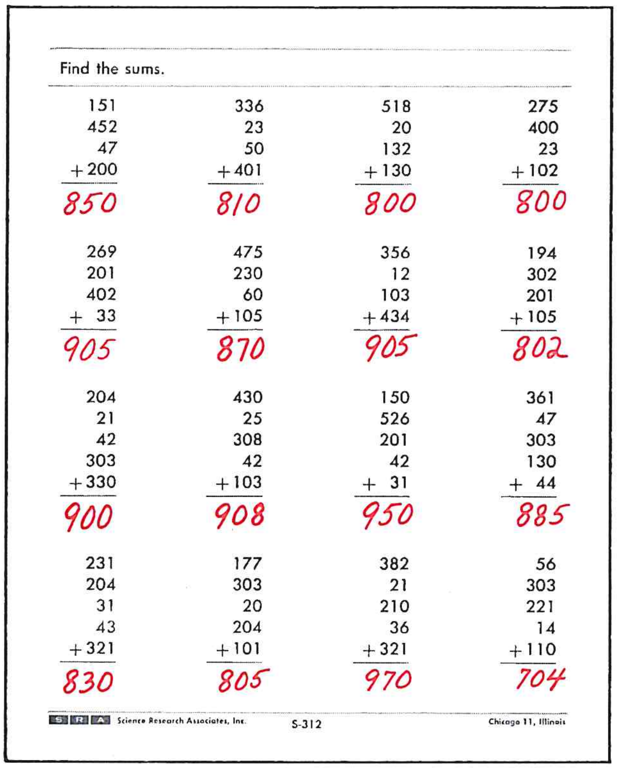

Find the sums.

151 452 47 +200 850	336 23 50 +401 810	518 20 132 +130 800	275 400 23 +102 800
269 201 402 + 33 905	475 230 60 +105 870	356 12 103 +434 905	194 302 201 +105 802
204 21 42 303 +330 900	430 25 308 42 +103 908	150 526 201 42 + 31 950	361 47 303 130 + 44 885
231 204 31 43 +321 830	177 303 20 204 +101 805	382 21 210 36 +321 970	56 303 221 14 +110 704

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▶ Work page S-309 provides a review of addition problems with carrying to tens and hundreds places when two- and three-digit addends are involved. Direct the children to find the sums.

Work page S-310 provides children with the opportunity to test themselves on their ability to work column-addition problems involving carrying to tens and hundreds places with three and four addends. Direct the children to find the sums.

▶ Work page S-311 gives children the opportunity to use their growing ability to solve addition problems with carrying in two places when two addends of three digits each are involved. Zero is included. Work the problem on the top of work page S-311 with the children, then have them complete the remainder of the work page on their own. Direct them to find the sums.

Direct the children to work the column-addition problems on work page S-312

Supplemental Experiences

▶ Reconstruction problems are an excellent activity to help children develop a good number sense. On the chalkboard, put several reconstruction problems that contain combinations of ten as well as

$$\begin{array}{r} 5 \\ \blacksquare \\ +6 \\ 5 \\ \hline 18 \end{array} \qquad \begin{array}{r} 16 \\ +2\blacksquare \\ \hline 40 \end{array} \qquad \begin{array}{r} 27 \\ +\blacksquare 5 \\ \hline 62 \end{array} \qquad \begin{array}{r} 38 \\ +47 \\ \hline \blacksquare 5 \end{array}$$

problems that involve carrying. Have children explain their thinking as they find the numerals for the missing numbers in the problems. In the first problem, for example, a child may say: "I added five and five, got ten, then added six and got sixteen. I knew that I must add two to sixteen to get the sum eighteen, so two must have been the missing number."

Subtraction—Addition

BORROWING AND CARRYING IN TWO PLACES

Work Pages S-313 Through S-322

Objectives

To extend the development of subtraction to borrowing from the tens and the hundreds places.

To strengthen understanding of the concept of borrowing in subtraction.

To help children develop skill in subtraction with borrowing.

To evaluate children's ability to work with addition and subtraction involving carrying and borrowing in two places.

Fundamentals

Subtraction that involves borrowing from both the hundreds and the tens places is the last step in the development of the concept of borrowing at the second-grade level. After the child has satisfactorily learned to borrow from the tens place, then from the hundreds place, the teacher will want to help him tie these two steps together in one problem.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Addition of Whole Numbers; Subtraction of Whole Numbers.

Readiness for Understanding

Understanding of the concept of place value.

Understanding of the concepts of borrowing and carrying.

Understanding of the concepts of commutativity and associativity.

Awareness of the inverse relation between addition and subtraction.

Ability to work with number combinations through eighteen.

Ability to use different numerals for numbers.

Ability to work with expanded notation and vertical notation.

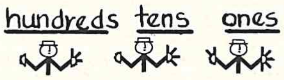
Developmental Experiences

MATERIALS

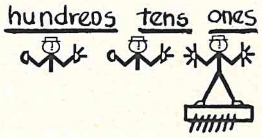
3 Countingmen

▶ Use the Countingmen to demonstrate problems that involve borrowing in two places. Begin with subtraction problems such as 262 − 94 = ☐ and then move to problems such as 347 − 189 = ☐.

(1)

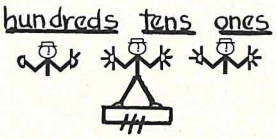


(2)

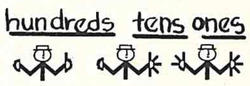


$$\begin{array}{r} \overset{2}{\cancel{3}}\,\overset{13}{\cancel{4}}\,{}^{1}7 \\ -1\,8\,9 \\ \hline 1\,5\,8 \end{array} \qquad \begin{array}{rcr} 300+40+\ 7 & = & 300+\ 30+17 \\ -(100+80+\ 9) & = & -(100+\ 80+\ 9) \\ \hline 300+30+17 & = & 200+130+17) \\ -(100+80+\ 9) & = & -(100+\ 80+\ 9) \\ \hline 8 & & 100+\ 50+\ 8 \\ & & =158 \end{array}$$

(3)



(4)

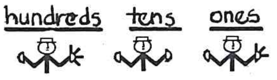


The teacher may manipulate the Countingmen as she asks questions pertinent to the action. Help children understand that two problems arise in subtractions of this kind and that they are taken care of one step at a time. First they must borrow from tens to take care of subtraction in ones place. Then they must borrow from hundreds to take care of subtraction in tens place. The expanded notation takes on a more complex aspect because of this situation. Do not expect children to work through these expanded notations on their own. Have them assist you in writing them on the chalkboard. As one child records the activity that occurs on the Countingmen, in expanded notation, have another child write the problem in algorism form.

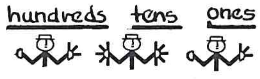
Summarize by relating ideas implied in the two notations.

Adapt this approach to problems such as 403 − 126 = ☐. Guide children to discover that in

(1)



(2)

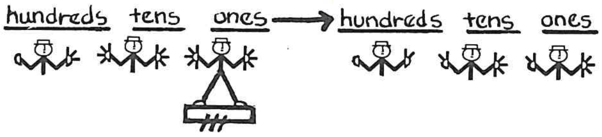


$$\begin{array}{rcl} 400+\ 0+3 &=& 300+100+\ 3) \\ -(100+\ 20+6) &=& -(100+\ 20+\ 6) \end{array}$$

$$\begin{array}{r} \overset{3}{\cancel{4}}{}^{1}\overset{9}{\cancel{0}}{}^{1}3 \\ -1\ 2\ 6 \\ \hline 2\ 7\ 7 \end{array} \qquad \begin{array}{rcl} 300+100+3 &=& 300+\ 90+13 \\ -(100+\ 20+6) &=& -(100+\ 20+\ 6) \\ && \overline{200+\ 70+\ 7} \\ && =277 \end{array}$$

(3)

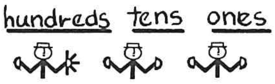
(4)



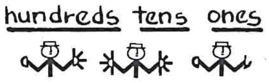
this type of subtraction, a different kind of problem arises. No tens can be borrowed from the tens place to make subtraction in the ones place possible, unless borrowing occurs from the hundreds place first. Step by step, one hundred must be changed to ten tens, then one of these tens must be changed to ten ones.

Continue with problems such as 500 − 342 = ☐.

(1)



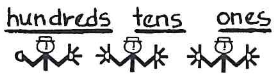
(2)



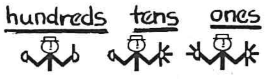
$$\begin{array}{rcl} 500+\ 0+0 &=& 400+100+\ 0 \\ -(300+\ 40+2) &=& -(300+\ 40+\ 2) \end{array}$$

$$\begin{array}{r} \overset{4}{\cancel{5}}{}^{1}\overset{9}{\cancel{0}}{}^{1}0 \\ -3\ 4\ 2 \\ \hline 1\ 5\ 8 \end{array} \qquad \begin{array}{rcl} 400+100+0 &=& 400+\ 90+10 \\ -(300+\ 40+2) &=& -(300+\ 40+\ 2) \\ && \overline{100+\ 50+\ 8} \\ && =158 \end{array}$$

(3)



(4)



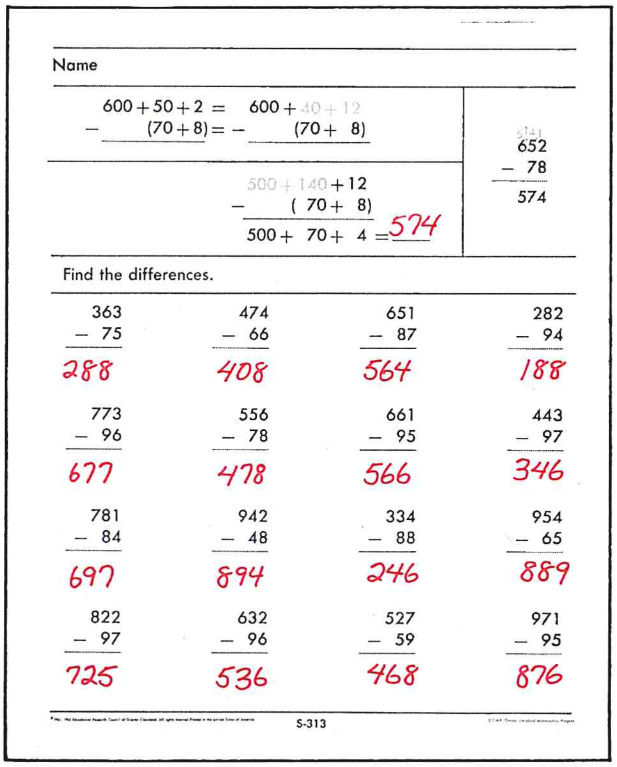
Demonstrate several addition problems involving carrying in two places. Have the related algorism worked out on the chalkboard. Have children "undo" these problems and place the corresponding subtraction algorisms on the chalkboard next to the additions.

▶ Use work page S-313 to provide practice in borrowing from both tens and hundreds places in subtraction. Work the first problem on the top of the work page with the children; then have them complete the page on their own.

Work page S-314 also provides practice in borrowing from both hundreds and tens places. Work the problem on the top of this work page with the class. Have them complete the remainder of the page independently.

▶ Use work page S-315 to provide practice in borrowing in two places with zero involved in the tens place. After working the problem at the top of work page S-315 with the class, direct them to find the differences for the problems on the remainder of the page.

Work page S-316 provides practice in borrowing from two places with zeros in both the ones and the tens places. After working the first problem at the



Name

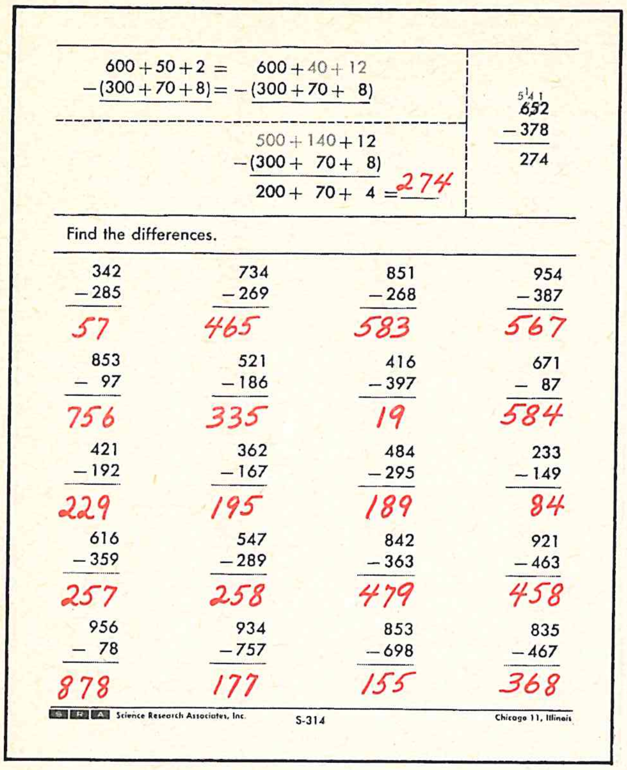
$$\begin{array}{rcl} 600+50+2 &=& 600+40+12 \\ -\ \ (70+8) &=& -\ \ (70+\ 8) \end{array}$$

$$\begin{array}{r} 500+140+12 \\ -\ \ (\ 70+\ 8) \\ \hline 500+\ 70+\ 4 = 574 \end{array} \qquad \begin{array}{r} {}^{5\,14\,1} \\ 652 \\ -\ 78 \\ \hline 574 \end{array}$$

Find the differences.

363 − 75 288	474 − 66 408	651 − 87 564	282 − 94 188
773 − 96 677	556 − 78 478	661 − 95 566	443 − 97 346
781 − 84 697	942 − 48 894	334 − 88 246	954 − 65 889
822 − 97 725	632 − 96 536	527 − 59 468	971 − 95 876

S-313



$$\begin{array}{rcl} 600+50+2 & = & 600+40+12 \\ -\underline{(300+70+8)} & = & -\underline{(300+70+\ 8)} \end{array}$$

$$\begin{array}{r} 500+140+12 \\ -\underline{(300+\ 70+\ 8)} \\ 200+\ 70+\ 4 = \underline{274} \end{array}$$

$$\begin{array}{r} \overset{5}{\cancel{6}}\,\overset{14}{\cancel{5}}\,\overset{1}{2} \\ -\underline{378} \\ 274 \end{array}$$

Find the differences.

342 −285 57	734 −269 465	851 −268 583	954 −387 567
853 − 97 756	521 −186 335	416 −397 19	671 − 87 584
421 −192 229	362 −167 195	484 −295 189	233 −149 84
616 −359 257	547 −289 258	842 −363 479	921 −463 458
956 − 78 878	934 −757 177	853 −698 155	835 −467 368

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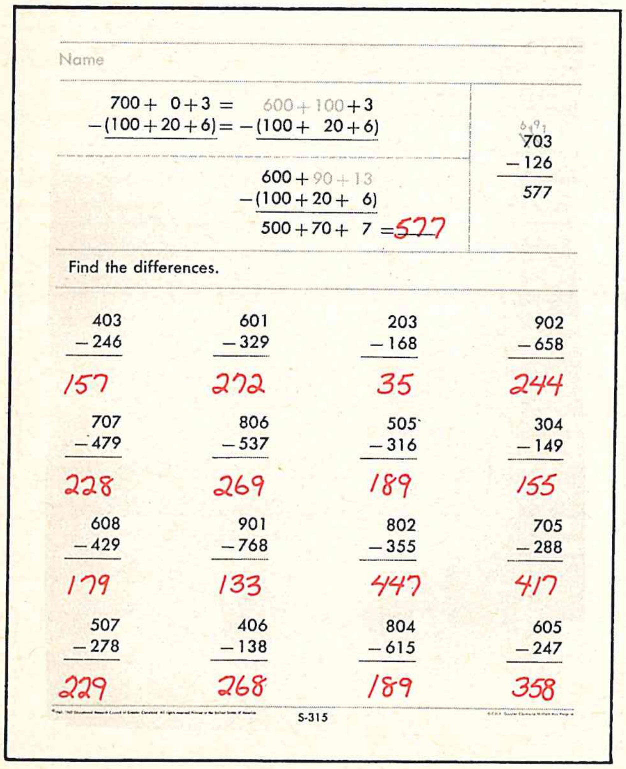
top of the work page with the class, direct them to complete the remainder of the problems by finding the differences. Give help only if needed.



Name

$$\begin{array}{rcl} 700+\ 0+3 & = & 600+100+3 \\ -\underline{(100+20+6)} & = & -\underline{(100+\ 20+6)} \end{array}$$

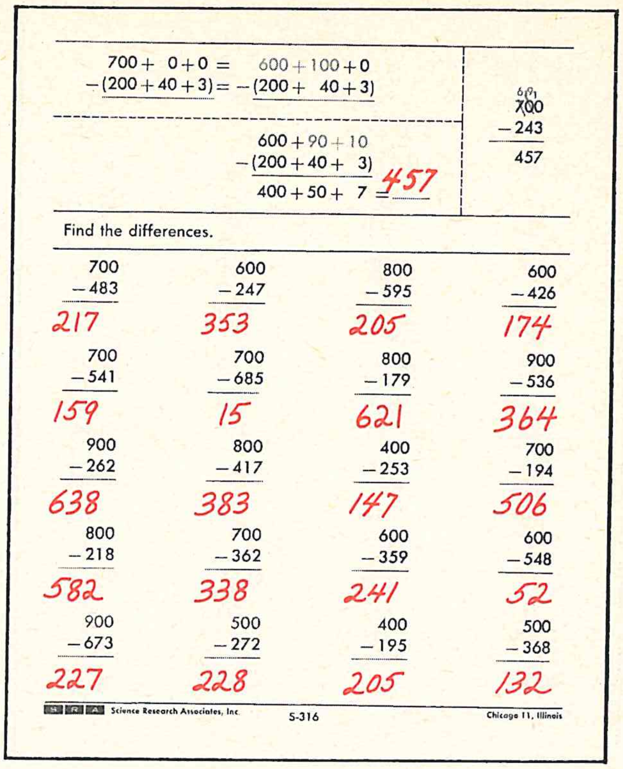
$$\begin{array}{r} 600+90+13 \\ -\underline{(100+20+\ 6)} \\ 500+70+\ 7 = \underline{577} \end{array}$$

$$\begin{array}{r} \overset{6}{\cancel{7}}\,\overset{9}{\cancel{0}}\,\overset{1}{3} \\ -\underline{126} \\ 577 \end{array}$$

Find the differences.

403 −246 157	601 −329 272	203 −168 35	902 −658 244
707 −479 228	806 −537 269	505 −316 189	304 −149 155
608 −429 179	901 −768 133	802 −355 447	705 −288 417
507 −278 229	406 −138 268	804 −615 189	605 −247 358

S-315



$$\begin{array}{rcl} 700+\ 0+0 & = & 600+100+0 \\ -\underline{(200+40+3)} & = & -\underline{(200+\ 40+3)} \end{array}$$

$$\begin{array}{r} 600+90+10 \\ -\underline{(200+40+\ 3)} \\ 400+50+\ 7 = \underline{457} \end{array}$$

$$\begin{array}{r} \overset{6}{\cancel{7}}\,\overset{9}{\cancel{0}}\,\overset{1}{0} \\ -\underline{243} \\ 457 \end{array}$$

Find the differences.

700 −483 217	600 −247 353	800 −595 205	600 −426 174
700 −541 159	700 −685 15	800 −179 621	900 −536 364
900 −262 638	800 −417 383	400 −253 147	700 −194 506
800 −218 582	700 −362 338	600 −359 241	600 −548 52
900 −673 227	500 −272 228	400 −195 205	500 −368 132

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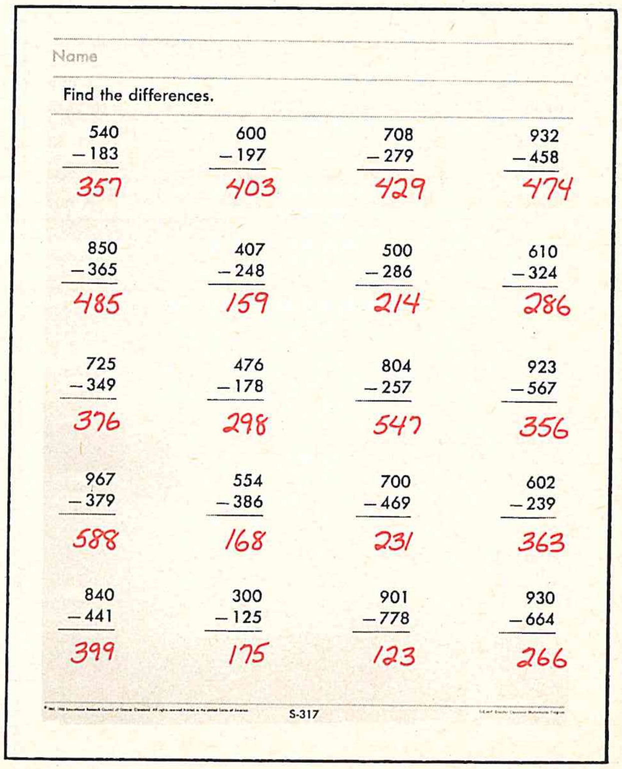
▶ Work pages S-317 and S-318 provide further practice with subtraction involving borrowing in two places. Direct the children to find the differences.

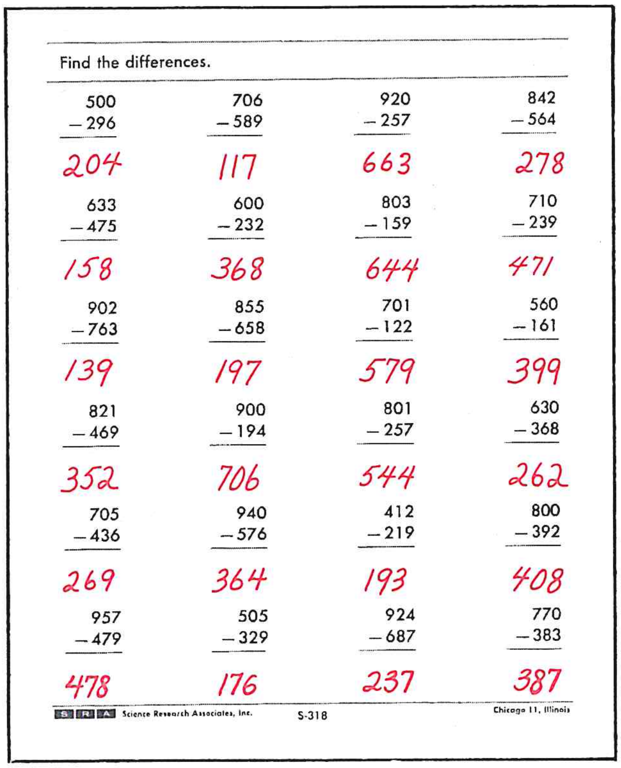


Name

Find the differences.

540 −183 357	600 −197 403	708 −279 429	932 −458 474
850 −365 485	407 −248 159	500 −286 214	610 −324 286
725 −349 376	476 −178 298	804 −257 547	923 −567 356
967 −379 588	554 −386 168	700 −469 231	602 −239 363
840 −441 399	300 −125 175	901 −778 123	930 −664 266

S-317



Find the differences.

500 – 296 204	706 – 589 117	920 – 257 663	842 – 564 278
633 – 475 158	600 – 232 368	803 – 159 644	710 – 239 471
902 – 763 139	855 – 658 197	701 – 122 579	560 – 161 399
821 – 469 352	900 – 194 706	801 – 257 544	630 – 368 262
705 – 436 269	940 – 576 364	412 – 219 193	800 – 392 408
957 – 479 478	505 – 329 176	924 – 687 237	770 – 383 387

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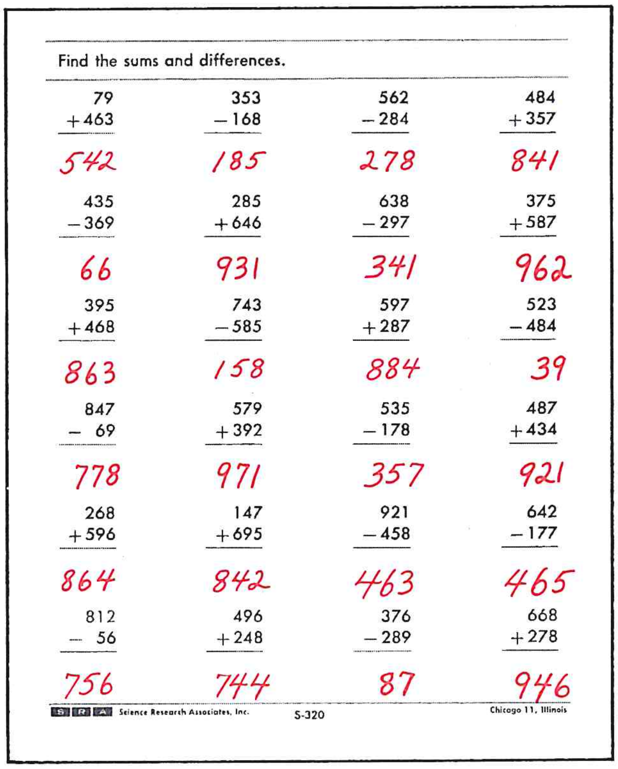

Find the sums and differences.

79 + 463 542	353 – 168 185	562 – 284 278	484 + 357 841
435 – 369 66	285 + 646 931	638 – 297 341	375 + 587 962
395 + 468 863	743 – 585 158	597 + 287 884	523 – 484 39
847 – 69 778	579 + 392 971	535 – 178 357	487 + 434 921
268 + 596 864	147 + 695 842	921 – 458 463	642 – 177 465
812 – 56 756	496 + 248 744	376 – 289 87	668 + 278 946

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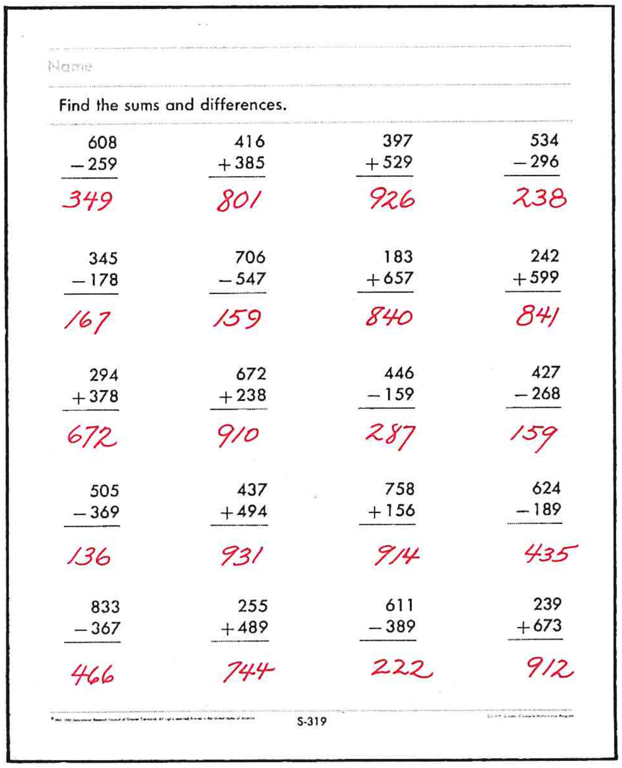
▶ Work pages S-319 through S-322 provide for a review of addition and subtraction with carrying and borrowing in two places. Direct the children to find the sums or differences.

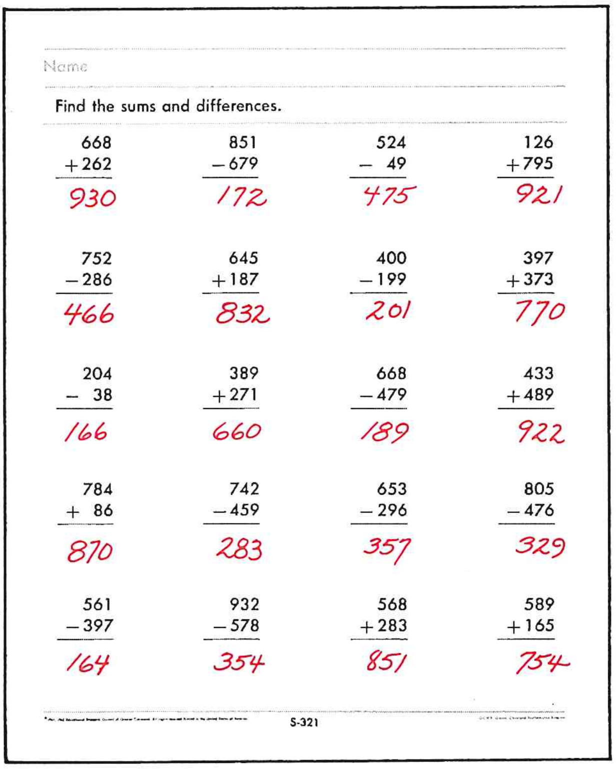


Name

Find the sums and differences.

608 – 259 349	416 + 385 801	397 + 529 926	534 – 296 238
345 – 178 167	706 – 547 159	183 + 657 840	242 + 599 841
294 + 378 672	672 + 238 910	446 – 159 287	427 – 268 159
505 – 369 136	437 + 494 931	758 + 156 914	624 – 189 435
833 – 367 466	255 + 489 744	611 – 389 222	239 + 673 912

S-319

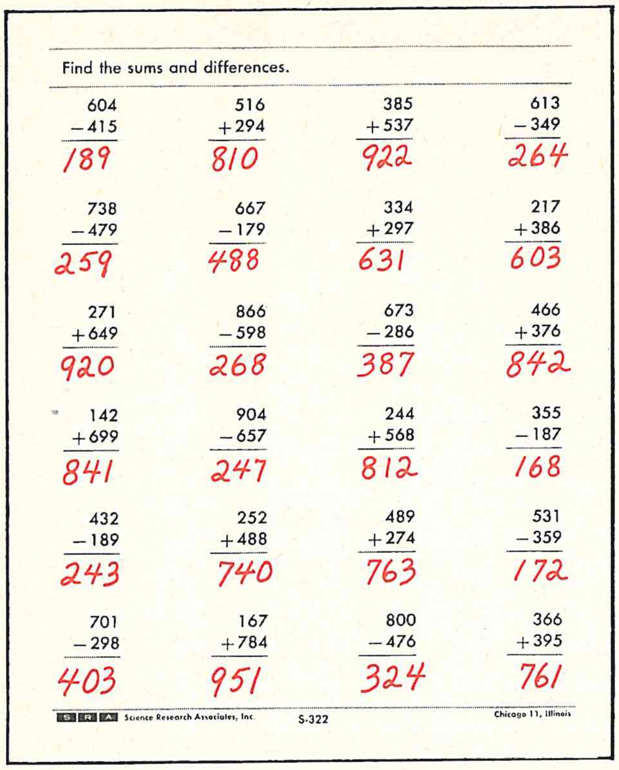


Name

Find the sums and differences.

668 + 262 930	851 – 679 172	524 – 49 475	126 + 795 921
752 – 286 466	645 + 187 832	400 – 199 201	397 + 373 770
204 – 38 166	389 + 271 660	668 – 479 189	433 + 489 922
784 + 86 870	742 – 459 283	653 – 296 357	805 – 476 329
561 – 397 164	932 – 578 354	568 + 283 851	589 + 165 754

S-321



Find the sums and differences.

$\begin{array}{r}604\\-415\\\hline 189\end{array}$	$\begin{array}{r}516\\+294\\\hline 810\end{array}$	$\begin{array}{r}385\\+537\\\hline 922\end{array}$	$\begin{array}{r}613\\-349\\\hline 264\end{array}$
$\begin{array}{r}738\\-479\\\hline 259\end{array}$	$\begin{array}{r}667\\-179\\\hline 488\end{array}$	$\begin{array}{r}334\\+297\\\hline 631\end{array}$	$\begin{array}{r}217\\+386\\\hline 603\end{array}$
$\begin{array}{r}271\\+649\\\hline 920\end{array}$	$\begin{array}{r}866\\-598\\\hline 268\end{array}$	$\begin{array}{r}673\\-286\\\hline 387\end{array}$	$\begin{array}{r}466\\+376\\\hline 842\end{array}$
$\begin{array}{r}142\\+699\\\hline 841\end{array}$	$\begin{array}{r}904\\-657\\\hline 247\end{array}$	$\begin{array}{r}244\\+568\\\hline 812\end{array}$	$\begin{array}{r}355\\-187\\\hline 168\end{array}$
$\begin{array}{r}432\\-189\\\hline 243\end{array}$	$\begin{array}{r}252\\+488\\\hline 740\end{array}$	$\begin{array}{r}489\\+274\\\hline 763\end{array}$	$\begin{array}{r}531\\-359\\\hline 172\end{array}$
$\begin{array}{r}701\\-298\\\hline 403\end{array}$	$\begin{array}{r}167\\+784\\\hline 951\end{array}$	$\begin{array}{r}800\\-476\\\hline 324\end{array}$	$\begin{array}{r}366\\+395\\\hline 761\end{array}$

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Supplemental Experiences

▶ Place several subtraction reconstruction problems on the chalkboard. Challenge children to use their expanding number sense to supply the numerals for the missing numbers. Have them explain their thinking as they find the numerals for the missing numbers in the problems.

$\begin{array}{r}92\\-4\square\\\hline 45\end{array}$ $\begin{array}{r}43\\-\square 6\\\hline 17\end{array}$ $\begin{array}{r}\square 5\\-28\\\hline 37\end{array}$ $\begin{array}{r}8\square\\-56\\\hline 26\end{array}$

Story Problems

Work Pages S-323 and S-324

Objectives

To provide practice in solving story problems.

To provide practice in three-digit addition and subtraction with carrying and borrowing.

Fundamentals

The story problems here give extra practice in three-digit addition with carrying to both the tens place and the hundreds place, and three-digit subtraction with borrowing from both the hundreds place and the tens place.

Readiness for Understanding

Ability to find the correct operation to solve a story problem.

Ability to check addition and subtraction problems.

An understanding of the steps used in addition involving carrying and in subtraction involving borrowing.

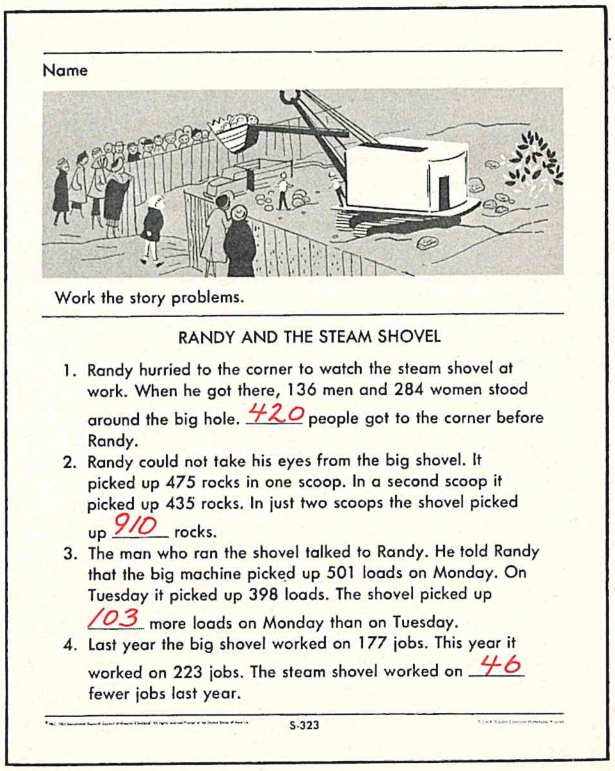
Developmental Experiences

MATERIALS

3 Countingmen
bundles of sticks

▶ Discuss the pictures on work pages S-323 and S-324 Have one child read aloud the first story problem and tell whether to add or subtract to find the answer. Ask one child to work at the board, another with the Countingmen, and a third with sticks bundled into sets of 10. Have each child give an explanation of his work.

Allow the children to work on their own. Provide individual help when needed. The teacher should check and return these papers so that the children can correct their errors.



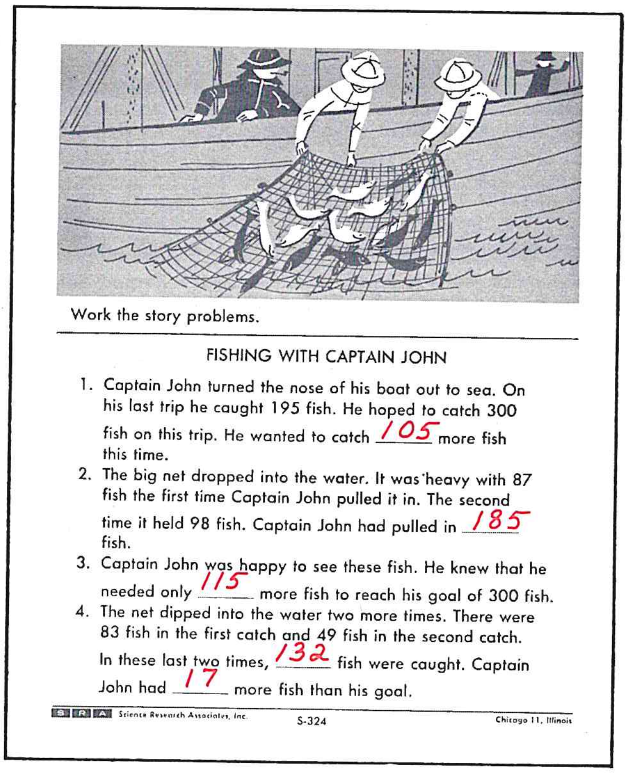
Name

Work the story problems.

RANDY AND THE STEAM SHOVEL

1. Randy hurried to the corner to watch the steam shovel at work. When he got there, 136 men and 284 women stood around the big hole. 420 people got to the corner before Randy.
2. Randy could not take his eyes from the big shovel. It picked up 475 rocks in one scoop. In a second scoop it picked up 435 rocks. In just two scoops the shovel picked up 910 rocks.
3. The man who ran the shovel talked to Randy. He told Randy that the big machine picked up 501 loads on Monday. On Tuesday it picked up 398 loads. The shovel picked up 103 more loads on Monday than on Tuesday.
4. Last year the big shovel worked on 177 jobs. This year it worked on 223 jobs. The steam shovel worked on 46 fewer jobs last year.

S-323



Work the story problems.

FISHING WITH CAPTAIN JOHN

1. Captain John turned the nose of his boat out to sea. On his last trip he caught 195 fish. He hoped to catch 300 fish on this trip. He wanted to catch 105 more fish this time.
2. The big net dropped into the water. It was heavy with 87 fish the first time Captain John pulled it in. The second time it held 98 fish. Captain John had pulled in 185 fish.
3. Captain John was happy to see these fish. He knew that he needed only 115 more fish to reach his goal of 300 fish.
4. The net dipped into the water two more times. There were 83 fish in the first catch and 49 fish in the second catch. In these last two times, 132 fish were caught. Captain John had 17 more fish than his goal.

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Supplemental Experiences

▶ Have children write the following numerals in decreasing numerical order:

468, 648, 402, 608, 486
396, 693, 963, 906, 369
107, 701, 376, 493, 824

▶ Have the children write the following numerals in increasing numerical order:

533, 335, 353, 553, 535
412, 214, 142, 421, 241
987, 879, 789, 798, 897

▶ Help the children make up a class story which contains both addition and subtraction problems. Begin the story with, "In our room there are fifteen boys and seventeen girls. There are ____ children in our room." The child who gives the correct answer may make up the next part of the story. He might say, "There are ____ more girls than boys." Or he might say, "Each of the children in our class has a pair of tennis shoes at school. They keep these ____ shoes in the coat closet." A third child then continues the story, moving on from the second child's story. For example, he might go on, "One day eight tennis shoes were lost. Then there were only ____ tennis shoes left." As such a cumulative story is continued it often becomes very funny. For example, "A mouse nibbled holes in three of the tennis shoes. He did not nibble any of the other ____ shoes."

Readiness for Multiplication

Work Pages S-325 and S-326

Objectives

To lay the foundation for the concept of multiplication.

To develop readiness for understanding of multiples of two.

Fundamentals

As with addition, the operation of multiplication will be defined in terms of an operation on sets. It is natural for children to think of several disjoint sets, each of which has the same number of objects. For example, each of three girls has two dolls. How many dolls do they have all together? Each of four boys has two cars. How many cars do these boys have?

Before the language and symbolism of multiplication is introduced, the children will be given an opportunity to consider groups of equivalent sets. In the development of place value, they became accustomed to working with sets of one and of ten; now, they will be given the opportunity to consider sets of two.

See Key Topics in Mathematics for the Primary Teacher: Multiplication of Whole Numbers.

Readiness for Understanding

Ability to recognize equivalent sets.

Knowledge of addition combinations through sums of ten.

Developmental Experiences

MATERIALS	VOCABULARY
flannel board	pair
felt cutouts	twins
counters	

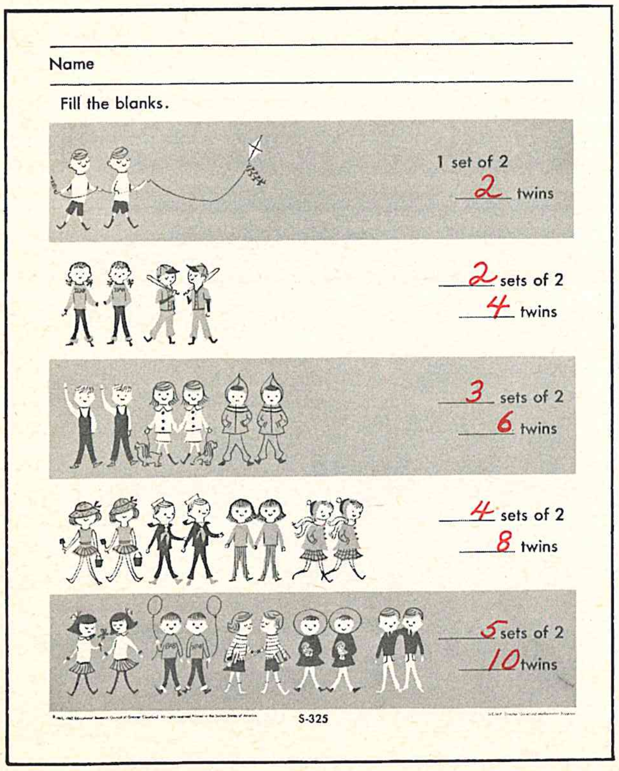
▶ Place five sets, each set containing one felt cutout, on the flannel board. Discuss each of these sets with the children. Loop a heavy piece of yarn around the display and ask the children to think of all these sets as forming a single new set. Ask, "What is the number of this new set?" Ask the children how they found the number of this new set. Someone may suggest that he counted.

Display on the flannel board five distinct sets, each set containing two objects. Have the children tell the number of each set. Ask them to consider all of these sets as making one new set. Call on a child to give the number of the new set. Ask how he found this number. Lead the children to discover that they can count by ones to find the number, but that it may be easier to count by twos.

Discuss the many things in the child's everyday environment which come in twos or in *pairs:* shoes, mittens, socks, and *twins.* Now direct one of the children to stand in the front of the room. Call on another child to tell how many eyes the first child has. Call on a second child to stand beside the first. Say, "Each of these children has two eyes. How many sets of two eyes do you see?" "How many eyes do you see altogether?"

As you have a third, fourth, and fifth child join the others at the front of the room, discuss the three sets of two, the four sets of two, and the five sets of two that are formed. Repeat this activity with different children, calling attention to sets of two ears or two feet.

As a variation, call on five children to stand before the class and hold out both hands. Have the others determine the total number of hands in these five sets of two. Now have the chosen children sit down, one at a time, leaving behind four sets of two, then three sets of two, two sets of two, one set of two, and finally no sets of two, or zero sets of two. In each case discuss the total number of hands in these sets of two.



Name

Fill the blanks.

1 set of 2
2 twins

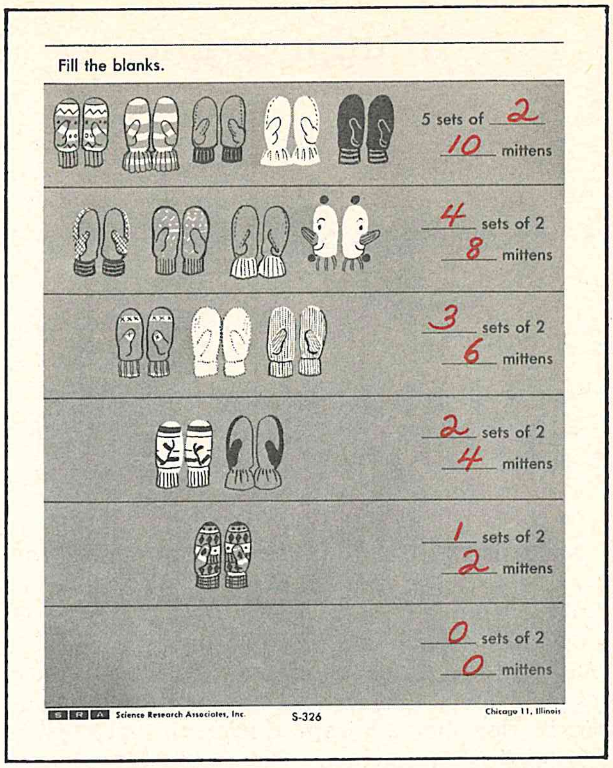
2 sets of 2
4 twins

3 sets of 2
6 twins

4 sets of 2
8 twins

5 sets of 2
10 twins

S-325



Fill the blanks.

5 sets of 2
10 mittens

4 sets of 2
8 mittens

3 sets of 2
6 mittens

2 sets of 2
4 mittens

1 sets of 2
2 mittens

0 sets of 2
0 mittens

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▶ Use work page S-325 to reinforce the idea of thinking of several sets of two as a single set. Discuss with the children the sets of twins that appear in each row. Read with them the words at the right. Call their attention to the fact that each set of twins has the number two. Ask how many sets there are in a given row. Then ask how many twins there are altogether. The children may arrive at their answer by counting by ones, or perhaps by counting by twos.

Work page S-326 may be used in a similar manner. Special attention should be called to the last row on this page, in which zero sets of two mittens appear.

Supplemental Experiences

▶ As the children line up to go to the playground or to the gym have them walk by twos. Call on two sets of two to move out from the line. Ask how many children are in this set. Have the next three sets of two step forward, and ask how many there are in this set. In a similar fashion determine the total number of children in four sets of two, five sets of two, and zero sets of two.

▶ Have the children work with sets of counters at their desks. Direct them to lay out three sets of two. Ask them how many they have altogether. Repeat this procedure with four sets of two, five sets of two, one set of two, two sets of two, and zero sets of two.

Multiplication

Work Pages S-327 and S-328

Objectives

To help develop the concept of multiplication.

To introduce the operational symbol × for multiplication.

Fundamentals

Multiplication can best be defined for children in terms of sets. The children have learned to find the cardinal number of a set formed from a number of equivalent disjoint sets. For example, they have found that a set formed from three sets, each of which has the cardinal number two, has the cardinal number six. Now they will learn to relate this set story to a multiplication equation. The multiplication symbol × will be introduced. The children will learn that 3 × 2 (read as, "three times two" or "three twos") is interpreted as three sets of two. In the multiplication equation 3 × 2 = 6, six is called the product of the factors three and two. Three and two are called factors of the product, six.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Multiplication of Whole Numbers.

Readiness for Understanding

Ability to determine the number of a set formed from zero through five sets of two.

Developmental Experiences

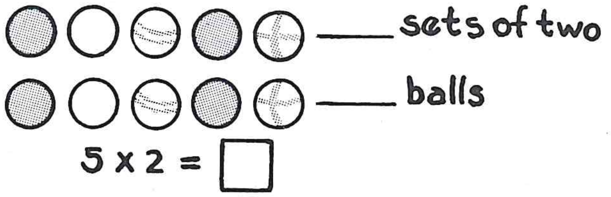
MATERIALS

- books
- pencils

VOCABULARY

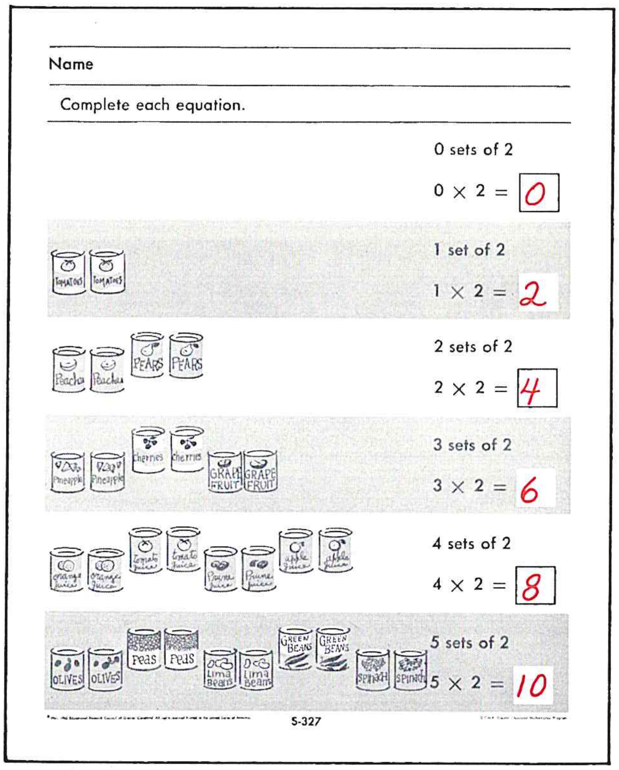
- times
- multiplication
- product
- factor

▶ On the chalkboard draw five sets of balls with two balls in each set. Beside the sets write "_____ sets of two" and "_____ balls." Have the children tell how many sets of two they see. Call on a child to write the correct numeral in the first blank. Now ask how many balls there are altogether. Have a child write the correct numeral in the second blank. Now ask if anyone knows of another way to write the story about the sets of two. Write on the chalkboard "5 × 2 = ☐." Read the equation as "Five *times* two is how many?" Point out to the children that the two tells how many are in each set and that the five tells how many sets of two there are. Ask how many there are altogether in five sets of two. Have a child write the numeral 10 in the placeholder. Say that *multiplication* is used to find the number of objects in a group of equivalent sets. Repeat this procedure with four sets of two, three sets of two, two sets of two, one set of two, and finally with zero sets of two. For each set story, have the children write the related multiplication equation. Speak in a casual manner about the *factors* and *products* in the multiplication equations. Do not insist that the children use these terms. Rather, encourage them to think in terms of sets.



▶ Arrange three sets of two books each on a low table. Ask a child to go to the chalkboard and write a multiplication equation which tells the story about the number of books he sees in the sets on the table. Vary the activity by laying out four sets of two pencils, two sets of two pieces of chalk, and so forth.

▶ Use work page S-327 to give the children additional practice in working with equivalent sets and multiplication equations. Discuss the sets on this



Name

Complete each equation.

0 sets of 2

0 × 2 = 0

1 set of 2

1 × 2 = 2

2 sets of 2

2 × 2 = 4

3 sets of 2

3 × 2 = 6

4 sets of 2

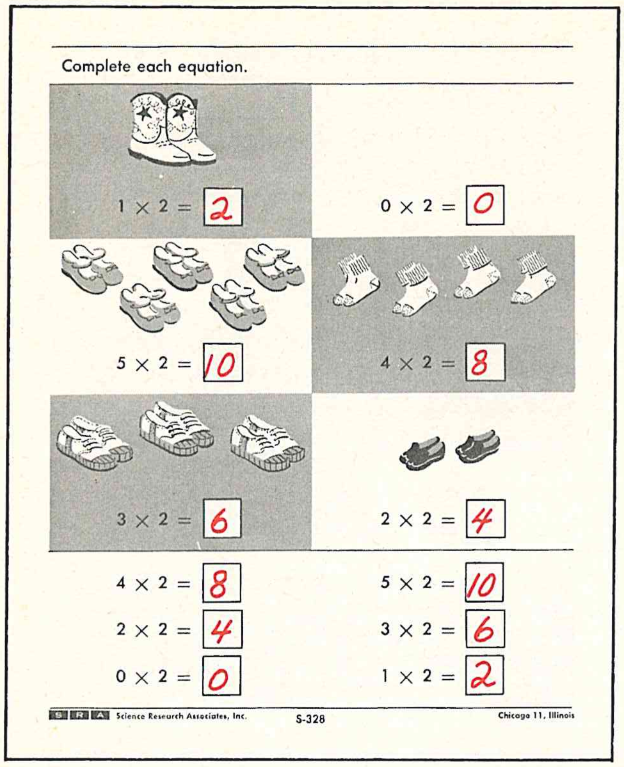
4 × 2 = 8

5 sets of 2

5 × 2 = 10

S-327

TOMATOES
TOMATOES
Peaches
Peaches
PEARS
PEARS
Pineapple
Pineapple
Cherries
Cherries
GRAPEFRUIT
GRAPEFRUIT
Orange Juice
Orange Juice
Tomato Juice
Tomato Juice
Prune Juice
Prune Juice
Apple Juice
Apple Juice
OLIVES
OLIVES
PEAS
PEAS
Lima Beans
Lima Beans
GREEN BEANS
GREEN BEANS
SPINACH
SPINACH



Complete each equation.

$1 \times 2 = \boxed{2}$ $0 \times 2 = \boxed{0}$

$5 \times 2 = \boxed{10}$ $4 \times 2 = \boxed{8}$

$3 \times 2 = \boxed{6}$ $2 \times 2 = \boxed{4}$

$4 \times 2 = \boxed{8}$ $5 \times 2 = \boxed{10}$

$2 \times 2 = \boxed{4}$ $3 \times 2 = \boxed{6}$

$0 \times 2 = \boxed{0}$ $1 \times 2 = \boxed{2}$

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page. Help them read the statement and equation in each row. Point out that the first numeral of each multiplication equation refers to the number of sets and the second numeral refers to the number in each set. Direct them to complete each equation.

Use work page S-328 in a similar fashion. If, when they reach the bottom of the page, any children have difficulty completing the equations without the help of sets, direct their attention to the sets at the top of the page or have them draw their own sets.

Supplemental Experiences

▶ Construct a set of cards which bear such incomplete equations as $1 \times 2 = \square$, $2 \times 2 = \square$, and so forth. Provide each child with counters, plastic numerals, and other symbols. Hold up one of the cards. Have each child lay out the correct number of sets with the correct number of counters in each set and then construct his own equation with the plastic numerals, and operation and relation symbols.

▶ Construct a second set of cards which bear pictures of from zero to five sets of two. Have the children match cards from this set with cards from the set bearing the incomplete multiplication equations. When they have successfully matched the cards, let them complete the equations.

Multiplication

Work Pages S-329 and S-330

Objectives

To teach the children to find the missing factors in multiplication equations.

To develop readiness for the understanding of division.

Fundamentals

After the child has learned to find the product of two factors, he will be asked to apply this knowledge to find one factor when the other factor and the product are given. Although a child can easily memorize multiplication facts for the twos, he must really understand the multiplication operation in order to find a missing factor. The teacher, aware of the need for complete understanding on the child's part, will encourage him to use all his faculties to develop this understanding. She will also recognize that in finding missing factors the child is developing readiness for the understanding of division.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Multiplication of Whole Numbers.

Readiness for Understanding

Understanding of multiplication in terms of sets.

Ability to determine the product of any two factors from one times two through five times two.

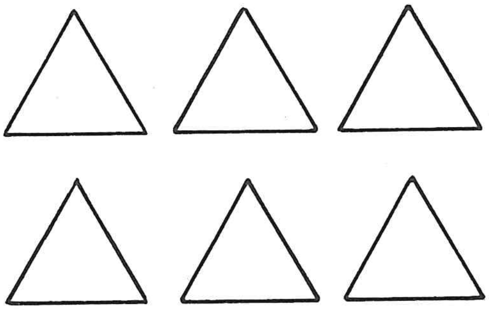
Developmental Experiences

MATERIALS	VOCABULARY
counters	missing factor
plastic numerals and other symbols	
flannel board	
felt cutouts	

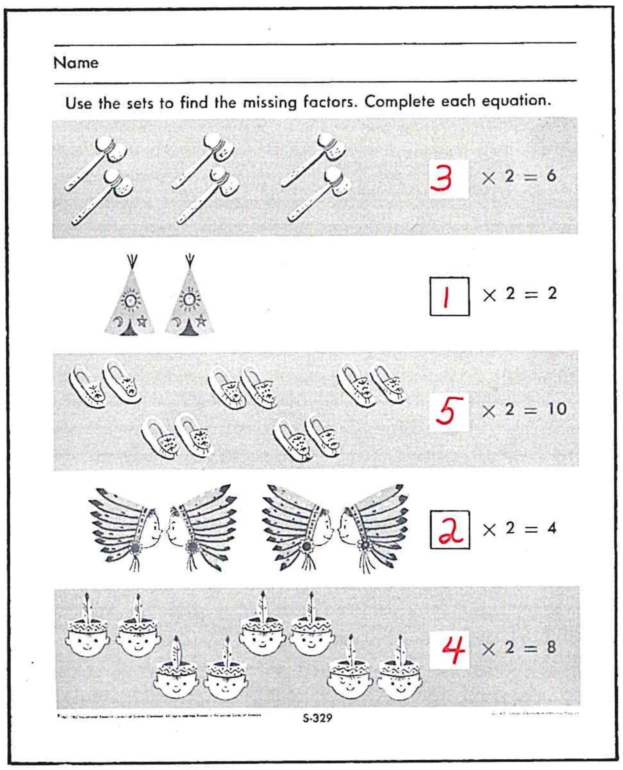
▶ Provide each child with a set of counters, and plastic numerals, and other symbols. Write the equation $\square \times 2 = 8$ on the chalkboard. Direct the children to lay out eight counters. Ask, "How many sets of two are there in a set of eight?" Help them see that they can find the answer to this question by arranging the set of eight into sets of two. Help them build the equation which goes with their set story: $4 \times 2 = 8$. Repeat this activity with other multiples of two.

Vary the activity. Direct the children to lay out ten counters. Say, "I would like you to make five sets, with the same number of counters in each set." This is a more difficult activity because a trial and error method is involved. After each child has succeeded in arranging his ten objects in five sets of two, have him build the multiplication equation which goes with this set story: $5 \times 2 = 10$. Repeat the activity with sets of eight, six, four, two, and zero.

▶ Draw a set of six triangles on the chalkboard. Beside the triangles write the equation $\square \times 2 = 6$.



Ask the children what each numeral in this equation stands for. The numeral which tells how many sets there are is missing; the 2 tells how many there are in each set; and the 6 tells how many there are altogether. Help the children determine the *missing factor*. Some children will already know the answer; others will find it helpful to circle sets of two within the set of six. They will then find that they have three sets of two and will write the numeral 3 in the placeholder. Now draw another set of six triangles or circles on the chalkboard. This time write the equation $3 \times \square = 6$ beside the set. Again have the children interpret each part of the equation. Ask, "How can we find the missing factor in this equation?" Some children will know the answer; others will draw rings around three sets by trial and error. Help them see that each set must have the same number for the multiplication operation to hold.



Name

Use the sets to find the missing factors. Complete each equation.

$\boxed{3} \times 2 = 6$

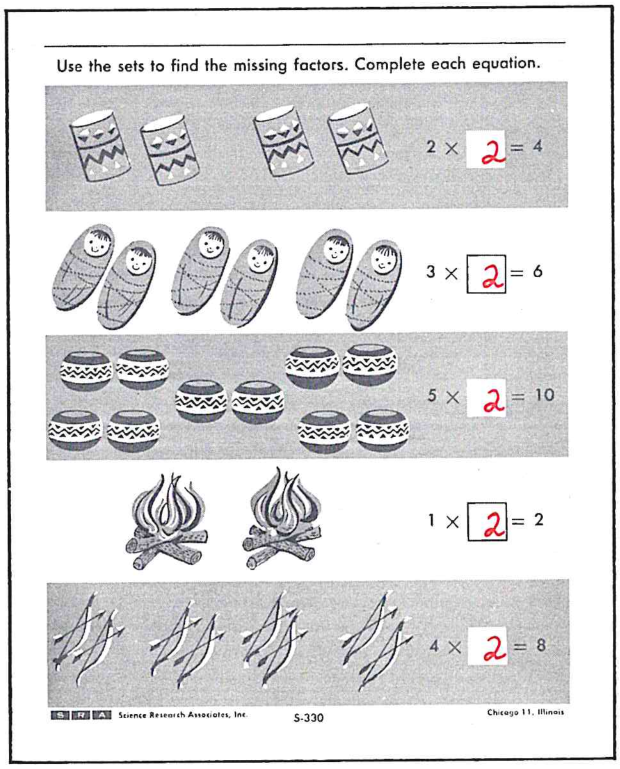
$\boxed{1} \times 2 = 2$

$\boxed{5} \times 2 = 10$

$\boxed{2} \times 2 = 4$

$\boxed{4} \times 2 = 8$

S-329



Use the sets to find the missing factors. Complete each equation.

$2 \times \boxed{2} = 4$

$3 \times \boxed{2} = 6$

$5 \times \boxed{2} = 10$

$1 \times \boxed{2} = 2$

$4 \times \boxed{2} = 8$

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▶ Use work page S-329 to give the children practice in finding the missing first factor. Direct the children to think of the meaning of each equation and then to use the sets to help them find the missing factor. After they have circled sets of two in each given set, direct them to write the correct numeral in the placeholder.

Use work page S-330 to give the children practice in finding the missing second factor. Have them read each equation and think about what each part of the equation means. Then, have them use the sets to find the missing factor. Remind them that all of the sets they circle must have the same number of objects.

Supplemental Experiences

▶ Construct a set of cards bearing multiplication equations with missing factors, in either the first or second position.

$$1 \times \square = 2 \qquad \square \times 2 = 2$$
$$\square \times 2 = 6 \qquad 3 \times \square = 6$$

Have the children come to the front of the room and line up to form two teams. Hold up one card and let the first child on one team supply the missing factor. If he gives the correct answer, let him return to his seat. If he fails to give the correct answer, he must go to the end of his team's line. Then hold up another card and let the first child on the other team supply the answer. As soon as all the children on one team have answered correctly, and returned to their seats, they are declared the winning team, and the game is over.

Multiplication

Work Pages S-331 and S-332

Objective

To help the children think of multiplication as repeated addition.

Fundamentals

When the children understand multiplication in terms of sets, they are ready to interpret certain multiplication combinations as repeated addition. The children have already learned that, although addition is a binary operation, the associative property of addition allows them to consider addition problems with more than two addends. For example, 3×2 can be interpreted as the sum of three twos, or $2 + 2 + 2$. Even though the repeated-addition interpretation of multiplication does not apply in such cases as 0×2 or 1×2, it does give the children a convenient method for finding products when they do not have sets available to help them. Further, it is related to something they already know. Although the children may at first find it easier to derive the sum of five twos by adding directly or by skip counting by twos, they will soon find that multiplication, 5×2, is faster and more convenient.

See Key Topics in Mathematics for the Primary Teacher: Multiplication of Whole Numbers.

Readiness for Understanding

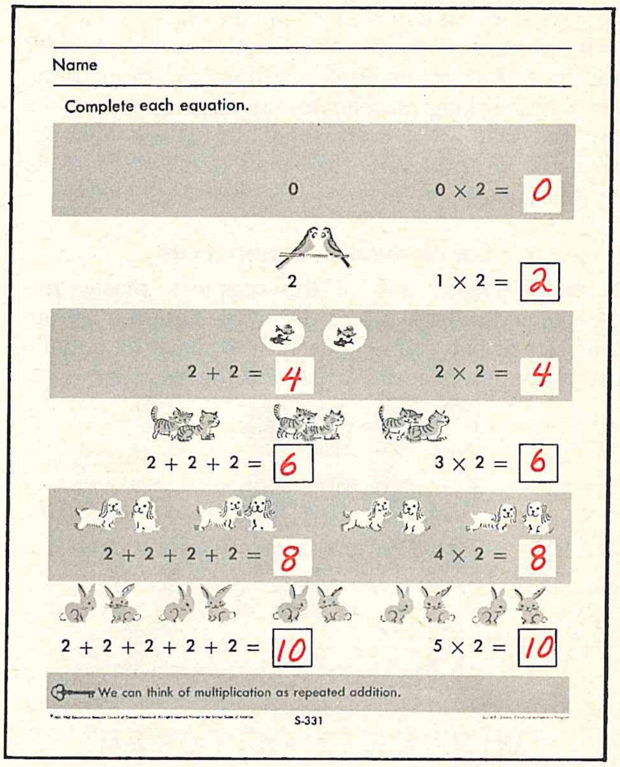
Ability to recognize equivalent sets.

Knowledge of addition combinations through sums of ten.

Developmental Experiences

MATERIALS	VOCABULARY
flannel board felt cutouts	repeated addition

▶ Put two sets of three cutouts each on the flannel board. Ask the children how many are in each set and how many there are altogether. Call on one of the children to write the corresponding multiplication equation on the chalkboard. Ask, "What other way could we have found out how many there are altogether?" Some child will suggest adding three and three. Have the equation $3 + 3 = 6$ written on the chalkboard. Repeat the activity with three sets of two. After the multiplication equation is written, ask, "Could we have added to find out how many there are here?" Lead the children to see that two plus two plus two also gives six. Ask the children, "What multiplication equation gives the same result?" "Which is easier to write?" "Which is easier to think about?" Anticipate that some children will prefer one method, some the other. Do not try to force the children into any one way of thinking.



Name

Complete each equation.

0 $\quad 0 \times 2 = \boxed{0}$

2 $\quad 1 \times 2 = \boxed{2}$

$2 + 2 = \boxed{4} \quad 2 \times 2 = \boxed{4}$

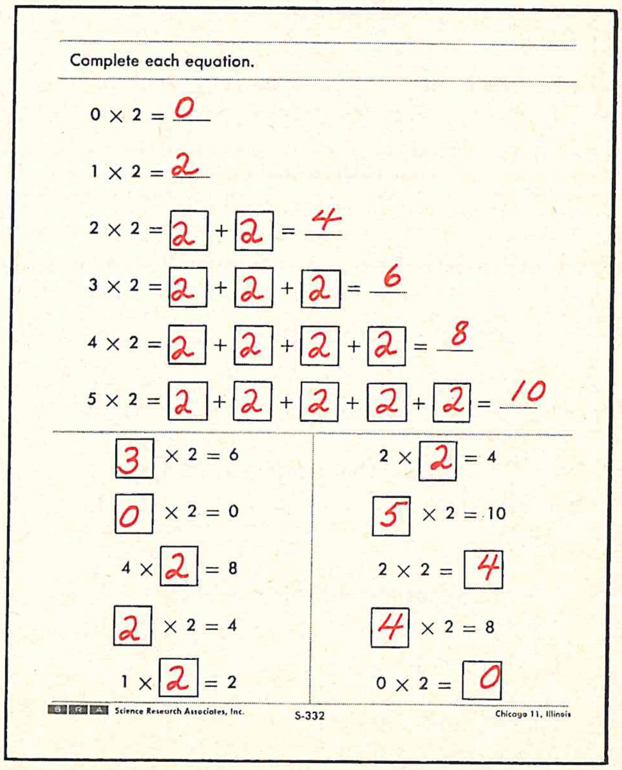
$2 + 2 + 2 = \boxed{6} \quad 3 \times 2 = \boxed{6}$

$2 + 2 + 2 + 2 = \boxed{8} \quad 4 \times 2 = \boxed{8}$

$2 + 2 + 2 + 2 + 2 = \boxed{10} \quad 5 \times 2 = \boxed{10}$

We can think of multiplication as repeated addition.

S-331



Complete each equation.

$0 \times 2 = \underline{0}$

$1 \times 2 = \underline{2}$

$2 \times 2 = \boxed{2} + \boxed{2} = \underline{4}$

$3 \times 2 = \boxed{2} + \boxed{2} + \boxed{2} = \underline{6}$

$4 \times 2 = \boxed{2} + \boxed{2} + \boxed{2} + \boxed{2} = \underline{8}$

$5 \times 2 = \boxed{2} + \boxed{2} + \boxed{2} + \boxed{2} + \boxed{2} = \underline{10}$

$\boxed{3} \times 2 = 6$

$\boxed{0} \times 2 = 0$

$4 \times \boxed{2} = 8$

$\boxed{2} \times 2 = 4$

$1 \times \boxed{2} = 2$

$2 \times \boxed{2} = 4$

$\boxed{5} \times 2 = 10$

$2 \times 2 = \boxed{4}$

$\boxed{4} \times 2 = 8$

$0 \times 2 = \boxed{0}$

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▶ Use work page S-331 to reinforce the idea that multiplication can, in some instances, be thought of as *repeated addition*. Point out that in the case of 0 × 2 and 1 × 2 there is no addition since addition can be performed only on two addends or, by using the associative property, on more than two addends. In each row, have the children use the sets to help them complete the equations.

Work page S-332 provides more experiences with multiplication as repeated addition and more practice with working placeholder equations. Remind the children that when more than one placeholder appears in an equation, the same numeral must be written in each.

Supplemental Experiences

▶ Make further use of the counters, plastic numerals, and other symbols. Have the children lay out four sets of two counters each. Now have them build two different equations which tell the set story.

$$2 + 2 + 2 + 2 = 8$$
$$4 \times 2 = 8$$

Multiplication

MULTIPLES OF THREE

Work Pages S-333 Through S-336

Objectives

To develop an understanding of multiples of three, from zero times three through five times three.

Fundamentals

As the next step in the careful development of multiplication, the child will work with sets of three. He will determine the cardinal number of a new set formed from a given number of sets of three. He will also interpret these products in terms of repeated addition.

See Key Topics in Mathematics for the Primary Teacher: Multiplication of Whole Numbers.

Readiness for Understanding

Understanding of the multiples of two, through five times two.

Developmental Experiences

MATERIALS	VOCABULARY
flannel board felt cutouts plastic numerals and other symbols	set of three

▶ Place one *set of three* objects on the flannel board. On the chalkboard, write "______ set of three." Have a child fill in the blank with the numeral which tells how many sets of three he sees, and then tell how many objects there are altogether. Let another child write the multiplication equation which shows what has been learned from the set story: 1 × 3 = 3.

Place two sets of three objects on the flannel board. On the chalkboard, write "______sets of three." Have a child tell the set story and fill in the blank with the correct numeral. Ask how many objects there are all together. Have a child write the related multiplication equation on the board to show what has been learned from the set story: 2 × 3 = 6. The class can use their plastic numerals and other symbols at their desks to show the same equation. Ask if anyone can show another way to write a number story about the sets. Guide them to give the related addition equation: 3 + 3 = 6.

To develop an understanding of the multiplication combination 3 × 3, have a child place three sets of three objects on the flannel board. Ask another child to write on the chalkboard the related multiplication equation which tells the set story: 3 × 3 = 9. Ask if anyone can write the story in another way: 3 + 3 + 3 = 9. Continue this procedure through the multiplication combinations 4 × 3 and 5 × 3.

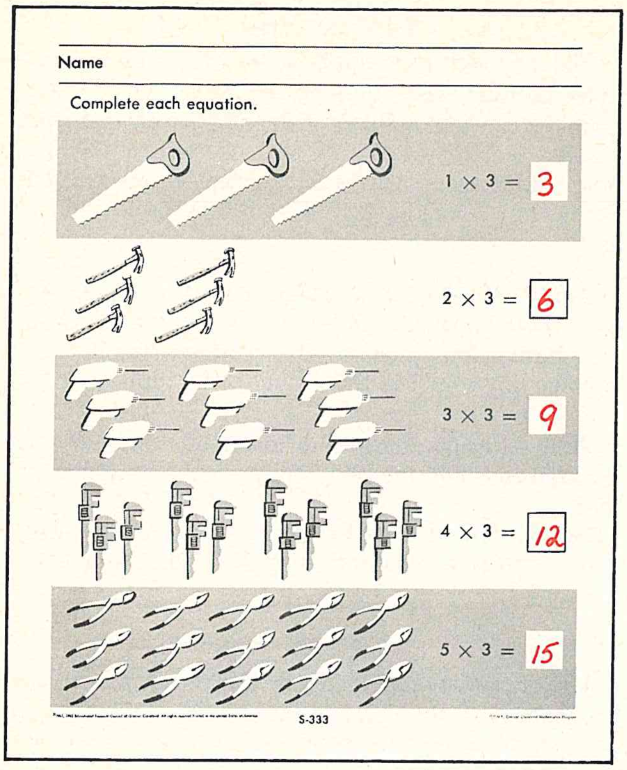
▶ To teach the concept of 0 × 3 = 0, ask questions such as "How many sets of three tigers are in the room?" Write the related multiplication equation which shows this: 0 × 3 = 0. Have the children use the plastic numerals and other symbols at their desks to show this combination.

▶ Use sets to develop skill in solving placeholder equations involving a missing factor. Write the equation ☐ × 2 = 6 on the chalkboard. Call on a child to select six objects and arrange them in sets of two. Ask him to tell how many sets of two he found in the set of six. Have another child complete the equation and read it aloud.

Write 2 × ☐ = 6 on the chalkboard. Let a child select six objects and arrange them into two equivalent sets to find how many are in each set. Have another child complete the equation and read it to the class.

▶ Use work page S-333 to give the children practice with multiplication combinations from 1 × 3 through 5 × 3. Remind them that the first numeral of the equation refers to the number of sets and the second numeral refers to the number in each set.

Work page S-334 provides practice in relating multiplication to repeated addition. Help the children see that 0 × 3 and 1 × 3 cannot be interpreted as repeated addition, since two numbers are needed for any addition.



Name

Complete each equation.

1 × 3 = 3

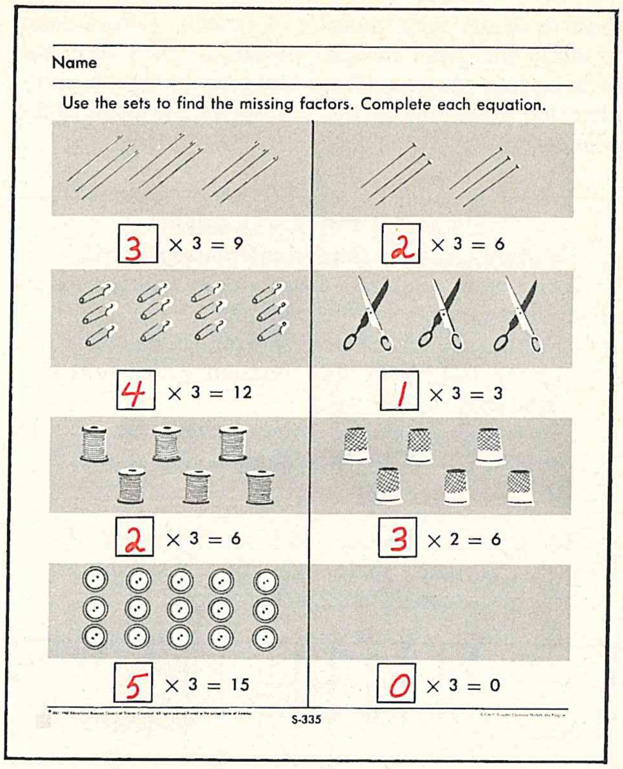
2 × 3 = 6

3 × 3 = 9

4 × 3 = 12

5 × 3 = 15

S-333



Name

Use the sets to find the missing factors. Complete each equation.

3 × 3 = 9

2 × 3 = 6

4 × 3 = 12

1 × 3 = 3

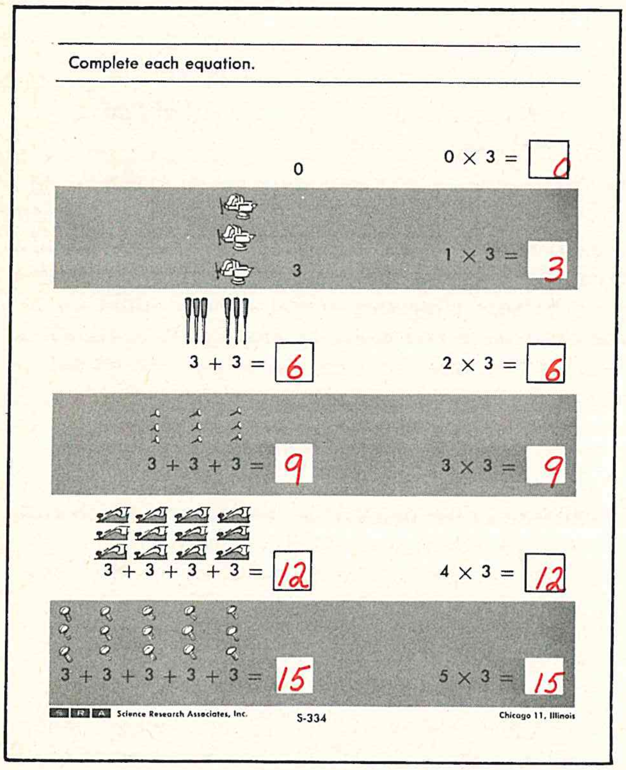
2 × 3 = 6

3 × 2 = 6

5 × 3 = 15

0 × 3 = 0

S-335



Complete each equation.

0

0 × 3 = 0

3

1 × 3 = 3

3 + 3 = 6

2 × 3 = 6

3 + 3 + 3 = 9

3 × 3 = 9

3 + 3 + 3 + 3 = 12

4 × 3 = 12

3 + 3 + 3 + 3 + 3 = 15

5 × 3 = 15

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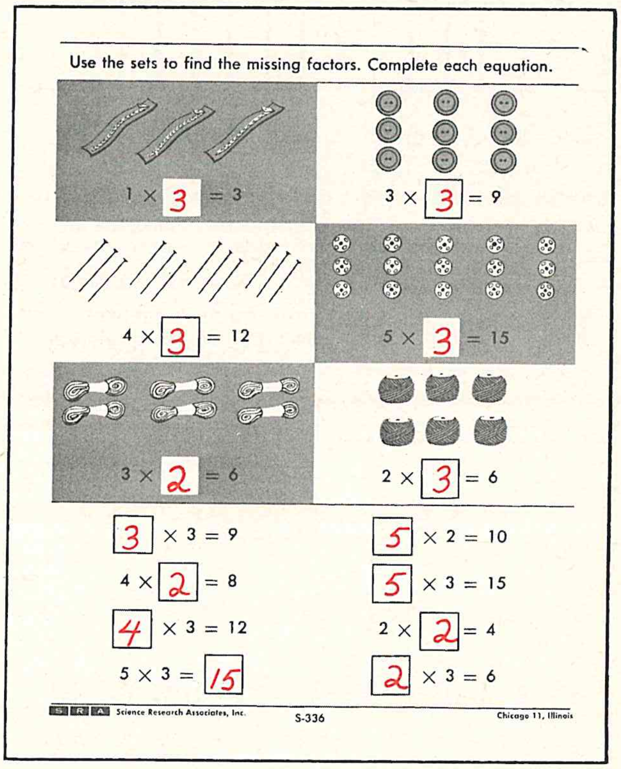

Use the sets to find the missing factors. Complete each equation.

1 × 3 = 3

3 × 3 = 9

4 × 3 = 12

5 × 3 = 15

3 × 2 = 6

2 × 3 = 6

3 × 3 = 9

5 × 2 = 10

4 × 2 = 8

5 × 3 = 15

4 × 3 = 12

2 × 2 = 4

5 × 3 = 15

2 × 3 = 6

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S-336

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▶ Use work page S-335 to develop skill in finding the missing first factor in a multiplication equation. Remind the children that they are trying to find the number of sets when the total number of objects and the number of objects in each set are given. Have them circle sets of the given number in each box and complete the equations. If some children have difficulty, let them manipulate concrete objects on their desks.

At the top of work page S-336, the second factor in each equation is missing. Tell the children that they know how many sets there are and how many objects there are all together; they must find how many are in each set. Remind them that each set must have the same number of objects. Direct them to circle the given number of sets in each box and to complete the equations. Have them try to complete the equations at the bottom of the page independently.

Supplemental Experiences

▶ To strengthen the important concept of 0 × 3 = 0, let some children create funny stories which illustrate the equation. For example:

> How many sets of three purple giants are in the room? (0) Write the equation which tells this fact: 0 × 3 = 0.
> How many sets of three whales are on the street? (0) Write the equation that shows this: 0 × 3 = 0.

▶ To provide practice with multiples of three, through five times three, play this baseball game: Draw a scoreboard on the chalkboard.

											TOTAL
Indians	/										
Yankees	0										
Giants	/										

Let three (or more) groups of children each select a name for their "baseball team" One child "pitches the ball" to the teams by calling out multiplication problems or by holding up cards on which are written placeholder equations such as 0 × 3 = □, 3 × □ = 6, or □ × 3 = 15. In order to score one run, the team must quickly answer each problem correctly. If one child answers incorrectly, his team scores 0 for that inning. After nine innings, total the scores. This game provides strong motivation for learning the multiplication facts.

Multiplication

MULTIPLES OF FOUR

Work Pages S-337 Through S-340

Objectives

To develop an understanding of multiples of four, from zero times four through five times four.

To review multiplication problems with a missing factor.

To develop skill in finding products through twenty.

Fundamentals

The next step in multiplication will deal with multiples of four. The children will be asked to determine the cardinal number of the new set formed from a given number of sets of four. Certain multiplication combinations will be interpreted as repeated addition. For example, 2 × 4 = 8 may be thought of as 4 + 4 = 8.

Again the children will be called upon to extend their knowledge to find the missing factor in a multiplication equation. The readiness for division is being developed in the setting of multiplication.

Skills in using multiples of two and three must be maintained while the children learn the multiples of four.

See Key Topics in Mathematics for the Primary Teacher: Multiplication of Whole Numbers.

Readiness for Understanding

Understanding the multiples of two and three, from zero times two through five times three.

Understanding that certain products can be shown as repeated addition.

Skill in finding the missing factor.

Developmental Experiences

MATERIALS	VOCABULARY
plastic numerals and other symbols	set of four

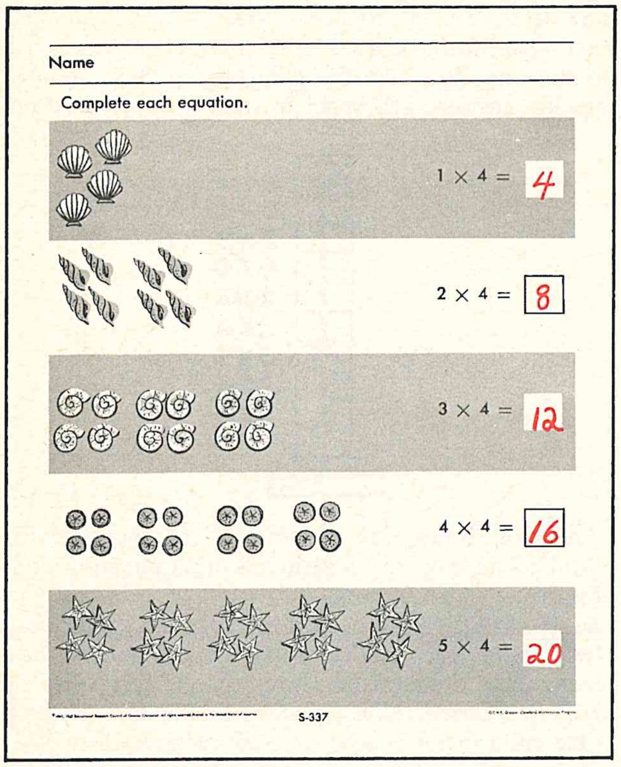
▶ Draw one set of four objects on the chalkboard. Write on the board "________ *set of four*." Direct a child to write in the blank the numeral which tells how many sets he sees, and to tell how many objects he sees altogether. Have another child go to the chalkboard and write the multiplication equation which shows what he has learned from the set story: 1 × 4 = 4.

Draw two sets of four objects on the chalkboard. Write beside the sets "________ sets of four." Have a child fill in the blank with the numeral which tells how many sets he sees: 2. Have another child write the related multiplication equation which shows what he has learned from the set story: 2 × 4 = 8. Ask a third child to show how to write the number story in another way: 4 + 4 = 8.

Continue this procedure in developing understanding of the multiplication combinations of three times four, four times four, five times four. Direct the children to write each number story in two ways, as a multiplication equation and as a repeated-addition equation.

▶ Use work page S-337 to provide practice in finding products of one times four through five times four. Direct the children to complete each multiplication equation.

Work page S-338 is designed to help the children relate certain multiplication combinations to repeated addition. Point out that repeated addition does not apply to the first two problems, since two numbers are needed for addition. Direct the children to complete each equation, being careful to read the operation signs correctly.


Name

Complete each equation.

1 × 4 = 4

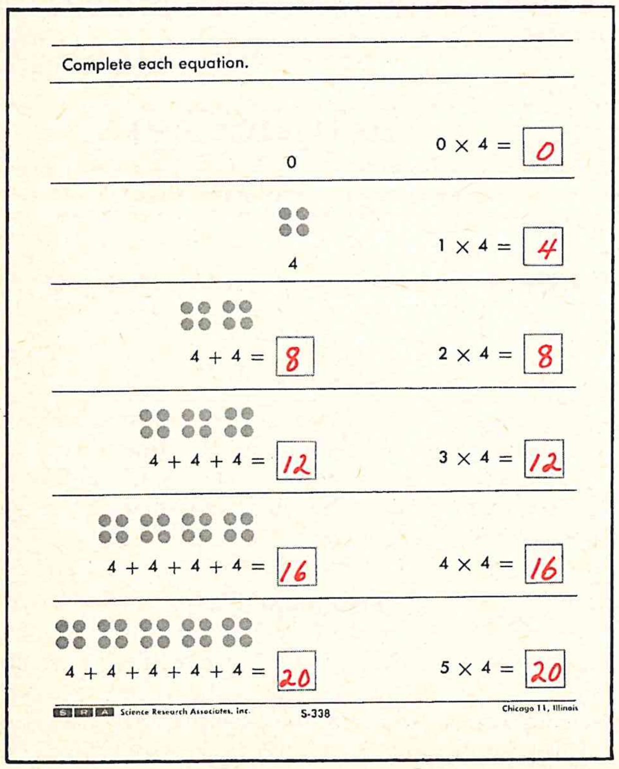
2 × 4 = 8

3 × 4 = 12

4 × 4 = 16

5 × 4 = 20

S-337


Complete each equation.

0

0 × 4 = 0

4

1 × 4 = 4

4 + 4 = 8

2 × 4 = 8

4 + 4 + 4 = 12

3 × 4 = 12

4 + 4 + 4 + 4 = 16

4 × 4 = 16

4 + 4 + 4 + 4 + 4 = 20

5 × 4 = 20

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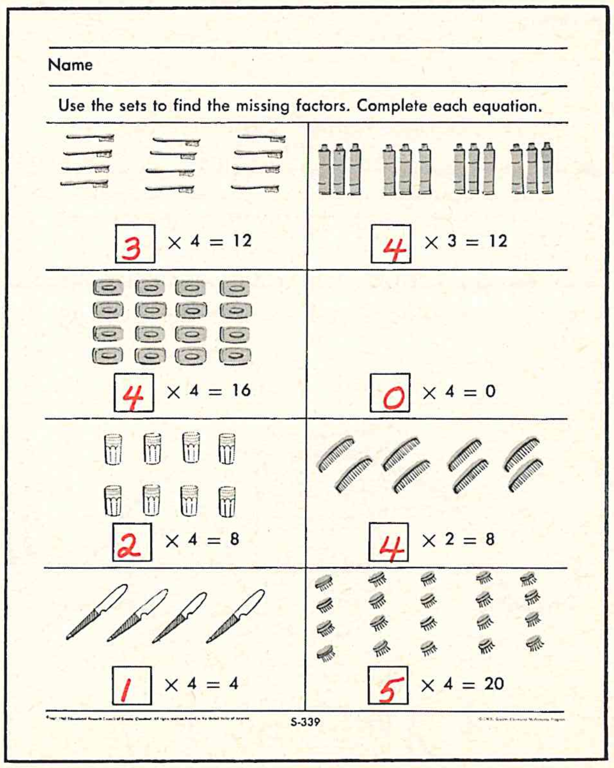
▶ Write the placeholder equation □ × 4 = 8 on the chalkboard. Ask what must be found to make this a true statement. Place eight books on a table and ask a child to arrange them in sets of four. Have him tell how many sets of four are in a set of eight and then write in the placeholder the numeral 2. Have another child read the completed equation. Repeat the activity with other equations related to multiples of two, three, and four; use other objects, such as pencils, crayons, children, and chairs.

Write the placeholder equation 4 × □ = 8 on the chalkboard. Ask the class to tell what they must find to complete this equation correctly. Place eight books on a table and ask a child to arrange them into four sets with the same number in each set. After he has made four equivalent sets, have him tell the number of books in each set. Have another child complete the equation and read it to the class. Repeat the activity with other equations in which the second factor is missing.

▶ Use work page S-339 to give practice in finding a missing first factor in multiplication equations. Direct the children's attention to the first box. Ask, "How many sets of four are in twelve?" Help them circle sets of four, determine how many they have, and complete the equation. Encourage the children to complete the page by themselves.


Name

Use the sets to find the missing factors. Complete each equation.

3 × 4 = 12

4 × 3 = 12

4 × 4 = 16

0 × 4 = 0

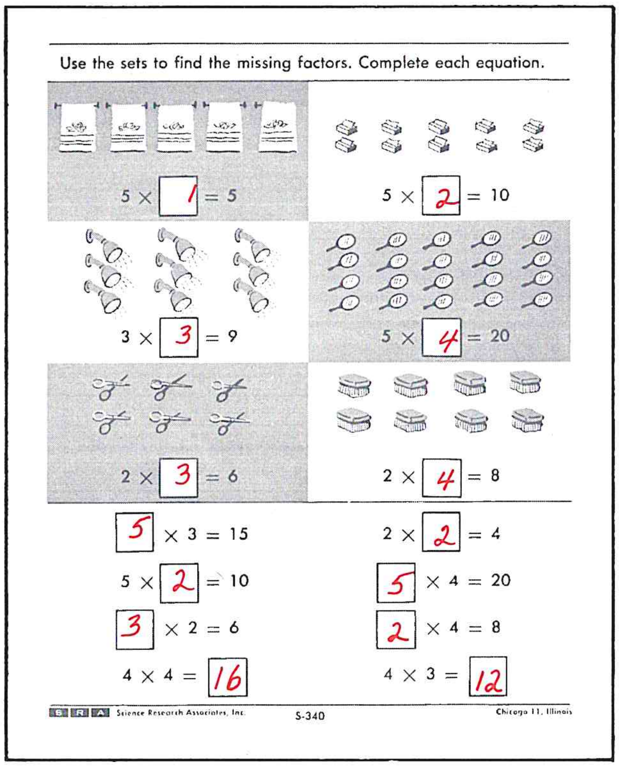
2 × 4 = 8

4 × 2 = 8

1 × 4 = 4

5 × 4 = 20

S-339



Use the sets to find the missing factors. Complete each equation.

5 × $\boxed{1}$ = 5

5 × $\boxed{2}$ = 10

3 × $\boxed{3}$ = 9

5 × $\boxed{4}$ = 20

2 × $\boxed{3}$ = 6

2 × $\boxed{4}$ = 8

$\boxed{5}$ × 3 = 15

2 × $\boxed{2}$ = 4

5 × $\boxed{2}$ = 10

$\boxed{5}$ × 4 = 20

$\boxed{3}$ × 2 = 6

$\boxed{2}$ × 4 = 8

4 × 4 = $\boxed{16}$

4 × 3 = $\boxed{12}$

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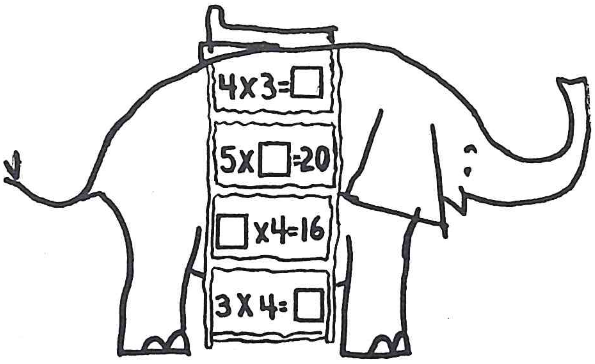
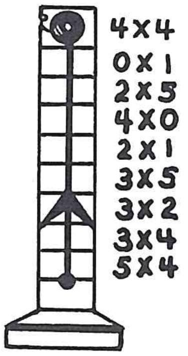
Work page S-340provides an opportunity for the children to find the missing second factor in a multiplication equation. In each box at the top of the page have the children circle the indicated number of equivalent sets, determine the number in each set, and complete the placeholder equation. At the bottom of the page have them complete each equation. If they have difficulty, have them draw pictures of sets on a separate piece of paper and circle subsets to find the missing factors or combine the sets to find products.

Supplemental Experiences

▶ Play the game "Ride the Elephant." Have a child draw an elephant on the board. Let another child draw a ladder up to the top of the animal's back. On the steps of the ladder write multiplication problems, with missing factors, which the class has studied. One child uses a pointer to help the rider up the steps. An incorrect answer carries two penalties—no ride and the order to use sets to find the products. If successful, the contestant then becomes the helper.



▶ On the chalkboard draw a picture of a carnival strength-testing machine with a gong on top. Direct the children to place multiplication problems on each line up to the top of the machine. Tell the children that power must be used to drive the sounding device to the top. To strike the gong, the contestant must quickly answer each problem starting at the bottom.



An error shows lack of strength and motivates his need to develop his skill in the multiplication combinations which he missed.

▶ If some children need more concrete examples to help them learn multiplication combinations, have them draw their own pictures of sets and write the related multiplication equation beside the set story. One child might choose to draw pictures of two sets of four cats, writing the related multiplication equation beside the set story: 2 × 4 = 8. Another child might choose to draw pictures of five sets of four dogs. After determining the total number of dogs in the set story, he then tells the story in a mathematical sentence, 5 × 4 = 20.

Multiplication

MULTIPLES OF FIVE

Work Pages S-341 Through S-344

Objectives

To develop an understanding of the multiples of five, from zero times five through five times five.

To interpret certain products as repeated addition.

To provide practice in solving multiplication equations with a missing factor.

Fundamentals

Now the child is ready to work with sets of five. As he combines different numbers of these sets, the child determines the cardinal number for each new set formed. He will find, for example, that three sets of five can be combined to form a set of fifteen. He will be asked to write the related multiplication equation, $3 \times 5 = 15$, and to interpret the set story as repeated addition: $5 + 5 + 5 = 15$.

He will be asked to find, for example, how many sets of five are contained in a set of twenty. Next the child will be asked to find a missing factor when one factor and the product are known. Such exercises serve to strengthen the child's understanding of the multiplication operation, and to build readiness for an understanding of division.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Multiplication of Whole Numbers.

Readiness for Understanding

Understanding of the multiples (from zero through twenty) of two, three, and four.

Skill in solving multiplication problems with one missing factor.

Developmental Experiences

MATERIALS

bead frame
beads

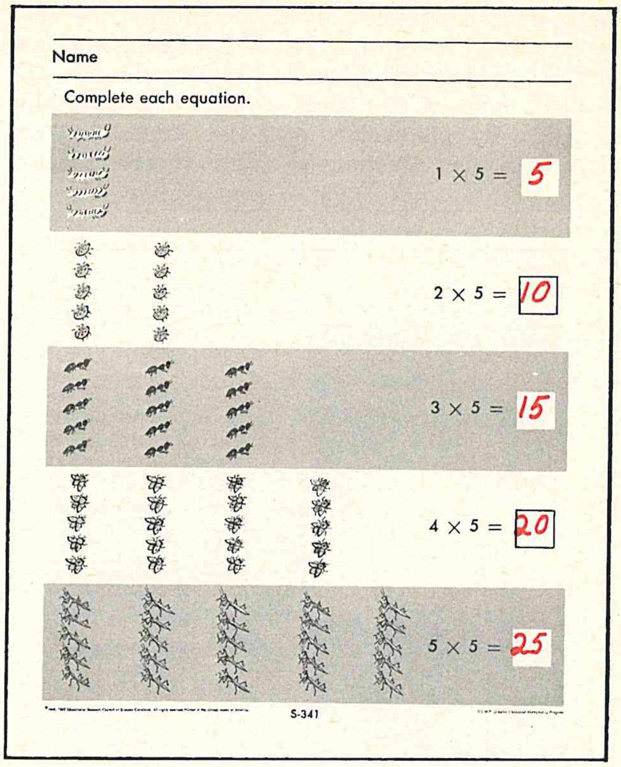
▶ Direct one child to place a set of five beads on one rod of the bead frame. Ask another child to write the related multiplication equation on the chalkboard: $1 \times 5 = 5$. Place one more set of five beads on another rod of the bead frame. Call on a child to tell how many sets of five he now sees and to write the related multiplication equation: $2 \times 5 = 10$. Ask if there is another way to show the set story. Call on a child to write the addition equation, $5 + 5 = 10$, beside the related multiplication equation on the chalkboard.

Continue the procedure with three, four, and five sets of five beads on the bead frame. Have the multiplication and repeated addition equations written each time. Now remove all the beads from the bead frame and ask a child how many sets of five he sees. Have the related equation, $0 \times 5 = 0$, written on the chalkboard.

▶ Use work page S-341 to provide additional practice in finding the products one times five through five times five. Discuss the pictures with the children and then tell them to complete each equation on the page.

Work page S-342 will give the children further opportunity to relate certain products to repeated addition. Direct them to complete each equation on the page.

▶ Write the equation $\square \times 5 = 15$ on the chalkboard. Ask a child to count out fifteen beads and to arrange them on the bead frame in sets of five. After he has completed this, ask how many sets he has, and have the equation completed. Repeat the activity with other equations in which the first factor is missing. Remind the children that this factor tells the number of sets that are involved.



Name

Complete each equation.

$1 \times 5 = \boxed{5}$

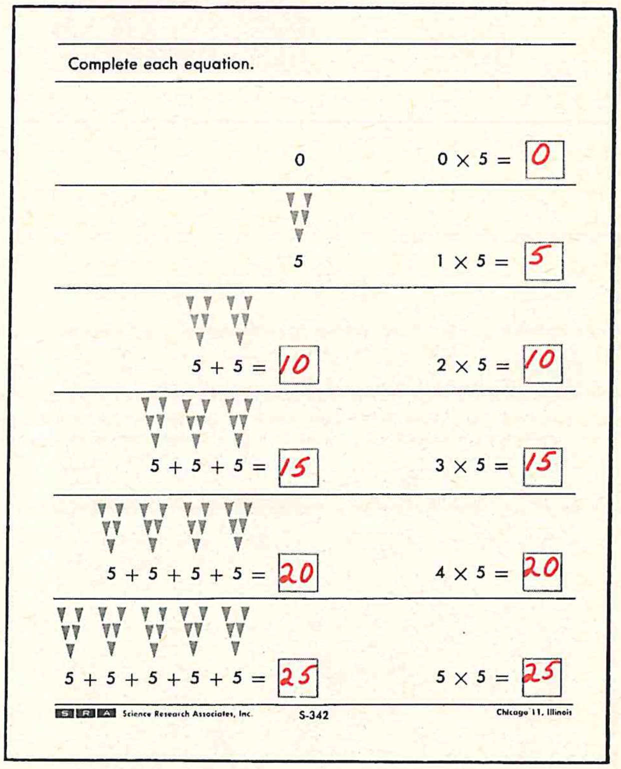
$2 \times 5 = \boxed{10}$

$3 \times 5 = \boxed{15}$

$4 \times 5 = \boxed{20}$

$5 \times 5 = \boxed{25}$

S-341



Complete each equation.

0 — $0 \times 5 = \boxed{0}$

5 — $1 \times 5 = \boxed{5}$

$5 + 5 = \boxed{10}$ — $2 \times 5 = \boxed{10}$

$5 + 5 + 5 = \boxed{15}$ — $3 \times 5 = \boxed{15}$

$5 + 5 + 5 + 5 = \boxed{20}$ — $4 \times 5 = \boxed{20}$

$5 + 5 + 5 + 5 + 5 = \boxed{25}$ — $5 \times 5 = \boxed{25}$

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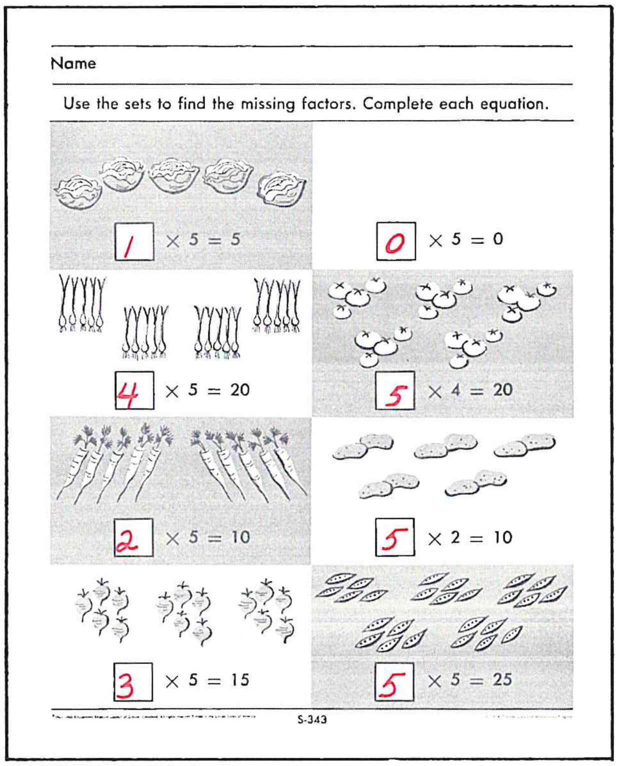
Write the equation $3 \times \square = 15$ on the chalkboard. Have a child count out fifteen beads and arrange them on the bead frame in such a way that he has three sets with the same number of beads in each set. If he knows his multiplication combination, he may immediately place five beads on each of three rods. If he is not sure of his combination, he may put one bead in turn on each of three rods until his supply is exhausted. When he has done this, he will see that there are five beads on each rod. Have him complete the placeholder equation. Repeat the activity with other equations in which the second factor is missing. Remind the children that this is the factor which tells how many are in each of the sets involved.


Name

Use the sets to find the missing factors. Complete each equation.

$\boxed{1} \times 5 = 5$

$\boxed{0} \times 5 = 0$

$\boxed{4} \times 5 = 20$

$\boxed{5} \times 4 = 20$

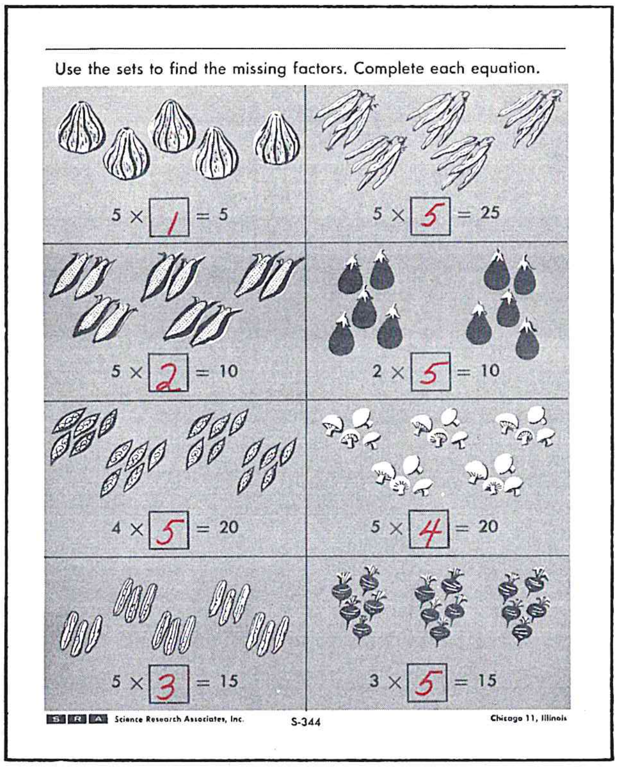
$\boxed{2} \times 5 = 10$

$\boxed{5} \times 2 = 10$

$\boxed{3} \times 5 = 15$

$\boxed{5} \times 5 = 25$

S-343


Use the sets to find the missing factors. Complete each equation.

$5 \times \boxed{1} = 5$

$5 \times \boxed{5} = 25$

$5 \times \boxed{2} = 10$

$2 \times \boxed{5} = 10$

$4 \times \boxed{5} = 20$

$5 \times \boxed{4} = 20$

$5 \times \boxed{3} = 15$

$3 \times \boxed{5} = 15$

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▶ Use work page S-343 to provide more practice in finding the missing first factor. Tell the children to circle sets of the given number in each box and to complete each equation.

On work page S-344 direct the children's attention to the first box. Tell them they are to circle five sets within the given set of five, and then have them complete the equation. Remind them that each set must have the same number of elements in it. Then direct them to circle the given number of equivalent sets in each of the remaining boxes and to complete each equation.

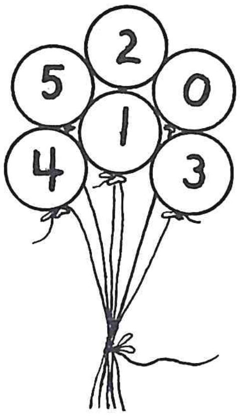
Supplemental Experiences

▶ Certain number sequences can be related to multiplication. It was seen that 3 × 4 can be thought of as 4 + 4 + 4. By extending this idea it can be seen that, by beginning with four and skip-counting by fours, the third term will be the product of three and four: 4, 8, 12.

Write several number sequences on the chalkboard. Have the children copy them and fill in the blanks.

1, 2, ____, ____, 5, ____.
2, 4, 6, ____, ____.
3, 6, ____, 12, ____.
4, 8, ____, ____, 20.
5, ____, 15, ____, 25.

▶ Have the children draw six large balloons on the chalkboard or cut them from colored paper and fasten them to the chalkboard. Write one numeral on each beginning with zero. Choose a number and tell the children that the number represented on each balloon must be multiplied by the number you selected. For example: tell the children that each number is to be multiplied by 5. Point to each balloon in turn. The child whose turn it is must try to pop the indicated balloon by quickly telling the product of five and the number indicated on that balloon. If he successfully pops all six balloons, he may choose a number for the next game and point to the balloons.



Multiplication

ONE—THE IDENTITY ELEMENT

Work Pages S-345 and S-346

Objective

To develop an understanding of one as the identity element of multiplication.

Fundamentals

The child has learned to regard one as a rather special number. One is the first counting number and it is also the number from which all other numbers can be built. In the natural order of numbers, each counting number is one more than the number which precedes it.

Now the child will learn that one has another special property. He will learn to think of it as the identity element of multiplication. He will generalize that the product of any number and one is that same number. The use of the term "identity element" is optional, but the child should become fully acquainted with the idea involved.

See Key Topics in Mathematics for the Primary Teacher: Multiplication of Whole Numbers.

Readiness for Understanding

Understanding of the set definition of multiplication.

Ability to interpret multiplication as repeated addition.

Developmental Experiences

MATERIALS

counters
plastic numerals
and other symbols

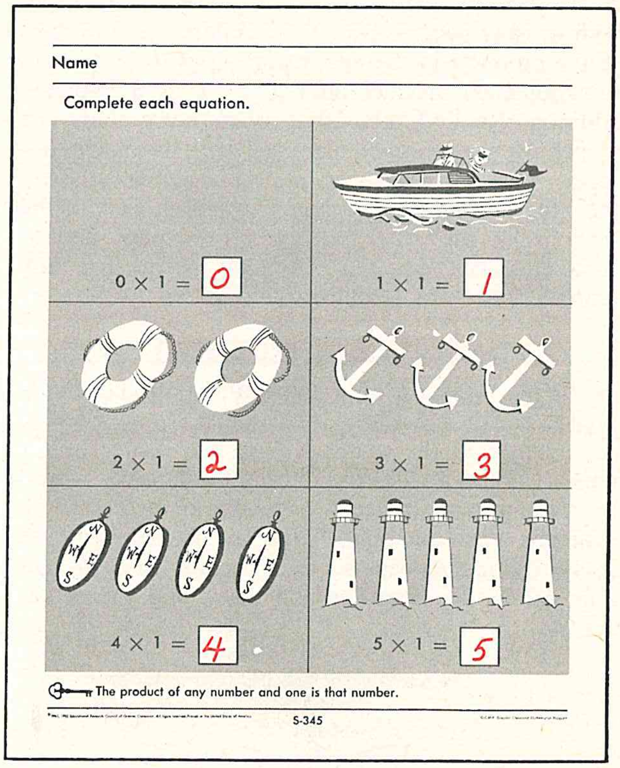
VOCABULARY

identity element
(optional)

▶ Direct the children to place one set of one counter on their desks. Have them use their plastic numerals and other symbols to build the related multiplication equation: 1 × 1 = 1. Now have the children lay out two sets with one counter in each set and determine how many counters are in both sets. Have them build the related multiplication equation: 2 × 1 = 2. Ask if there is another way to show the set story. Lead them to build the addition equation 1 + 1 = 2. Continue the procedure with three, four, five, and zero sets of one. Have the related multiplication and repeated-addition equations shown each time.

▶ Write placeholder equations on the chalkboard as shown. Have the children complete each equation by recording the product in the placeholder. When all of the equations have been completed, discuss the pattern that has been formed. Lead them to generalize that any number times one is that same number. Some children may wish to try out this idea with numbers greater than five. Encourage them to do so by using sets of counters. Have them use their plastic numerals and other symbols to make related multiplication equations.

0 × 1 = ☐
1 × 1 = ☐
2 × 1 = ☐
3 × 1 = ☐
4 × 1 = ☐
5 × 1 = ☐


Name

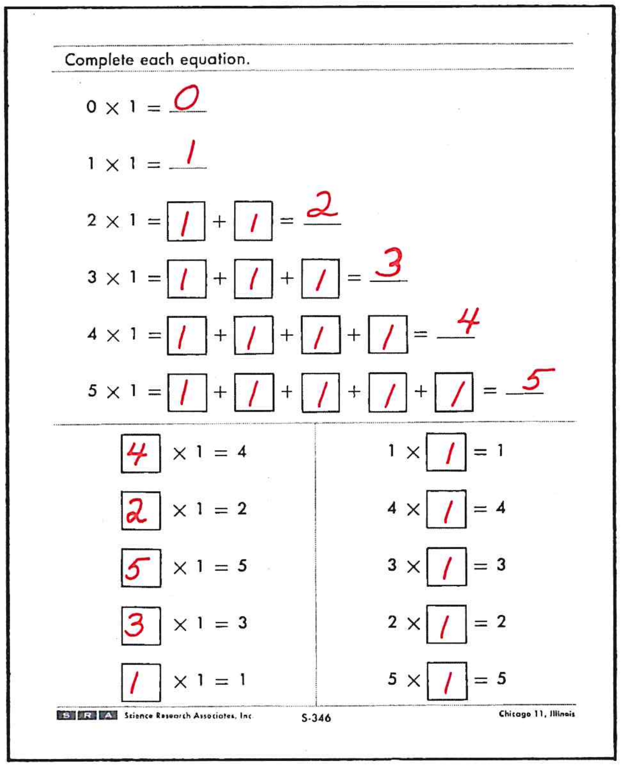
Complete each equation.

0 × 1 = 0
1 × 1 = 1
2 × 1 = 2
3 × 1 = 3
4 × 1 = 4
5 × 1 = 5

The product of any number and one is that number.

S-345

▶ Use work page S-345 to emphasize the idea that the product of any number and one is that number. Direct the children to complete each equation.



Complete each equation.

$0 \times 1 = 0$

$1 \times 1 = 1$

$2 \times 1 = \boxed{1} + \boxed{1} = 2$

$3 \times 1 = \boxed{1} + \boxed{1} + \boxed{1} = 3$

$4 \times 1 = \boxed{1} + \boxed{1} + \boxed{1} + \boxed{1} = 4$

$5 \times 1 = \boxed{1} + \boxed{1} + \boxed{1} + \boxed{1} + \boxed{1} = 5$

$\boxed{4} \times 1 = 4$

$\boxed{2} \times 1 = 2$

$\boxed{5} \times 1 = 5$

$\boxed{3} \times 1 = 3$

$\boxed{1} \times 1 = 1$

$1 \times \boxed{1} = 1$

$4 \times \boxed{1} = 4$

$3 \times \boxed{1} = 3$

$2 \times \boxed{1} = 2$

$5 \times \boxed{1} = 5$

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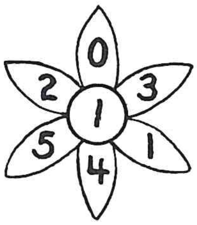
Use work page S-346 to give the children an opportunity to interpret certain multiplications as repeated additions and to apply their knowledge of one as the *identity element* to the solution of equations with missing factors. Direct the children to find each of the products in the upper portion of the page and to write each product after 1×1 as a repeated addition. Remind them that when more than one placeholder appears in an equation, the same numeral must be written in each placeholder. Direct the children to find the missing factors in each of the sections on the lower portion of the page and to complete each equation. Remind them that any number times one is that same number.

Supplemental Experiences

▶ Draw a picture of a large daisy on the chalkboard. Write a numeral, 0 through 5, on each of six petals and the numeral 1 in the center of the daisy. Call on a child to multiply the number named by the numeral on each petal by one. Success is achieved when one child "picks all the petals off the daisy" by naming each product correctly. Change the numeral in the center of the daisy and repeat the activity.



Multiplication

THE COMMUTATIVE OR ORDER PROPERTY

Work Pages S-347 and S-348

Objectives

To develop the concept of commutativity of multiplication.

To strengthen the understanding of multiplication combinations through five times five.

Fundamentals

The child has been informally exposed to the idea of commutativity of multiplication on many occasions. Perhaps he has discovered that two sets of three contain as many elements as three sets of two and that 2×3 names the same number as 3×2. The teacher will want to guide him to examine this idea more closely to determine whether it is just a coincidence. The child knows that addition is commutative and that multiplication can be thought of as repeated addition. It is therefore logical for him to think that multiplication might also be commutative. He should be given many opportunities to try out different possibilities. Eventually, he will be led to generalize that, given any two numbers, the product is the same whatever their order: that is, $a \times b = b \times a$.

The teacher will want to help the child realize that the commutative property is helpful in work with multiplication combinations. For example, if the child knows that five times three equals fifteen, he also knows that three times five equals fifteen.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Multiplication of Whole Numbers.

Readiness for Understanding

Understanding of multiplication combinations through five times five.

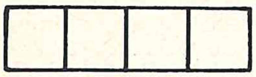
Developmental Experiences

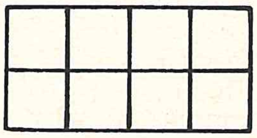
MATERIALS	VOCABULARY
tagboard	commutative property
flannel board	order property
felt cut-outs	commute
felt numerals	
felt ×, +, and = signs	

▶ Review the *commutative property* of addition. Place two felt cut-outs on the flannel board. Have a set of three cut-outs joined to this set of two. Have the children tell how many are in this set union and have them build the related addition equation:

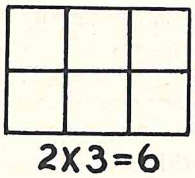
2 + 3 = 5. Now have a set of two joined to a set of three and have the related equation built: 3 + 2 = 5. Discuss with the children the meaning of the commutative property of addition. Have the children give other examples of the commutative property of addition: 4 + 1 = 5 and 1 + 4 = 5, and so forth. Ask the children whether they think multiplication might be commutative. Some child may suggest that it is, since addition is commutative and multiplication can be thought of as repeated addition. Suggest that the children use some set cards to help them find out whether or not multiplication is commutative.

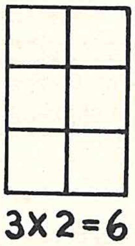
Construct a set of tagboard cards to illustrate sets related to different multiplication combinations. Mark off rows of 3″ x 3″ squares. To show the product 1 × 4, for instance, cut a card 3″ x 12″ and mark off four blocks as illustrated. To show 2 × 4, cut a card 6″ x 12″ and mark off two rows of four blocks each. Glue bits of flannel or sandpaper on the back of each card so that they will adhere to the flannel board.





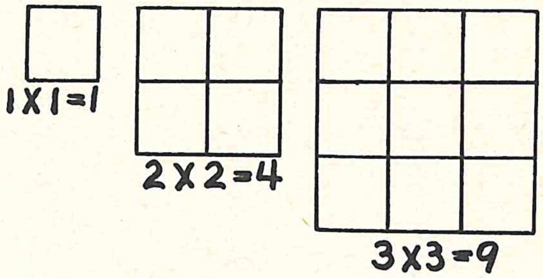
Place a card which has two rows of three blocks on the flannel board. Ask a child to tell how many rows of squares he sees and how many squares are in each row. Have him use the felt numerals and times and equal signs to build the related multiplication equation: 2 × 3 = 6. Rotate the card ninety degrees so that three rows of two blocks each are displayed. Ask a child to build the multiplication equation which relates to this set picture: 3 × 2 = 6.



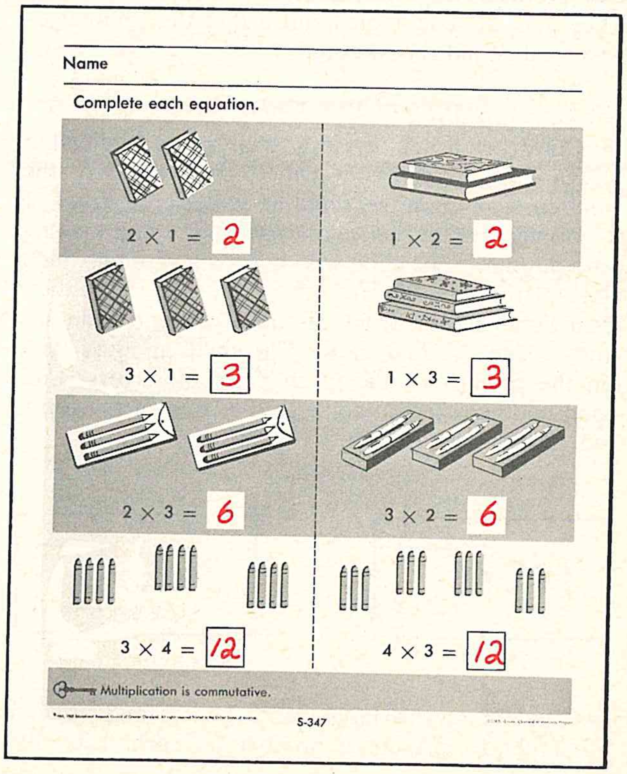


Remove the card from the flannel board and have both equations read and discussed. Some child might point out that the order of the factors has been changed but that the product is the same. Ask the children if they think this same idea is true of other pairs of numbers. Display different set cards and have the two related multiplication equations built as before. Lead the children to conclude that the order of the factors does not affect the product: multiplication is commutative.

Use the square cards from the set just described to show that in some multiplications, when both factors are the same, there is only one form of the written equation. For example, the card which has two rows of two squares each can be turned in many directions and still the equation 2 × 2 = 4 is the only one which relates to this particular set picture.



▶ Use work page S-347 to reinforce the understanding of the commutativity of multiplication. Discuss the set pictures in the two boxes in the first row and help the children complete each equation. Then direct them to complete the page independently. When this has been done discuss the order of the factors in each equation.



Name

Complete each equation.

2 × 1 = 2
1 × 2 = 2

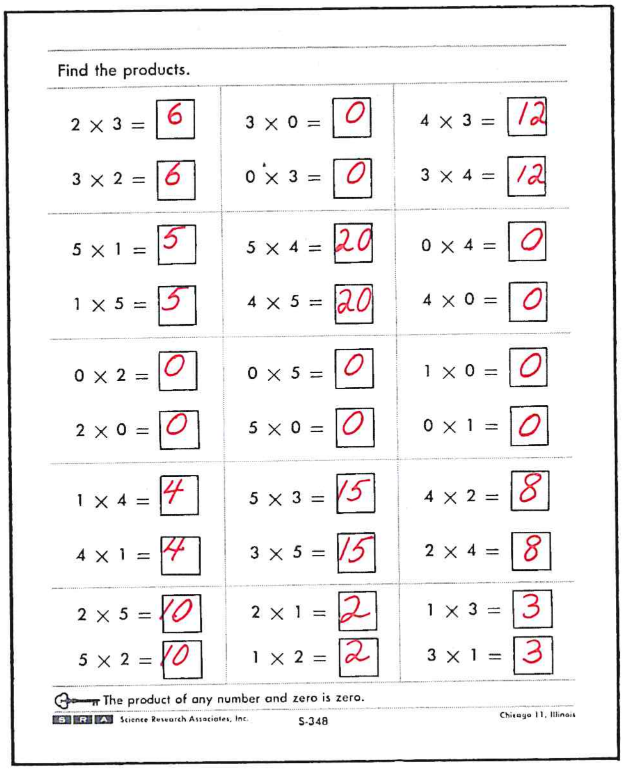
3 × 1 = 3
1 × 3 = 3

2 × 3 = 6
3 × 2 = 6

3 × 4 = 12
4 × 3 = 12

Multiplication is commutative.

S-347



Find the products.

2 × 3 = 6	3 × 0 = 0	4 × 3 = 12
3 × 2 = 6	0 × 3 = 0	3 × 4 = 12
5 × 1 = 5	5 × 4 = 20	0 × 4 = 0
1 × 5 = 5	4 × 5 = 20	4 × 0 = 0
0 × 2 = 0	0 × 5 = 0	1 × 0 = 0
2 × 0 = 0	5 × 0 = 0	0 × 1 = 0
1 × 4 = 4	5 × 3 = 15	4 × 2 = 8
4 × 1 = 4	3 × 5 = 15	2 × 4 = 8
2 × 5 = 10	2 × 1 = 2	1 × 3 = 3
5 × 2 = 10	1 × 2 = 2	3 × 1 = 3

The product of any number and zero is zero.

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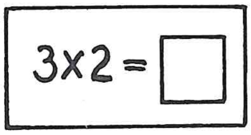
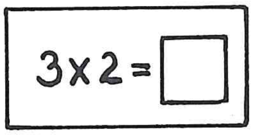
On work page S-348, direct the children to find the products of the two factors in each equation. Many children will immediately see that if they find the answer to one problem in each box, they have also found the other answer. After the page has been completed, discuss once again the commutative property of multiplication. Some of the children may also have noticed a very interesting fact about multiplication problems in which zero is one of the factors. They may be able to generalize that the product of any number and zero is zero.

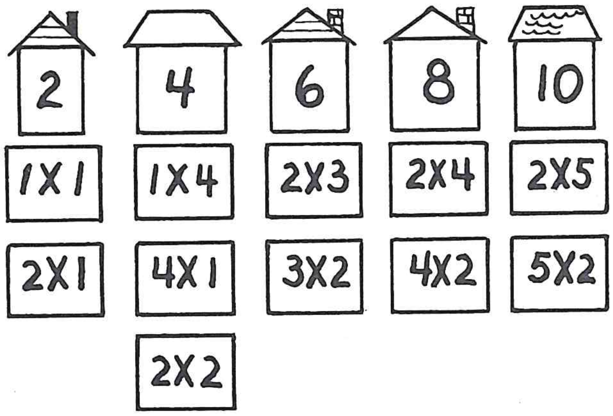
Supplemental Experiences

▶ Construct a set of cards bearing multiplication placeholder equations. Distribute these cards to the children. Call upon one child to come to the front of the room, display his card, and say, "I'm looking for a partner." The child who holds the card on which appears the *commuted* form of the given equation must come to the front of the room and hold his card beside the first card. The children must then tell the product of the given pairs of factors. Call upon another child to come to the front of the room and continue the activity.





▶ Cut out several large paper houses and write a numeral to represent a product on each. Line the houses up along the chalk tray. Distribute cards on which have been written multiplication combinations. Have the children pretend they are mailmen and tell them they must deliver their cards to the proper houses.



Call upon the children, one at a time, to stand, read the combination on their card, and then deliver their card to the house whose number is a product of the two factors represented on the card. Continue until all the "mail" has been delivered. Then review the factors of each of the products; call particular attention to the commuted pairs.

Multiplication

COMPARISON

Work Pages S-349 and S-350

Objectives

To strengthen skills in solving multiplication problems.

To reinforce the children's understanding of the commutative property of multiplication.

To compare numbers in the environment of multiplication.

Fundamentals

The teacher will want to help the child develop skills in working with the multiplication combinations from zero times one through five times five. She will want to help him utilize these skills and abilities as he compares two numbers in the setting of multiplication.

The child should be encouraged to apply his understanding of the commutative property to determine the relation symbol needed in an equation such as 3 × 4 ○ 4 × 3. His ability to interpret multiplication as repeated addition should help when he is asked to determine whether 2 + 2 or 3 × 2 represents the greater number. In many cases the application of known concepts will lead to a quicker decision than that obtained by computation. For instance, a child might very well reason that 4 × 5 is greater than 3 × 5 (since it represents one more five) without having to think of the products.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Multiplication of Whole Numbers.

Readiness for Understanding

Familiarity with multiplication combinations from zero times one through five times five.

Ability to interpret multiplication as repeated addition.

Understanding of the commutative property of multiplication.

Ability to compare numbers.

Developmental Experiences

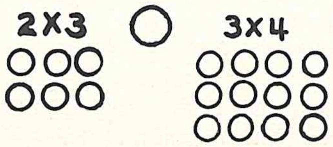
MATERIALS

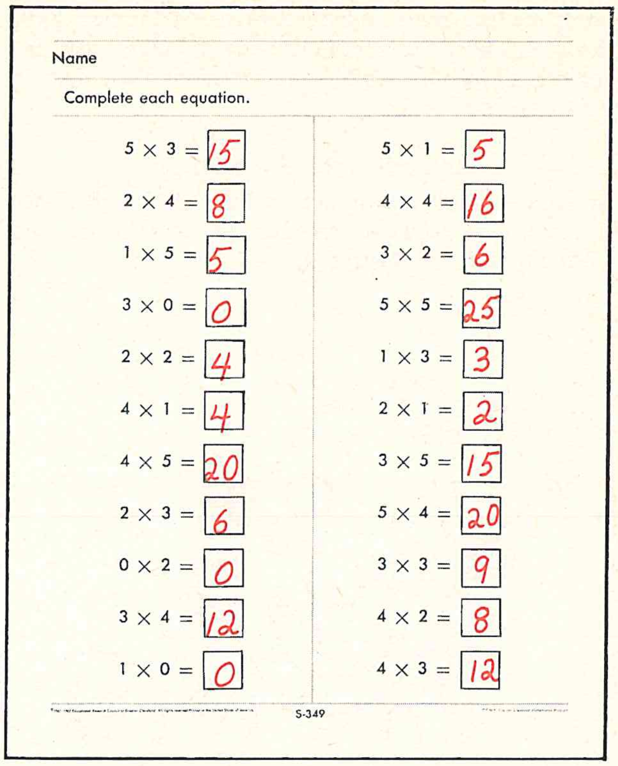
flannel board
felt numerals
felt operation and relation signs
felt circle
felt disks

▶ Build an equation requiring a relation symbol, on the flannel board: 5 × 3 ○ 3 × 5. Ask the children to quickly tell which symbol, >, <, or =, belongs in the circle to make this a true statement. Lead the children to see that they can find the answer without computing the products. Examination of the factors on either side of the circle will reveal that they have been commuted and therefore that both numerals represent the same number. Have a child place the = symbol in the circle and read the completed statement.

Place an equation such as 2 + 2 + 2 ○ 3 × 2 on the flannel board. Ask a child to tell which relation symbol is needed and to explain his reasoning. He might say that 3 × 2 is another way of thinking of 2 + 2 + 2 and that, therefore, the = symbol is needed.

Place 2 × 3 ○ 3 × 4 on the flannel board. Have a child tell what relation symbol is needed and explain his thinking. If he has difficulty in deciding which numeral represents the greater number, have him use felt disks on the flannel board to aid in making his decision. Continue the activity. Include some equations such as 4 × 2 ○ 3 + 3 in which the child must compute the answer before he can make his decision. In such a case remind the child that he must be very careful to think of the meaning of the × sign and the meaning of the + sign so that he carries out the correct operations.





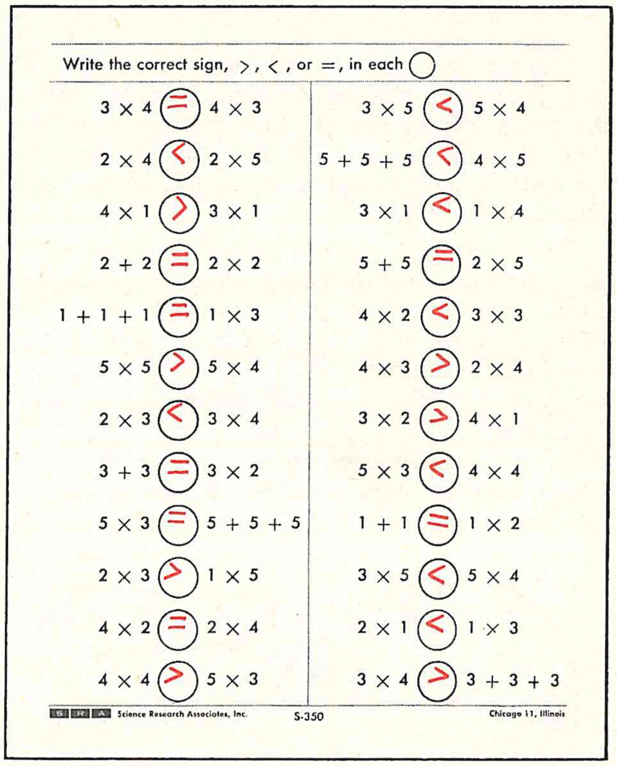
Name

Complete each equation.

5 × 3 = 15
2 × 4 = 8
1 × 5 = 5
3 × 0 = 0
2 × 2 = 4
4 × 1 = 4
4 × 5 = 20
2 × 3 = 6
0 × 2 = 0
3 × 4 = 12
1 × 0 = 0

5 × 1 = 5
4 × 4 = 16
3 × 2 = 6
5 × 5 = 25
1 × 3 = 3
2 × 1 = 2
3 × 5 = 15
5 × 4 = 20
3 × 3 = 9
4 × 2 = 8
4 × 3 = 12

S-349



Write the correct sign, >, <, or =, in each ○

3 × 4 = 4 × 3
2 × 4 < 2 × 5
4 × 1 > 3 × 1
2 + 2 = 2 × 2
1 + 1 + 1 = 1 × 3
5 × 5 > 5 × 4
2 × 3 < 3 × 4
3 + 3 = 3 × 2
5 × 3 = 5 + 5 + 5
2 × 3 > 1 × 5
4 × 2 = 2 × 4
4 × 4 > 5 × 3

3 × 5 < 5 × 4
5 + 5 + 5 < 4 × 5
3 × 1 < 1 × 4
5 + 5 = 2 × 5
4 × 2 < 3 × 3
4 × 3 > 2 × 4
3 × 2 > 4 × 1
5 × 3 < 4 × 4
1 + 1 = 1 × 2
3 × 5 < 5 × 4
2 × 1 < 1 × 3
3 × 4 > 3 + 3 + 3

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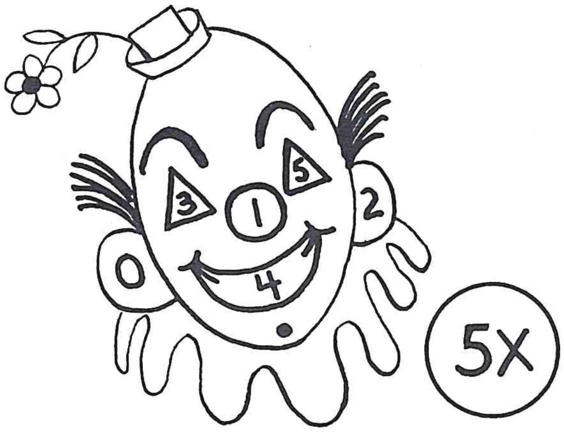
▶ Use work page S-349 to evaluate the children's knowledge of multiplication combinations through five times five. Direct them to quickly complete each equation by recording the product in the placeholder.

Work page S-350 provides another opportunity for the children to use their ability to reason. Work the first problem with the group, then have them complete the page independently. After they have done so, check the problems with the class; ask different children to explain their reasons for selecting each relation symbol.

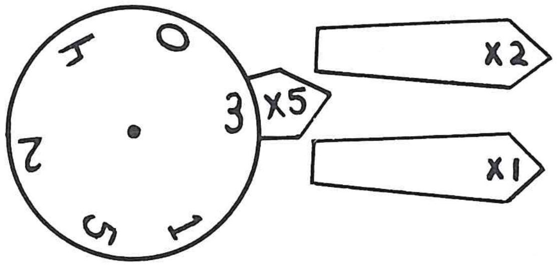
Supplemental Experiences

▶ Draw a large clown face on the chalkboard. Place a numeral on each eye, on each ear, on the nose, and on the mouth, as illustrated.



Near the clown draw a ball and write on it, "5 × __." The object of the game is to "toss" the ball at the clown and to give the correct answer. As the child pretends to throw at the clown's nose, for example, he must complete the multiplication equation, 5 × 1 = □.

▶ Construct a multiplication wheel for individual work or for work with small groups. Cut out a circle with a diameter of 8″ from heavy cardboard or tagboard. Write the numerals 0 through 5 around the edge of the wheel. Cut several tagboard or cardboard strips approximately 6″ in length and write × 5, × 3, × 1, and so forth, on the end of each. Attach one of the strips to the wheel with a paper fastener as



illustrated. As the strip is moved around the wheel, the child must give the product of each pair of factors. Change the strip to provide practice with different combinations.

Multiplication

COMBINATIONS FROM 0 x 0 THROUGH 5 x 5

Work Pages S-351 Through S-354

Objectives

To help children organize known multiplication combinations.

To teach children to record known products in a multiplication table.

Fundamentals

After the child has become familiar with many different multiplication combinations, he should stop and organize what he has learned. One way to do this is to make listings of related multiplication equations. Perhaps the most convenient form is the multiplication table.

The child should be guided to record the product of different pairs of numbers in the body of the table and then to examine the patterns which appear within the table. To some extent the completed table can be used as a reference for looking up forgotten combinations. It is far better, however, to have the forgetful child work out the solution to multiplication problems by using sets or repeated addition.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Multiplication of Whole Numbers.

Readiness for Understanding

Understanding of the operation of multiplication.

Knowledge of multiplication combinations from zero times zero through five times five.

Developmental Experiences

MATERIALS	VOCABULARY
chart paper	multiplication table
black crayon	
2 pointers	
easel	

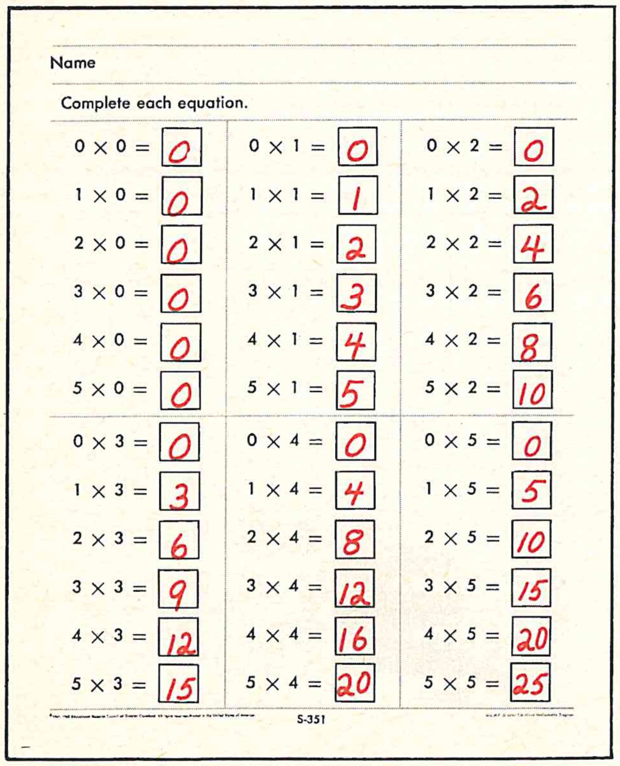
▶ Tell the children you want them to write on the chalkboard all of the multiplication combinations they have learned thus far. Call on several children to name multiplication combinations and write the equations on the chalkboard. After several have been written, ask if the children can think of a better way to arrange these combinations so that they can be sure they have recorded all of the combinations they know. Some child may suggest that all equations in which the first factor is the same might be grouped. Another child might suggest that it would be possible to group equations with the same second factor. Help the class to choose one of these groupings and then have all multiplication combinations from 0 × 0 through 5 × 5 recorded. Have the children count the number of equations to see how many combinations they have learned (36). If time allows, have all of the combinations listed again but arranged in the other grouping. Contrast such listings as those illustrated below.

$0 \times 4 = 0$	$4 \times 0 = 0$
$1 \times 4 = 4$	$4 \times 1 = 4$
$2 \times 4 = 8$	$4 \times 2 = 8$
$3 \times 4 = 12$	$4 \times 3 = 12$
$4 \times 4 = 16$	$4 \times 4 = 16$
$5 \times 4 = 20$	$4 \times 5 = 20$

▶ Use work page S-351 to give the children an opportunity to tabulate multiplication combinations they have learned. Direct them to complete each equation.

Work page S-352 provides another opportunity for the children to record in orderly form the multiplication combinations zero times zero through five times five. Direct them to complete each equation.

After work pages S-351 and S-352 have been completed, have the children discuss the patterns that are evident on these two pages. Remind them that the product of any number and zero is zero and that the product of any number and one is that same number.

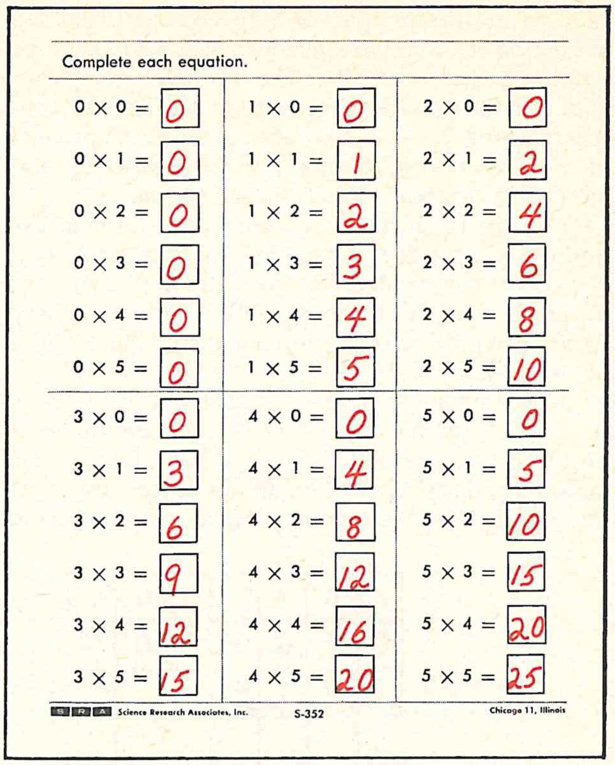


Name

Complete each equation.

0 × 0 =	0	0 × 1 =	0	0 × 2 =	0
1 × 0 =	0	1 × 1 =	1	1 × 2 =	2
2 × 0 =	0	2 × 1 =	2	2 × 2 =	4
3 × 0 =	0	3 × 1 =	3	3 × 2 =	6
4 × 0 =	0	4 × 1 =	4	4 × 2 =	8
5 × 0 =	0	5 × 1 =	5	5 × 2 =	10
0 × 3 =	0	0 × 4 =	0	0 × 5 =	0
1 × 3 =	3	1 × 4 =	4	1 × 5 =	5
2 × 3 =	6	2 × 4 =	8	2 × 5 =	10
3 × 3 =	9	3 × 4 =	12	3 × 5 =	15
4 × 3 =	12	4 × 4 =	16	4 × 5 =	20
5 × 3 =	15	5 × 4 =	20	5 × 5 =	25

S-351



Complete each equation.

0 × 0 =	0	1 × 0 =	0	2 × 0 =	0
0 × 1 =	0	1 × 1 =	1	2 × 1 =	2
0 × 2 =	0	1 × 2 =	2	2 × 2 =	4
0 × 3 =	0	1 × 3 =	3	2 × 3 =	6
0 × 4 =	0	1 × 4 =	4	2 × 4 =	8
0 × 5 =	0	1 × 5 =	5	2 × 5 =	10
3 × 0 =	0	4 × 0 =	0	5 × 0 =	0
3 × 1 =	3	4 × 1 =	4	5 × 1 =	5
3 × 2 =	6	4 × 2 =	8	5 × 2 =	10
3 × 3 =	9	4 × 3 =	12	5 × 3 =	15
3 × 4 =	12	4 × 4 =	16	5 × 4 =	20
3 × 5 =	15	4 × 5 =	20	5 × 5 =	25

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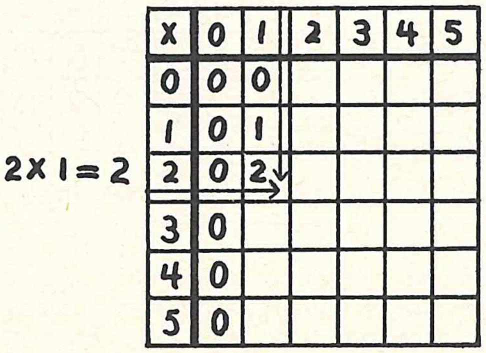
▶ Construct a large *multiplication table* on a piece of chart paper. Have seven rows of seven squares each. In the first box in the first row write the operational sign for multiplication, ×. In the remaining squares in the top row write 0, 1, 2, 3, 4, and 5. Down the first column write 0, 1, 2, 3, 4, 5. Display the multiplication table on an easel. Tell the children that this table will be used to record the thirty-six multiplication combinations they have learned.

Point to the box with the times sign in it and review the meaning of the symbol. Point to the numerals in the first column and tell the children that these numerals represent the first factor in a multiplication combination. Point to the numerals in the top row and say that these numerals represent the second factor in a multiplication combination. Point to the empty boxes and tell the class that this is where they are to record the products. Ask some



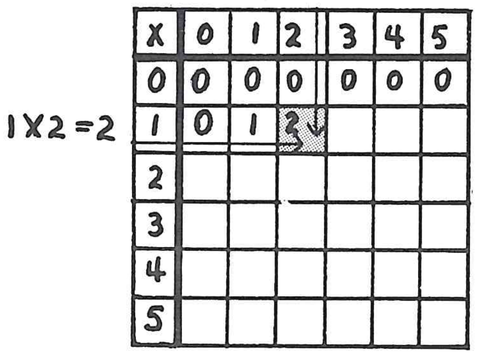
2 × 1 = 2

X	0	1	2	3	4	5
0	0	0				
1	0	1				
2	0	2				
3	0					
4	0					
5	0					

child to tell the product of zero and zero. Place a pointer on the 0 in the first row and a pointer on the 0 in the first column. Move the two pointers together until they meet in the first empty box. Tell the children that this is where they are to record the product of zero and zero. Call upon a child to write the numeral 0 in this box with the black crayon.

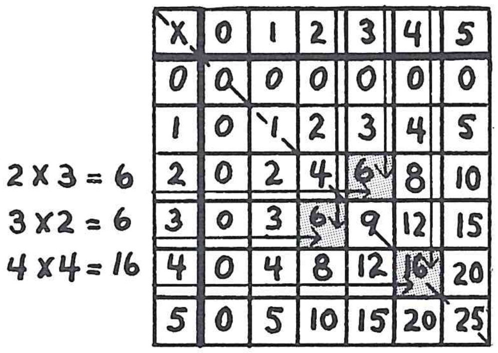
Have the products of one times zero through five times zero recorded in the table in this same way. Next have the products of zero times one through five times one recorded in the table. Continue to have the children record other products down the columns of the table.

▶ Place a second unfilled multiplication table on the easel. Vary the procedure for recording the products. Call upon several children to help. Begin again with zero times zero; then fill in across the rows instead of down the columns. Have children record the products zero times one through zero times five. Move on to one times zero through one times five. Continue in this way to have the products recorded in the rows in the table.



X	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2			
2						
3						
4						
5						

▶ As a third possible approach to filling in the table have children choose any multiplication combination with products zero through twenty-five, find the box for the product, and write the correct numeral in this space. Perhaps a child may wish to record the product 3 × 2. Call upon a second child to record the product represented by the commuted form, 2 × 3. A third child may wish to record the product 4 × 4. Point out that the commutative property holds for this pair of numbers as it does for all other factors, although we cannot observe a difference in the commuted form of the numerals. Help the children observe the position in the table of the numerals representing these particular products; they are all located on the main diagonal. Some child may note that the products represented by commuted pairs of numerals are symmetric about the main diagonal.

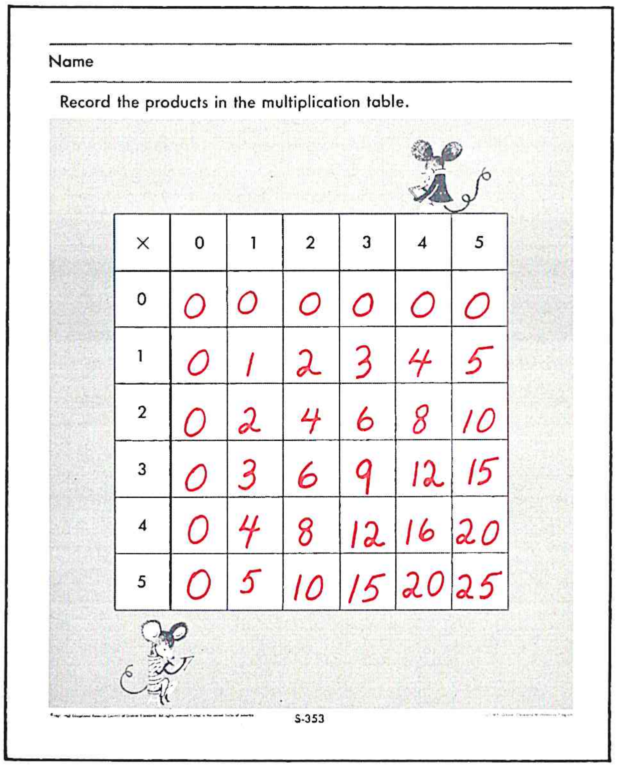


X	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25

If a child comments on the fact that there are all 0's in the first row and the first column of the body of the table, remind him that the product of any number and zero is zero. Another child may note that the headings are repeated in the second row and the second column in the body of the table. Remind him that the product of any number and one is that same number. Other children may note that corresponding rows and columns have the same entries and that many number sequences, such as 0, 3, 6, 9, 12, 15, appear in the table.

▶ Work page S-353 provides each child with his own multiplication table. Have the children work independently to record the products in the body of the table, then call on several children to explain the method they used to fill in the table. Some may have filled it by moving down the columns, some by moving across the rows, and others by recording the products in random fashion. Discuss the many patterns that are evident in the completed table.

Use work page S-354 to evaluate the children's understanding of the multiplication combinations through five times five. Tell the children they are to complete each equation as quickly as possible.

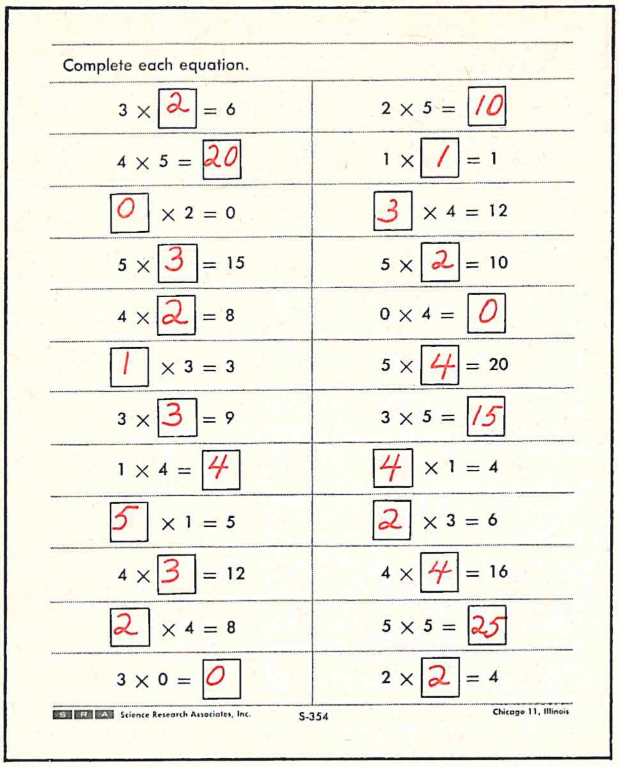


Name

Record the products in the multiplication table.

×	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25

S-353



Complete each equation.

$3 \times \boxed{2} = 6$

$4 \times 5 = \boxed{20}$

$\boxed{0} \times 2 = 0$

$5 \times \boxed{3} = 15$

$4 \times \boxed{2} = 8$

$\boxed{1} \times 3 = 3$

$3 \times \boxed{3} = 9$

$1 \times 4 = \boxed{4}$

$\boxed{5} \times 1 = 5$

$4 \times \boxed{3} = 12$

$\boxed{2} \times 4 = 8$

$3 \times 0 = \boxed{0}$

$2 \times 5 = \boxed{10}$

$1 \times \boxed{1} = 1$

$\boxed{3} \times 4 = 12$

$5 \times \boxed{2} = 10$

$0 \times 4 = \boxed{0}$

$5 \times \boxed{4} = 20$

$3 \times 5 = \boxed{15}$

$\boxed{4} \times 1 = 4$

$\boxed{2} \times 3 = 6$

$4 \times \boxed{4} = 16$

$5 \times 5 = \boxed{25}$

$2 \times \boxed{2} = 4$

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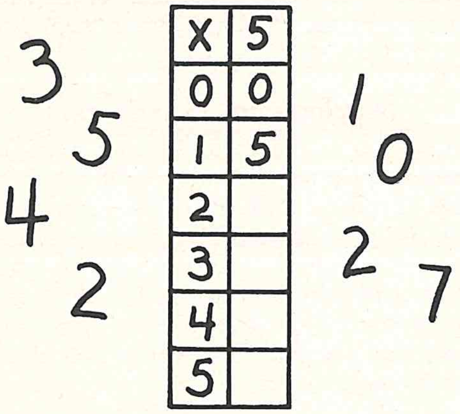
Supplemental Experiences

▶ Make a set of partial multiplication tables from tagboard or construction paper. Distribute these to the children and have them use their plastic numerals to record the products of the given factors. After each card has been checked, have the children exchange cards and repeat the activity.



▶ Write numerals 0 through 25 in a long horizontal row on the chalkboard. Give one child a piece of colored chalk. Have him begin with 1 and circle the numeral which represents each multiple of one through five times one. As he circles each numeral in turn he should say, "One times one equals one, two times one equals two, three times one equals three," and so forth. Give a second child a different-colored piece of chalk. Have him begin by circling 2 and the numeral for each successive multiple of two through five times two. Have three other children follow the same procedure by beginning with 3 and counting by threes, beginning at 4 and counting by fours, and beginning at 5 and counting by fives until they have circled five multiples of each number. In some cases different-colored circles will appear around the same numeral. Ask the children why they think this happens. They may say, for example, that four is the product four times one, two times two, and one times four. There are three different combinations of four so three circles appear.

Refer back to a completed multiplication table and find how many times 4 appears in the body of the table. Repeat with each of the other numerals that has been circled.

Multiplication

VERTICAL NOTATION

Work Pages S-355 and S-356

Objectives

To introduce vertical notation for multiplication problems.

To provide practice in working with multiplication combinations through five times five.

Fundamentals

The child has become familiar with multiplication combinations through five times five. Following his work with sets of objects he learned to work with numbers and to complete multiplication placeholder equations. He will now be introduced to the vertical-notation form for writing multiplication problems. In the third grade, multiplication work will include two-digit factors. Then, vertical notation will be more efficient for computational purposes than the equation form. Vertical notation is introduced here to familiarize the child with the form, although it provides no advantage for multiplying with one-digit factors.

The child will be asked to relate each equation to a particular vertical notation in which the second factor of the equation appears in the top position. Later, for ease of computation, he will learn to apply the commutative property in certain cases and write his multiplication problem with the greater factor in the top position.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Multiplication of Whole Numbers.

Readiness for Understanding

Understanding of the multiplication combinations through five times five.

Developmental Experiences

MATERIALS

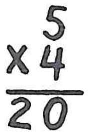
plastic numerals and other numerals

▶ Write the equation 2 × 3 = □ on the chalkboard. Have a child complete the equation and read it aloud, "Two times three equals six." Now write the same combination in vertical form: Read this

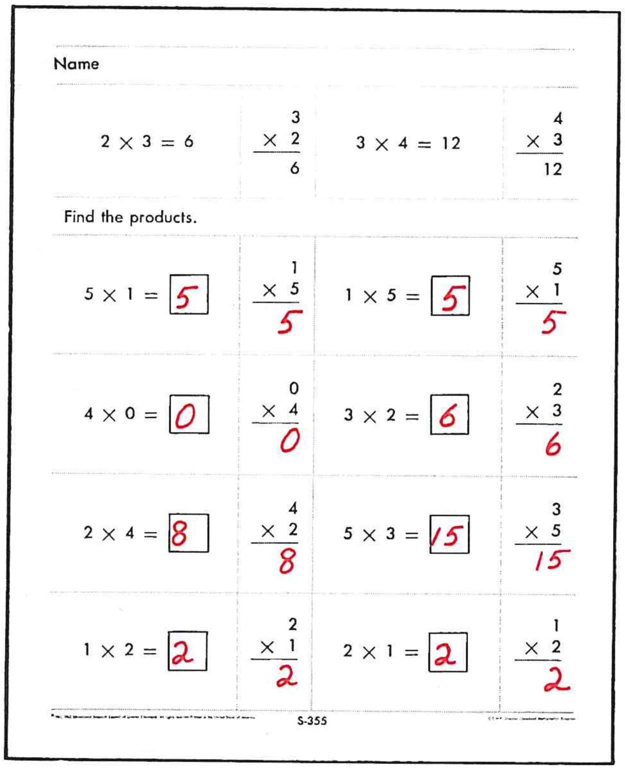
$$\begin{array}{r} 3 \\ \times 2 \\ \hline \end{array}$$

combination to the children, "two times three"; point to each numeral as it is read. Have a child give the product and write the numeral 6 below the line. Point out that in the vertical form the second factor is on the top and the first factor on the bottom. Write several other multiplication equations and have the children write the related vertical notation for each.

▶ Direct the children to use their plastic numerals and times and equals signs on their desks to show the equation 4 × 5 = □. After they have completed this equation, ask them to show the vertical form for the same problem. Repeat the activity with other multiplication combinations.



▶ Use work page S-355 to help the children make the transition from equation form to vertical-notation form. Discuss the two examples at the top of the page with the children. Then direct them to find the products in each pair of problems on the remainder of the page. Encourage them to work all these problems on their own.



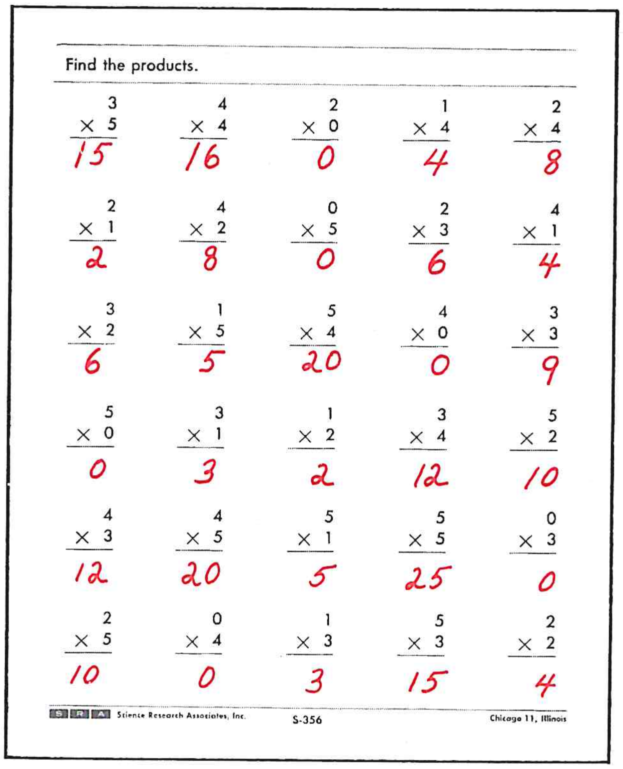
Name

2 × 3 = 6	3 × 2 6	3 × 4 = 12	4 × 3 12

Find the products.

5 × 1 = $\boxed{5}$	1 × 5 5	1 × 5 = $\boxed{5}$	5 × 1 5
4 × 0 = $\boxed{0}$	0 × 4 0	3 × 2 = $\boxed{6}$	2 × 3 6
2 × 4 = $\boxed{8}$	4 × 2 8	5 × 3 = $\boxed{15}$	3 × 5 15
1 × 2 = $\boxed{2}$	2 × 1 2	2 × 1 = $\boxed{2}$	1 × 2 2

S-355



Find the products.

3 × 5 15	4 × 4 16	2 × 0 0	1 × 4 4	2 × 4 8
2 × 1 2	4 × 2 8	0 × 5 0	2 × 3 6	4 × 1 4
3 × 2 6	1 × 5 5	5 × 4 20	4 × 0 0	3 × 3 9
5 × 0 0	3 × 1 3	1 × 2 2	3 × 4 12	5 × 2 10
4 × 3 12	4 × 5 20	5 × 1 5	5 × 5 25	0 × 3 0
2 × 5 10	0 × 4 0	1 × 3 3	5 × 3 15	2 × 2 4

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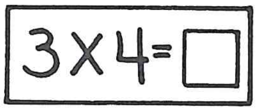
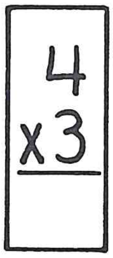
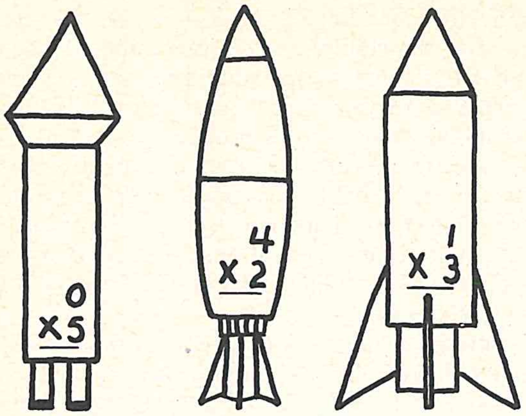
Work page S-356 provides practice in working multiplication problems in vertical form. Direct the children to find the products. Have them complete the page independently.

Supplemental Experiences

▶ Construct a set of cards bearing multiplication placeholder equations. On another set write the same problems in vertical form. Place the vertical-notation cards on the chalk tray. Display one of the equation cards to two children. The one who first finds the related vertical-notation card on the chalk tray and tells the product of the given factors is the winner of the game. He becomes the leader and displays a card for the next two contestants.

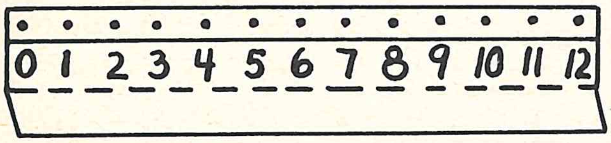






▶ Have children draw pictures of rockets at different altitudes on the chalkboard. Write a multiplication combination in vertical form on each rocket. Have one child act as rocket booster by pointing to one rocket at a time. Each child in turn has a chance to fire a rocket by quickly reading the problem and giving the product. If he misses he must use sets to determine the product. His reward for success is an imaginary space shot.

▶ Place a number-strip on the flannel board. Construct the strip in such a way that it is possible to fold the lower section up and down to cover and uncover the numerals but not the dots. Begin by covering the numerals; leave only the dots on the line exposed. Place your finger on the dot that represents zero. "Jump" your finger two dots at a time for three times. Hold your index finger on the stopping point. Ask children to tell how many spaces you jumped each time, how many times you jumped, and where you are on the number-strip now. After the location has been given, uncover the numerals on the strip and have the children check their answer. Show them the path you jumped by starting again at zero and repeating the three jumps. Have the children watch the action and relate it to the numbers they see represented on the strip. Repeat the activity once more and have the children skip-count as you make each jump. Continue in this way to use the number-strip to demonstrate other multiplication combinations through five times five.



Division

Work Pages S-357 and S-358

Objectives

To introduce division as the process of finding a missing factor.

To introduce the division symbol for the division operation.

To develop an understanding of the inverse relationship between multiplication and division.

Fundamentals

The child has learned to find the missing factor in an equation such as □ × 2 = 8. Referring to the meaning of multiplication in set terms, he asks himself, "How many sets of two in a set of eight?" He separates the set of eight into sets of two and finds that he has four such sets. He then concludes that four is the missing factor.

Now the child will be guided to think of the same situation in terms of a new operation—division. He knows the product and one factor in the problem □ × 2 = 8; the other factor must be found. A new notation, 8 ÷ 2 = □, will be introduced for this situation. An advantage of this notation for the child is that the known numbers are represented on one side and the placeholder stands by itself. He can now think, "A set of eight partitioned into sets of two will give how many sets?"

The new notation is reminiscent of a parallel situation in addition. Given the problem □ + 3 = 8, the child has become accustomed to writing a related equation, 8 − 3 = □. In this case 3 represents the known addend, 8 represents the sum, and the □ holds the place for the missing addend. The minus sign is a symbol for a new operation called subtraction, the operation of finding a missing addend.

The teacher will use the parallel to its fullest in the new situation. In the example □ × 2 = 8, 2 represents the known factor, 8 represents the product, and the □ holds the place for the missing factor. The sign ÷ is the symbol for a new operation called division, the operation of finding a missing factor.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Division of Whole Numbers.

Readiness for Understanding

Understanding of the multiplication combinations zero times zero through five times five.

Understanding of the inverse relation between addition and subtraction.

Developmental Experiences

MATERIALS	VOCABULARY
flannel board	division
felt cutouts	divide
felt numerals	÷
felt ×, ÷, and = signs	partition
felt placeholders	

▶ Place twelve felt cutouts on the flannel board. Build the equation $\square \times 4 = 12$ beside them. Ask some child to read this equation: "Some number times four equals twelve." Ask another child to explain how he might use sets to help him find the number that is missing. Have him rearrange the set of felt cutouts into sets of four and tell how many such sets he has. Have another child complete the equation.

Again place the equation $\square \times 4 = 12$ on the flannel board. Tell the children to think about the meaning of each part of this equation. Point to each symbol in turn and ask the children to tell what it stands for. They should say that 12 represents the product in this problem; that 4 represents a known factor; that the placeholder holds the place for the missing factor. Review with the children the way they found the missing factor: they started with a set of twelve and *partitioned* it into sets of four. Say to the children that there is another way to show what happened: $\square \times 4 = 12$; $12 \div 4 = \square$. Build the equation $12 \div 4 = \square$ on the flannel board beneath the multiplication placeholder equation. Read the equation to the children, "Twelve *divided* by four equals some number." Help the children see that they can find this number in the same way that they found the missing factor in the multiplication equation. Have a child complete the *division* equation. Say, "We started with the product and divided by the known factor to find the missing factor."

Repeat the activity with other division combinations related to products through five times five. In each case, stress that division is the process by which the missing factor in the related multiplication equation is found.

▶ Review the inverse relation between addition and subtraction. Write the equation $\square + 3 = 5$ on the chalkboard. Ask the children to explain what is needed in this equation to complete it. They should explain that one addend is missing. Ask some child to explain how he can find the missing addend. He should say that he starts with the sum and subtracts the known addend. Write the equation $5 - 3 = \square$ beneath the addition equation. Remind the children that subtraction is used to find the missing addend in an addition equation.

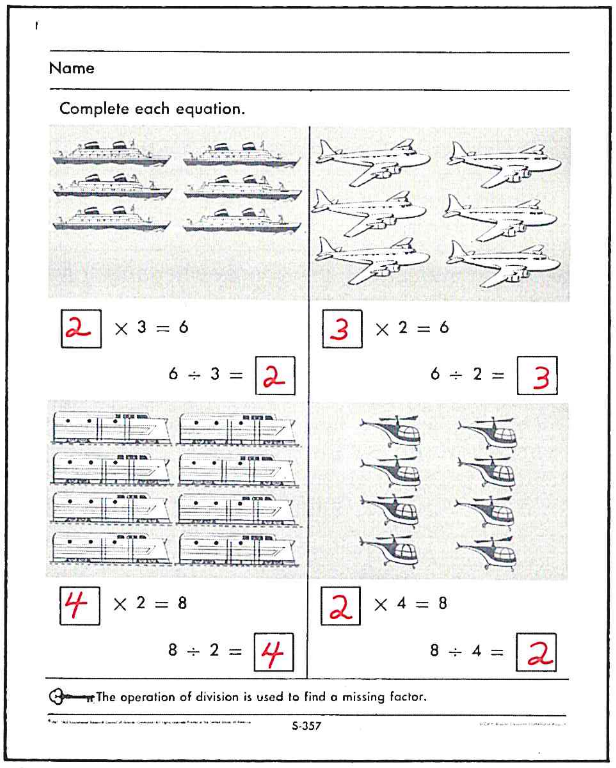
$$\square + 3 = 5 \qquad \square \times 3 = 6$$
$$5 - 3 = \square \qquad 6 \div 3 = \square$$

Write the equation $\square \times 3 = 6$ on the chalkboard. Ask the children to tell two things: what will complete this equation and what information is given. They should say that they must find a missing factor and that one factor and the product are known. Lead some child to say that in order to find the missing factor he must divide the product by the known factor. Write the equation $6 \div 3 = \square$ beneath the related multiplication equation. Say that division is the operation of finding the missing factor in multiplication equations.

Summarize by pointing out the parallel between the related addition and subtraction equations and the related multiplication and division equations.

▶ Use work page S-357 to reinforce the children's understanding of division as the operation which is used to find a missing factor in a multiplication equation. Call the children's attention to the first box on the page and have the two placeholder equations read. Direct the children to circle sets of three in the set of six and to complete each equation. Now call attention to the second box on the page. Have the children tell what they must find in each of the equations. Direct them to circle sets of two in the set of six and to complete each equation. Discuss with the children the relation between the multiplication equations in these two boxes and the relation between the multiplication equations and the corresponding division equations.

Direct the children to complete each equation on work page S-358; then discuss the relation between each multiplication equation and the inverse division equation.



Name

Complete each equation.

$\boxed{2} \times 3 = 6$

$6 \div 3 = \boxed{2}$

$\boxed{3} \times 2 = 6$

$6 \div 2 = \boxed{3}$

$\boxed{4} \times 2 = 8$

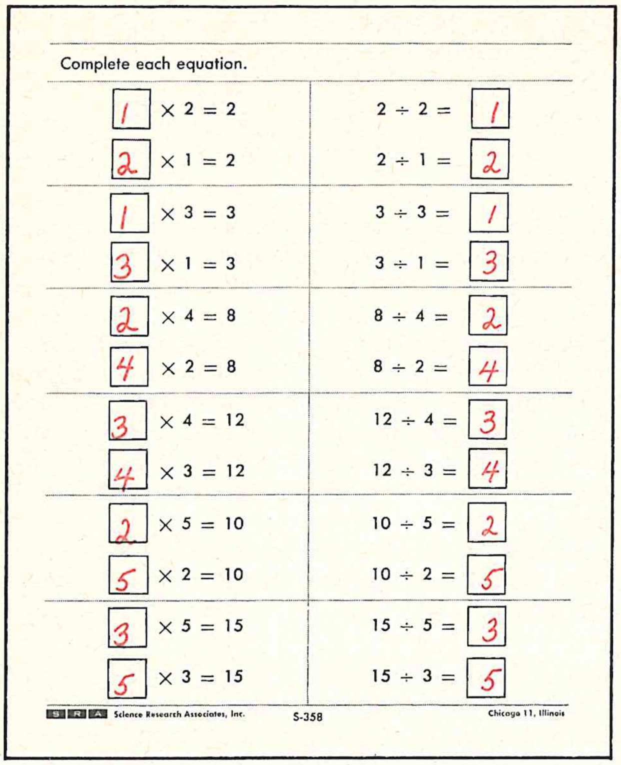
$8 \div 2 = \boxed{4}$

$\boxed{2} \times 4 = 8$

$8 \div 4 = \boxed{2}$

The operation of division is used to find a missing factor.

S-357



Complete each equation.

$\boxed{1} \times 2 = 2$	$2 \div 2 = \boxed{1}$
$\boxed{2} \times 1 = 2$	$2 \div 1 = \boxed{2}$
$\boxed{1} \times 3 = 3$	$3 \div 3 = \boxed{1}$
$\boxed{3} \times 1 = 3$	$3 \div 1 = \boxed{3}$
$\boxed{2} \times 4 = 8$	$8 \div 4 = \boxed{2}$
$\boxed{4} \times 2 = 8$	$8 \div 2 = \boxed{4}$
$\boxed{3} \times 4 = 12$	$12 \div 4 = \boxed{3}$
$\boxed{4} \times 3 = 12$	$12 \div 3 = \boxed{4}$
$\boxed{2} \times 5 = 10$	$10 \div 5 = \boxed{2}$
$\boxed{5} \times 2 = 10$	$10 \div 2 = \boxed{5}$
$\boxed{3} \times 5 = 15$	$15 \div 5 = \boxed{3}$
$\boxed{5} \times 3 = 15$	$15 \div 3 = \boxed{5}$

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Supplemental Experiences

▶ Construct a set of cards on which are written multiplication placeholder equations such as $\square \times 2 = 4$. Have two children at a time play "detective." Display one of the cards. The child who "finds" the missing factor first is the best detective. The loser must write the related inverse equation on the chalkboard: $4 \div 2 = 2$.

▶ Make up story problems in which children can dramatize situations related to division. Say, for instance, "Sally had eight books. She wanted to give two to each of several friends. How many of Sally's friends would receive books?" Have one child write the related division equation on the chalkboard: $8 \div 2 = \square$. Call on a child to pretend to be Sally. Have her distribute eight books among several children to find how many children might get two each. Finally, have the placeholder equation on the chalkboard completed. Repeat with other stories which call for a division interpretation.

On some occasions write a division equation on the chalkboard first and have the children create a story which might relate to this equation.

▶ Place two columns of placeholder equations on the chalkboard. Put multiplication placeholder equations in the column on the left and division equations in the column on the right. Call upon a child to complete the first multiplication equation. Have a second child draw a line from this completed equation to a division equation he thinks belongs with the multiplication equation. When the line has been drawn, have a third child complete the related division equation. Have the class discuss what they have discovered about the numbers that were used to complete the two equations. Have the equations read. Continue in this way to have all equations matched, completed, and read.

$\square \times 4 = 16$
$\square \times 5 = 20$
$\square \times 2 = 8$
$\square \times 1 = 5$
$\square \times 3 = 9$
$\square \times 4 = 12$

$9 \div 3 = \square$
$12 \div 4 = \square$
$20 \div 5 = \square$
$16 \div 4 = \square$
$5 \div 1 = \square$
$8 \div 2 = \square$

Division

REPEATED SUBTRACTION

Work Pages S-359 and S-360

Objectives

To interpret division as repeated subtraction.

To strengthen the children's understanding of the inverse relation between multiplication and division.

Fundamentals

The child has found that he can solve the problem $\square \times 2 = 8$ or, as it is indicated in the new notation, $8 \div 2 = \square$, by answering the question, "How many sets of two are in a set of eight?" He will now find that this problem can be reduced to a subtraction problem.

When the child worked with equations such as $4 \times 2 = \square$, he found that he could find the answer by combining four sets of two objects each and determining the number of the new set. He also found that he could use repeated addition to find this product. The sums were related to the step-by-step

$$\begin{array}{r}2\\+\,2\\\hline 4\end{array}\nearrow\begin{array}{r}4\\+\,2\\\hline 6\end{array}\nearrow\begin{array}{r}6\\+\,2\\\hline 8\end{array}$$

by joining of four sets of two each.

Now the child should be led to discover that by "undoing" this process, that is, by separating sets of two from a set of eight until the supply is exhausted, he can solve the equation $8 \div 2 = \square$. Each time a set of two is separated, two is subtracted:

$$\begin{array}{r}8\\-\,2\\\hline 6\end{array}\nearrow\begin{array}{r}6\\-\,2\\\hline 4\end{array}\nearrow\begin{array}{r}4\\-\,2\\\hline 2\end{array}\nearrow\begin{array}{r}2\\-\,2\\\hline 0\end{array}$$

The child finds that this can be done four times and concludes that $8 \div 2 = 4$. This method is known as "repeated subtraction."

See Key Topics in Mathematics for the Primary Teacher: Division of Whole Numbers.

Readiness for Understanding

An understanding of multiplication and division.

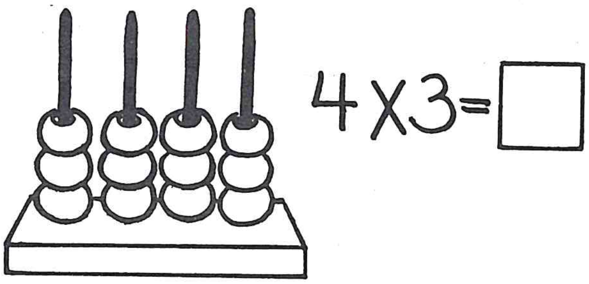
Ability to interpret multiplication as repeated addition.

Understanding of the inverse relation between addition and subtraction.

Developmental Experiences

MATERIALS	VOCABULARY
bead frame	repeated subtraction
beads	

▶ Review the interpretation of multiplication as repeated addition. Write the equation $4 \times 3 = \square$ on the chalkboard. Call upon a child to place four sets of three beads each on the bead frame. Ask another child to tell how many beads there are all together and to complete the multiplication equation.



Ask some other child to tell another way to find the product four times three. When he suggests using repeated addition, have the related equation $3 + 3 + 3 + 3 = \square$ written on the chalkboard. Remind the children that only two threes can be added at a time. Have some child tell how he arrives at the sum of the four threes. He should say, "$3 + 3 = 6$, $6 + 3 = 9$, $9 + 3 = 12$." Tell the children that this repeated addition can be written in another way. Demonstrate the use of vertical notation as illustrated. Conclude by pointing out that four threes

$$3 + 3 + 3 + 3 = \square \qquad \begin{array}{r} 3 \\ +\,3 \\ \hline 6 \end{array} \nearrow \begin{array}{r} 6 \\ +\,3 \\ \hline 9 \end{array} \nearrow \begin{array}{r} 9 \\ +\,3 \\ \hline 12 \end{array}$$

have been added to give the sum of twelve.

▶ Write the equation $12 \div 3 = \square$ on the chalkboard. Display four sets of three beads each on the bead frame. Ask a child to tell how many sets of three there are in the set of twelve. After the answer has been given, ask a child to complete the placeholder equation.

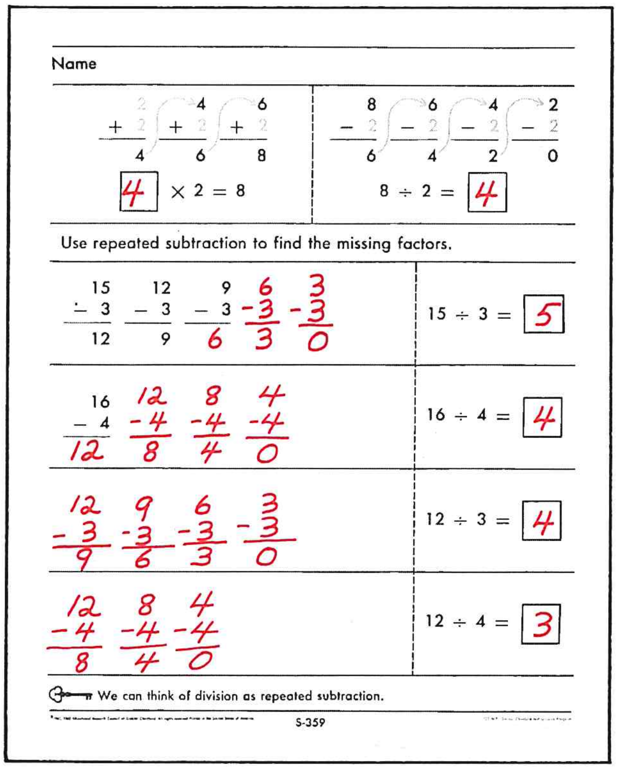
Say to the class, "We have found that we can find the product four times three by adding four threes. Can someone suggest a different way to find the answer to the problem $12 \div 3 = \square$?" Some child may suggest that you try subtracting three. Have one child remove sets of three beads from the set of twelve beads. As each set of three is removed, write the related subtraction problem in vertical form as illustrated. When all beads have been removed and

$$12 \div 3 = \square \qquad \begin{array}{r} 12 \\ -\,3 \\ \hline 9 \end{array} \qquad \begin{array}{r} 9 \\ -\,3 \\ \hline 6 \end{array} \qquad \begin{array}{r} 6 \\ -\,3 \\ \hline 3 \end{array} \qquad \begin{array}{r} 3 \\ -\,3 \\ \hline 0 \end{array}$$

the difference zero has been reached, ask the children how many sets of three were removed from the set of twelve. Ask them to tell how many times three was subtracted. As the answer four is given, refer back to the completed division equation. Summarize by telling the children that another way of saying, "how much is twelve divided by three?" is "how many times can three be subtracted from twelve?" Tell them that this method is called *repeated subtraction* because the same number is subtracted each time.

Repeat the activity with other division equations. Have the children interpret each division as repeated subtraction.

▶ Use work pages S-359 and S-360 to give the children further practice in using repeated subtraction to find the missing factors in division equations. Discuss the examples at the top of each page with the children. Remind them that while multiplication can be thought of as repeated addition, division can be thought of as repeated subtraction. Encourage them to complete the remainder of the problems on each page by themselves.



Name

$$\begin{array}{r} 2 \\ +\,2 \\ \hline 4 \end{array} \quad \begin{array}{r} 4 \\ +\,2 \\ \hline 6 \end{array} \quad \begin{array}{r} 6 \\ +\,2 \\ \hline 8 \end{array} \qquad \boxed{4} \times 2 = 8$$

$$\begin{array}{r} 8 \\ -\,2 \\ \hline 6 \end{array} \quad \begin{array}{r} 6 \\ -\,2 \\ \hline 4 \end{array} \quad \begin{array}{r} 4 \\ -\,2 \\ \hline 2 \end{array} \quad \begin{array}{r} 2 \\ -\,2 \\ \hline 0 \end{array} \qquad 8 \div 2 = \boxed{4}$$

Use repeated subtraction to find the missing factors.

$$\begin{array}{r} 15 \\ -\,3 \\ \hline 12 \end{array} \quad \begin{array}{r} 12 \\ -\,3 \\ \hline 9 \end{array} \quad \begin{array}{r} 9 \\ -\,3 \\ \hline 6 \end{array} \quad \begin{array}{r} 6 \\ -3 \\ \hline 3 \end{array} \quad \begin{array}{r} 3 \\ -3 \\ \hline 0 \end{array} \qquad 15 \div 3 = \boxed{5}$$

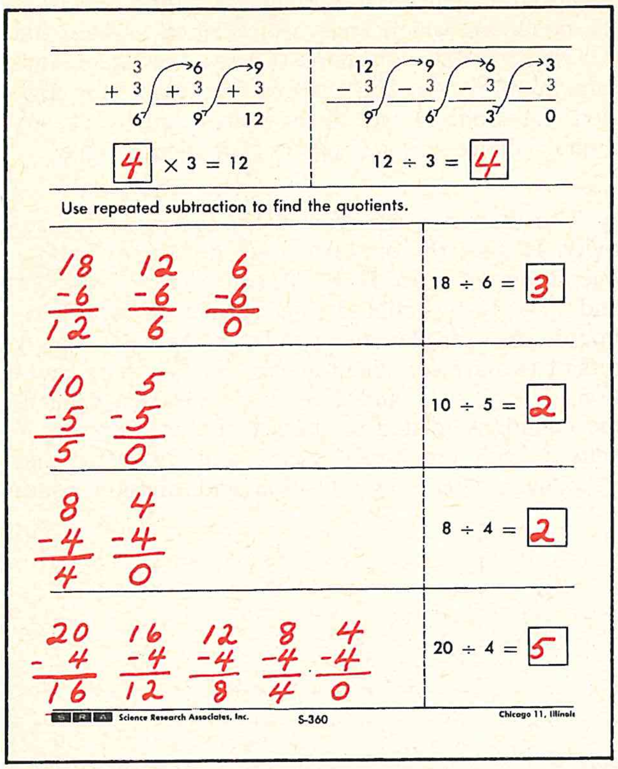
$$\begin{array}{r} 16 \\ -\,4 \\ \hline 12 \end{array} \quad \begin{array}{r} 12 \\ -4 \\ \hline 8 \end{array} \quad \begin{array}{r} 8 \\ -4 \\ \hline 4 \end{array} \quad \begin{array}{r} 4 \\ -4 \\ \hline 0 \end{array} \qquad 16 \div 4 = \boxed{4}$$

$$\begin{array}{r} 12 \\ -3 \\ \hline 9 \end{array} \quad \begin{array}{r} 9 \\ -3 \\ \hline 6 \end{array} \quad \begin{array}{r} 6 \\ -3 \\ \hline 3 \end{array} \quad \begin{array}{r} 3 \\ -3 \\ \hline 0 \end{array} \qquad 12 \div 3 = \boxed{4}$$

$$\begin{array}{r} 12 \\ -4 \\ \hline 8 \end{array} \quad \begin{array}{r} 8 \\ -4 \\ \hline 4 \end{array} \quad \begin{array}{r} 4 \\ -4 \\ \hline 0 \end{array} \qquad 12 \div 4 = \boxed{3}$$

We can think of division as repeated subtraction.

S-359



$$\begin{array}{r} 3\\ +\,3\\ \hline 6\end{array} \rightarrow \begin{array}{r} 6\\ +\,3\\ \hline 9\end{array} \rightarrow \begin{array}{r} 9\\ +\,3\\ \hline 12\end{array}$$

$\boxed{4} \times 3 = 12$

$$\begin{array}{r} 12\\ -\,3\\ \hline 9\end{array} \rightarrow \begin{array}{r} 9\\ -\,3\\ \hline 6\end{array} \rightarrow \begin{array}{r} 6\\ -\,3\\ \hline 3\end{array} \rightarrow \begin{array}{r} 3\\ -\,3\\ \hline 0\end{array}$$

$12 \div 3 = \boxed{4}$

Use repeated subtraction to find the quotients.

$$\begin{array}{r} 18\\ -6\\ \hline 12\end{array} \quad \begin{array}{r} 12\\ -6\\ \hline 6\end{array} \quad \begin{array}{r} 6\\ -6\\ \hline 0\end{array}$$

$18 \div 6 = \boxed{3}$

$$\begin{array}{r} 10\\ -5\\ \hline 5\end{array} \quad \begin{array}{r} 5\\ -5\\ \hline 0\end{array}$$

$10 \div 5 = \boxed{2}$

$$\begin{array}{r} 8\\ -4\\ \hline 4\end{array} \quad \begin{array}{r} 4\\ -4\\ \hline 0\end{array}$$

$8 \div 4 = \boxed{2}$

$$\begin{array}{r} 20\\ -4\\ \hline 16\end{array} \quad \begin{array}{r} 16\\ -4\\ \hline 12\end{array} \quad \begin{array}{r} 12\\ -4\\ \hline 8\end{array} \quad \begin{array}{r} 8\\ -4\\ \hline 4\end{array} \quad \begin{array}{r} 4\\ -4\\ \hline 0\end{array}$$

$20 \div 4 = \boxed{5}$

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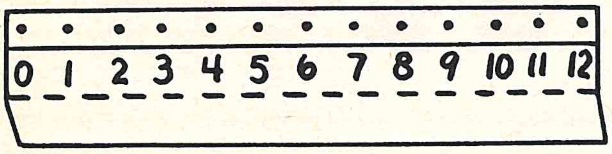
Supplemental Experiences

▶ Put a number-strip on the flannel board. Place your index finger on the dot that represents ten. Fold the lower section of the strip up over the numerals, leaving only the dots exposed. "Jump" your finger two dots at a time toward the left until zero is reached. Ask the children to tell where you started, how many spaces you jumped each time, and how many times you jumped to reach zero. Now unfold the lower section so that the numerals can be seen. Show them the path you jumped by starting again at 10 and repeating the jumps. Have the children watch the action and relate it to the numbers they see represented on the strip. Repeat the activity once more and have the children skip count as you make each jump—"eight, six, four, two, zero." Relate the activity to the equation 10 ÷ 2 = ☐. Continue in this way to use the number-strip to demonstrate other division problems.



▶ Separate the children into teams with five children on each team to play a number-sequence game. As each team has its turn, announce a starting point and a number-sequence pattern which this team must follow. For example, tell one team to begin with one and add one each time. The first child must stand and say "one"; the second child, "two"; the third child, "three"; the fourth child, "four"; and the last child, "five." In another instance you might choose to have a team start at ten and subtract two each time: the successive responses would be "ten," "eight," "six," "four," "two." If each member of the team responds correctly, the team scores one point.

Vary the game by having the children call out multiplication or division combinations. For example, the members of one team might give multiples of three. In this case the first member would say, "one times three equals three"; the second member, "two times three equals six," and so forth. On another occasion a team might give division combinations, 20 ÷ 4 through 4 ÷ 4.

Division

COMPARISON

Work Pages S-361 Through S-364

Objectives

To develop the concept that any number divided by one is that same number.

To develop the concept that any number divided by itself is one.

To provide practice in work with division equations.

To provide an opportunity to compare numbers in the setting of multiplication and division.

Fundamentals

The child has discovered that one and zero play special roles in multiplication. He found that the product of any number and one is that same number, and that the product of any number and zero is zero.

Now the child will find that one and zero also play special roles in division. He will discover that any number divided by one is that same number, and that any number divided by itself is one. The situation for zero is more complicated: zero divided by any number gives zero; it is *not possible* to divide any number by zero.

Although division problems involving zero do not appear on the work pages, the teacher should be ready to answer questions concerning such problems. She will want to refer to the relation between multiplication and division to fully understand what is involved. The statement 0 ÷ 6 = 0 is true, since

0 × 6 = 0. Note, however, that 6 ÷ 0 = □ and □ × 0 = 6 have no solution, since there is no number which multiplied by zero gives 6 (any number times zero is zero).

As the child's skills in working with multiplication and division combinations increase, the teacher will want to help him apply these skills as he compares two numbers. In determining the relation between numbers represented by such numerals as 4 ÷ 1 and 4 × 1, the child should be guided to call on his understanding of the role played by one in multiplication and division. He should also be encouraged to look for patterns in such problems as 9 ÷ 3 ○ 3 ÷ 1 and 2 ÷ 1 ○ 4 ÷ 2.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Division of Whole Numbers.

Readiness for Understanding

Understanding of the inverse relation between multiplication and division.

Understanding of one as the identity element of multiplication.

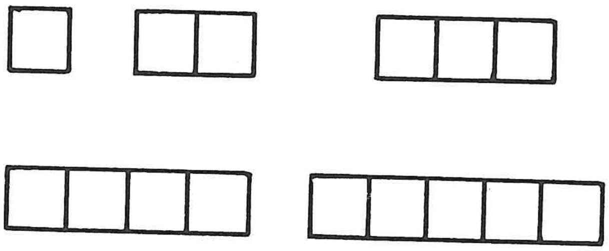
Ability to work with multiplication and division combinations related to products through five times five.

Developmental Experiences

MATERIALS

construction paper

▶ Cut from construction paper a set of number-strips to represent multiples of one from 1 × 1 through 5 × 1. Mark each strip into 3″ x 3″ squares as illustrated. Display the one-strip. Ask how many sets of one there are in this strip. As the answer is given, have some child write the related multiplication equation on the chalkboard: 1 × 1 = 1. Ask another child to write a related division equation beside the multiplication equation: 1 ÷ 1 = 1. Display the two-strip and ask how many sets of one there are in this set of two. When the answer has been given ask children to write the related multiplication and division equations. Repeat with the three-, four-, and five-strips. Have the related multiplication and division equations written each time. Refer to the equations which have been written and remind the children that any number times one equals that same number. Ask the children to examine the division equations. Lead them to conclude that any number divided by one equals that same number.

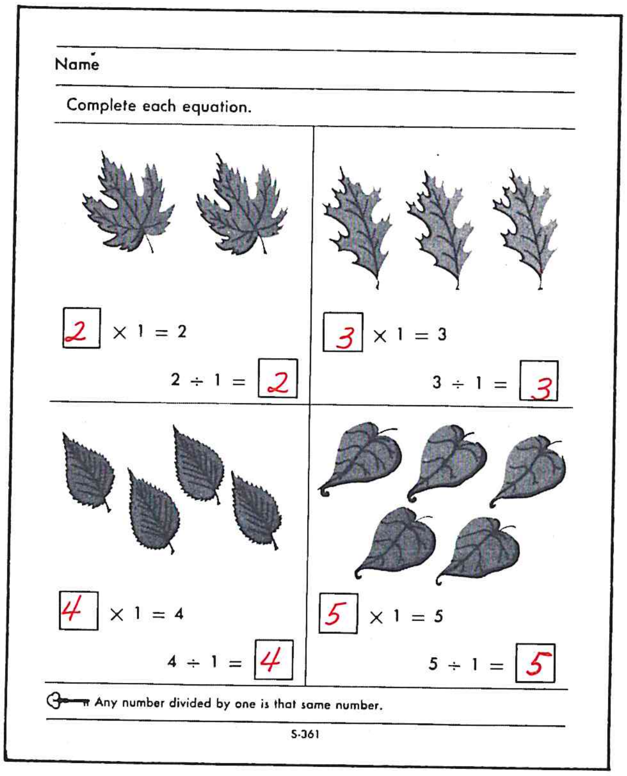


1 × 1 = 1   1 ÷ 1 = 1
2 × 1 = 2   2 ÷ 1 = 2
3 × 1 = 3   3 ÷ 1 = 3
4 × 1 = 4   4 ÷ 1 = 4
5 × 1 = 5   5 ÷ 1 = 5

▶ Use the strips constructed for the preceding activity. Display the one-strip. Ask how many sets of one are in this strip. Have the equations 1 × 1 = 1 and 1 ÷ 1 = 1 placed side by side on the chalkboard. Now display the two-strip. Ask how many sets of two are contained in this set. Have the equations 1 × 2 = 2 and 2 ÷ 2 = 1 written beneath the equations related to the set of one. Repeat the activity with the three-, four-, and five-strips and have the related multiplication and division equation written each time. Refer to the multiplication equations and remind the children that one times any number equals that number. Ask them to examine the related division equations. Lead them to conclude that any number divided by itself equals one.

1 × 1 = 1   1 ÷ 1 = 1
1 × 2 = 2   2 ÷ 2 = 1
1 × 3 = 3   3 ÷ 3 = 1
1 × 4 = 4   4 ÷ 4 = 1
1 × 5 = 5   5 ÷ 5 = 1

▶ Use work page S-361 to strengthen the understanding that any number divided by one is that same



Name

Complete each equation.

$\boxed{2} \times 1 = 2$
$2 \div 1 = \boxed{2}$

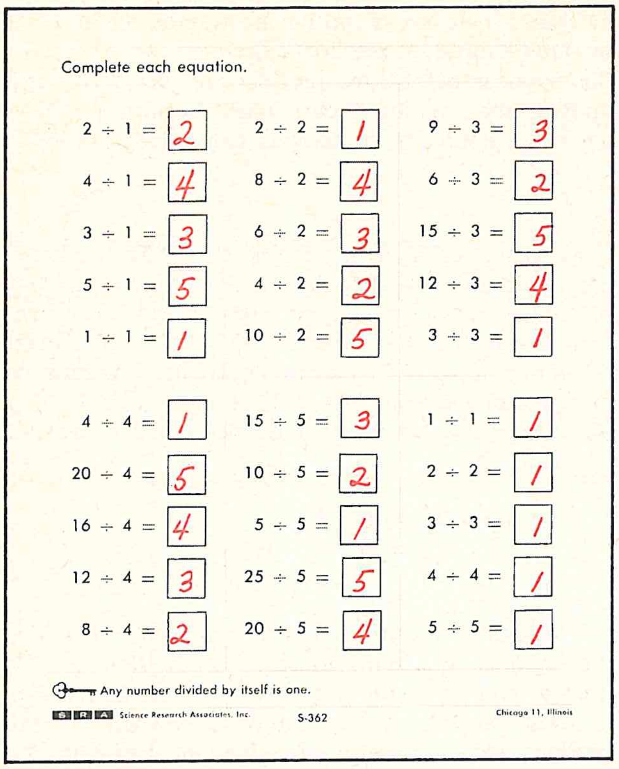
$\boxed{3} \times 1 = 3$
$3 \div 1 = \boxed{3}$

$\boxed{4} \times 1 = 4$
$4 \div 1 = \boxed{4}$

$\boxed{5} \times 1 = 5$
$5 \div 1 = \boxed{5}$

Any number divided by one is that same number.

S-361



Complete each equation.

$2 \div 1 = \boxed{2}$
$4 \div 1 = \boxed{4}$
$3 \div 1 = \boxed{3}$
$5 \div 1 = \boxed{5}$
$1 \div 1 = \boxed{1}$

$4 \div 4 = \boxed{1}$
$20 \div 4 = \boxed{5}$
$16 \div 4 = \boxed{4}$
$12 \div 4 = \boxed{3}$
$8 \div 4 = \boxed{2}$

$2 \div 2 = \boxed{1}$
$8 \div 2 = \boxed{4}$
$6 \div 2 = \boxed{3}$
$4 \div 2 = \boxed{2}$
$10 \div 2 = \boxed{5}$

$15 \div 5 = \boxed{3}$
$10 \div 5 = \boxed{2}$
$5 \div 5 = \boxed{1}$
$25 \div 5 = \boxed{5}$
$20 \div 5 = \boxed{4}$

$9 \div 3 = \boxed{3}$
$6 \div 3 = \boxed{2}$
$15 \div 3 = \boxed{5}$
$12 \div 3 = \boxed{4}$
$3 \div 3 = \boxed{1}$

$1 \div 1 = \boxed{1}$
$2 \div 2 = \boxed{1}$
$3 \div 3 = \boxed{1}$
$4 \div 4 = \boxed{1}$
$5 \div 5 = \boxed{1}$

Any number divided by itself is one.

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number. Direct the children to circle sets of one in each given set and to complete the related multiplication and division equations.

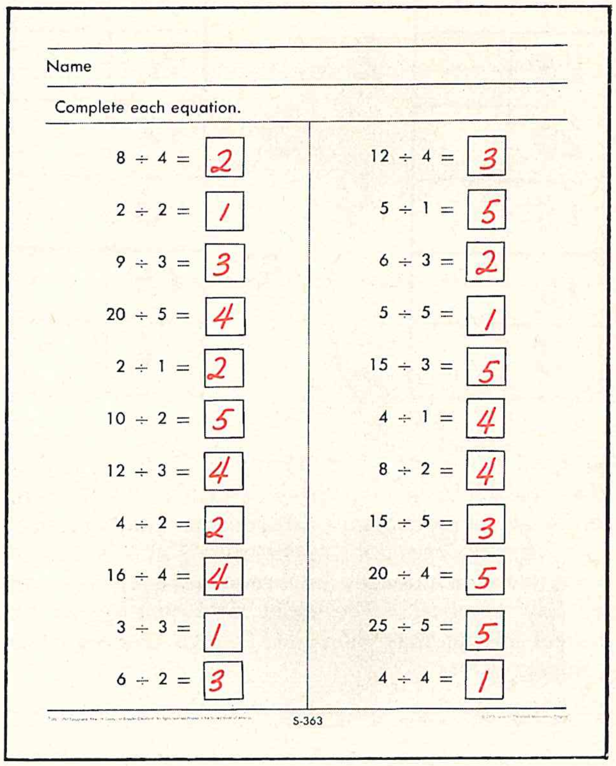
Work page S-362 can be used for a review of all the division combinations learned thus far. Tell the children to quickly complete each equation. When they have done so, call their attention to the last box on the page and lead them to say that any number divided by itself is one.

▶ Write a number sentence requiring a relation symbol, on the chalkboard: 3 × 1 ○ 3 ÷ 1. Ask the children to quickly tell which symbol, >, <, or =, belongs in the circle to make this a true statement. As the answer "equals" is given, ask the children to tell what they know about multiplying by one and dividing by one.

Present a problem such as 5 × 5 ○ 5 ÷ 5. After the correct answer has been supplied discuss with the children the difference between multiplying a number by itself and dividing a number by itself.

Repeat the activity with other problems such as 20 ÷ 4 ○ 4 × 5, 6 ÷ 2 ○ 6 ÷ 3, and 10 ÷ 5 ○ 5 ÷ 1.

▶ Use work page S-363 to evaluate the children's knowledge of division combinations related to products through five times five. Direct them to quickly complete each equation by recording the missing factor in the placeholder. Work page S-364 provides an opportunity for the children to use their ability to reason. Work the first problem with the group, then have them complete the page independently. After they have done so, check the problems with the class; ask different children to explain their reasons for selecting each relation symbol.



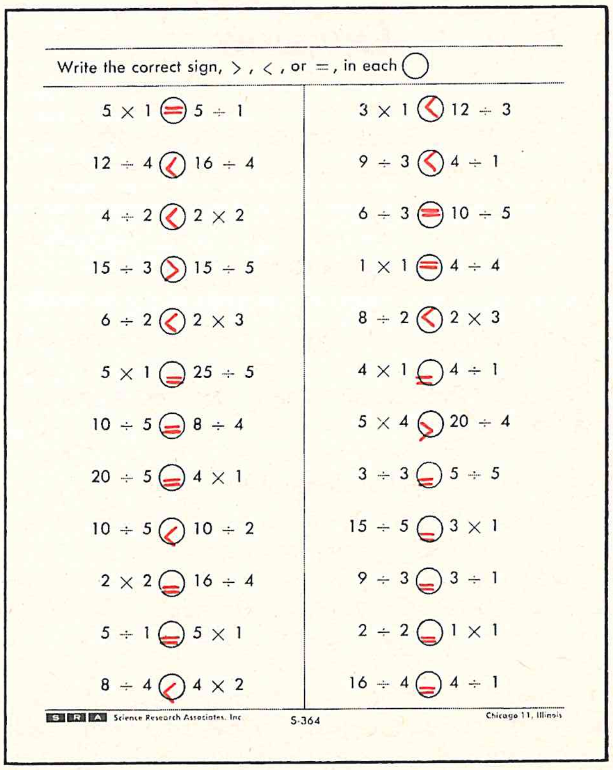
Name

Complete each equation.

$8 \div 4 = \boxed{2}$
$2 \div 2 = \boxed{1}$
$9 \div 3 = \boxed{3}$
$20 \div 5 = \boxed{4}$
$2 \div 1 = \boxed{2}$
$10 \div 2 = \boxed{5}$
$12 \div 3 = \boxed{4}$
$4 \div 2 = \boxed{2}$
$16 \div 4 = \boxed{4}$
$3 \div 3 = \boxed{1}$
$6 \div 2 = \boxed{3}$

$12 \div 4 = \boxed{3}$
$5 \div 1 = \boxed{5}$
$6 \div 3 = \boxed{2}$
$5 \div 5 = \boxed{1}$
$15 \div 3 = \boxed{5}$
$4 \div 1 = \boxed{4}$
$8 \div 2 = \boxed{4}$
$15 \div 5 = \boxed{3}$
$20 \div 4 = \boxed{5}$
$25 \div 5 = \boxed{5}$
$4 \div 4 = \boxed{1}$

S-363



Write the correct sign, >, <, or =, in each ○.

$5 \times 1 = 5 \div 1$
$12 \div 4 < 16 \div 4$
$4 \div 2 < 2 \times 2$
$15 \div 3 > 15 \div 5$
$6 \div 2 < 2 \times 3$
$5 \times 1 = 25 \div 5$
$10 \div 5 = 8 \div 4$
$20 \div 5 = 4 \times 1$
$10 \div 5 < 10 \div 2$
$2 \times 2 = 16 \div 4$
$5 \div 1 = 5 \times 1$
$8 \div 4 < 4 \times 2$

$3 \times 1 < 12 \div 3$
$9 \div 3 < 4 \div 1$
$6 \div 3 = 10 \div 5$
$1 \times 1 = 4 \div 4$
$8 \div 2 < 2 \times 3$
$4 \times 1 = 4 \div 1$
$5 \times 4 > 20 \div 4$
$3 \div 3 = 5 \div 5$
$15 \div 5 = 3 \times 1$
$9 \div 3 = 3 \div 1$
$2 \div 2 = 1 \times 1$
$16 \div 4 = 4 \div 1$

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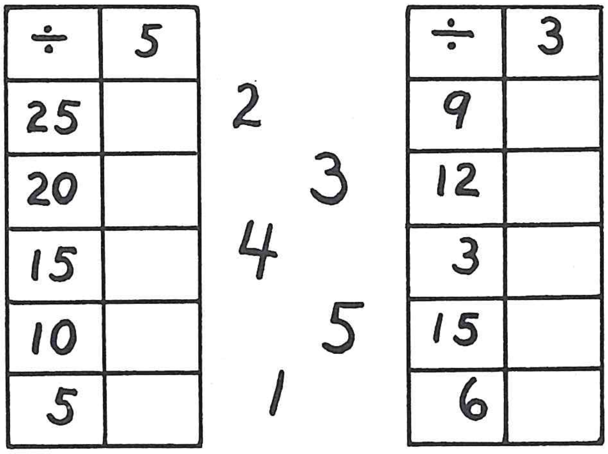
Supplemental Experiences

▶ Construct miniature division tables as illustrated. Have children place their plastic numerals in the empty boxes to indicate the missing factor.



▶ Play the game "I'm Thinking of a Number." Have the children take turns describing numbers in terms of multiplication and division combinations. For instance, one child might say, "I'm thinking of a number which divided by three gives five. Who can tell my number?" The child who first gives the correct answer may then have a turn to describe a number.

Division

Work Pages S-365 and S-366

Objectives

To extend understanding of the operation of division.

To stress the inverse relation between multiplication and division.

Fundamentals

Thus far the child has been using division to find the missing factor in equations such as $\square \times 2 = 6$. He has been thinking in terms of finding how many sets of two are in a set of six. He thinks about finding a missing factor and writes the equation $6 \div 2 = \square$.

Now the child will be asked to re-examine another type of situation which calls for finding a missing factor. When he sees the equation $3 \times \square = 6$, he thinks in terms of finding how many are in each of three equivalent sets formed from a set of six. In this case, as before, the product and one factor are known; the missing factor must be found. Once again the operation of division can be used: $6 \div 3 = \square$.

Mathematically, it is possible to justify the latter use of division by appealing to the inverse concept the commutative property of multiplication. We have seen that $6 \div 2 = \square$ because $\square \times 2 = 6$. Given the equation $3 \times \square = 6$, we apply the commutative property of multiplication and obtain $\square \times 3 = 6$. Because of the way we have defined division, we can now see that $6 \div 3 = \square$.

$\square \times 2 = 6$ $\qquad$ $6 \div 2 = \square$
$3 \times \square = 6$
$\square \times 3 = 6$ $\qquad$ $6 \div 3 = \square$

Through carefully planned classroom activities the child should learn to see that division can be used to find either the number of sets or the number in each set. He should know that whenever the product and one factor are given he can find the other factor by dividing.

See KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER: Division of Whole Numbers.

Readiness for Understanding

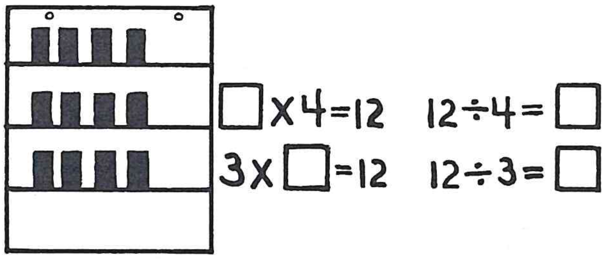
Knowledge of multiplication and division combinations related to products through five times five.

Understanding of the inverse relation between multiplication and division.

Developmental Experiences

MATERIALS
pocket chart
3″ x 5″ cards

▶ Hold up twelve 3″ x 5″ cards or slips of colored paper. Tell the children you would like to have the set of these twelve cards partitioned in such a way that four are placed in each of several rows in the pocket chart. Say that you want to find out how



many sets of four there are in the set of twelve. Have some child place the cards, four in a row, in the pocket chart. Have some child write the placeholder multiplication equation which applies to this activity: $\square \times 4 = 12$. Call on another child to write the related division equation: $12 \div 4 = \square$.

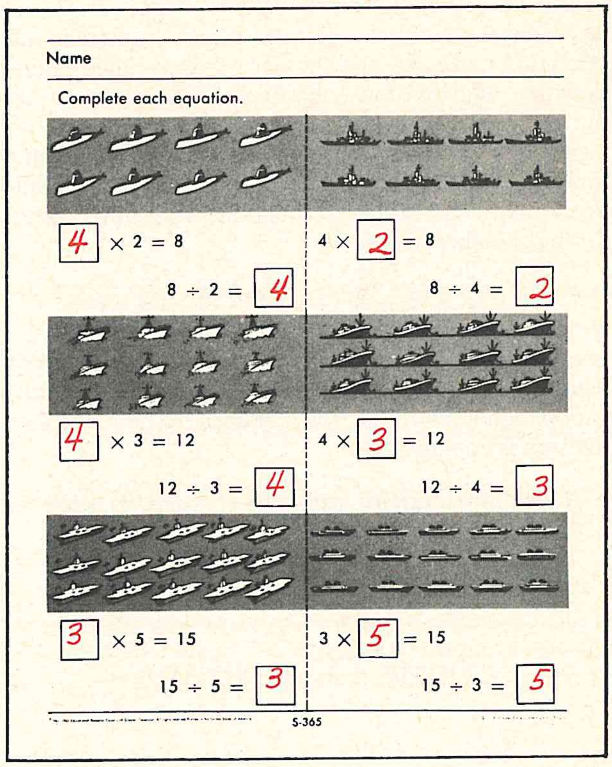
Remove the twelve cards from the pocket chart and pose a different type of question. Tell the children you would like the set of twelve cards partitioned in such a way that you have three sets with the same number in each set. Write the equation $3 \times \square = 12$ on the chalkboard and call on a child to arrange the cards in three different rows in the pocket chart. When he has completed his assignment, ask another child to be sure that there are the same number in each set and to give this number. Have the placeholder equation completed. Tell the children that they can also write a division equation to relate to this activity. Write $12 \div 3 = \square$ on the chalkboard. Refer back to the multiplication equation $3 \times \square = 12$. Point out that the product and one factor are known in this equation and that division can be used to find the missing factor.

Repeat the activity with different multiplication and division combinations. Stress the fact that division can be used to find either the number in each set or the number of sets.

▶ Present story problem situations which call for the children to use division. In each case have them decide whether they have found the number of sets or the number in each set. For example, say, "Ruth bought twelve balloons for her birthday party. She gave away all of these balloons to her guests. Each child received four. How many children came to the party?" Have the equation $12 \div 4 = \square$ written on the chalkboard and completed. Lead the children to say that three children came to the party. Use other examples such as, "Dan gave away twelve balloons at his birthday party. If three children came to the party and Dan gave each one the same number of balloons, how many did each child receive?" In this case the children would have to write the equation $12 \div 3 = \square$ on the chalkboard and complete it. They should conclude that they found how many were in each set; each child received four balloons.

▶ Use work page S-365 to reinforce the concept that either factor in a multiplication equation can be found by dividing. Direct the children's attention to the first box on the page. Tell them to circle sets of two in the set of eight and to complete each of the equations. Now direct them to look at the equations in the second box. Tell them they are to circle four sets within the set of eight and that each set must contain the same number of objects. Then direct them to complete each equation. Have the children work the remaining problems on the page independently.

Work page S-366 gives the children another op-



Name

Complete each equation.

$\boxed{4} \times 2 = 8$

$8 \div 2 = \boxed{4}$

$4 \times \boxed{2} = 8$

$8 \div 4 = \boxed{2}$

$\boxed{4} \times 3 = 12$

$12 \div 3 = \boxed{4}$

$4 \times \boxed{3} = 12$

$12 \div 4 = \boxed{3}$

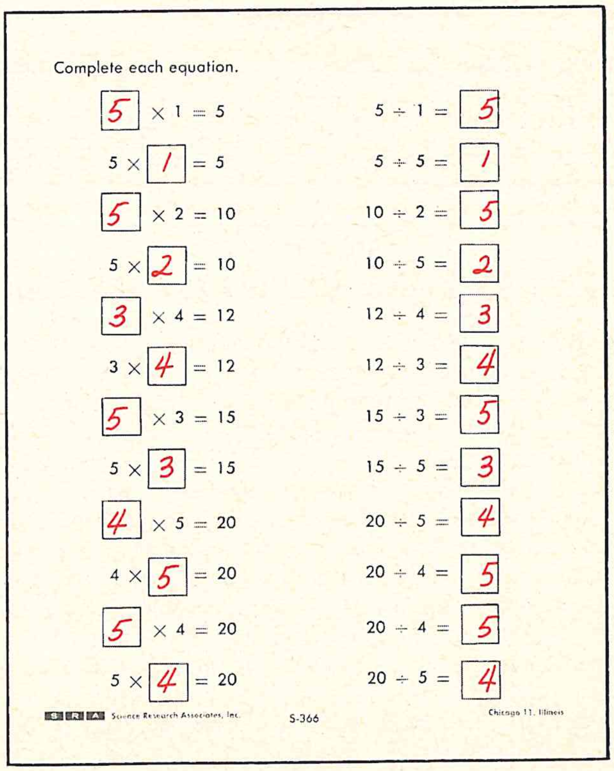
$\boxed{3} \times 5 = 15$

$15 \div 5 = \boxed{3}$

$3 \times \boxed{5} = 15$

$15 \div 3 = \boxed{5}$

S-365



Complete each equation.

$\boxed{5} \times 1 = 5$	$5 \div 1 = \boxed{5}$
$5 \times \boxed{1} = 5$	$5 \div 5 = \boxed{1}$
$\boxed{5} \times 2 = 10$	$10 \div 2 = \boxed{5}$
$5 \times \boxed{2} = 10$	$10 \div 5 = \boxed{2}$
$\boxed{3} \times 4 = 12$	$12 \div 4 = \boxed{3}$
$3 \times \boxed{4} = 12$	$12 \div 3 = \boxed{4}$
$\boxed{5} \times 3 = 15$	$15 \div 3 = \boxed{5}$
$5 \times \boxed{3} = 15$	$15 \div 5 = \boxed{3}$
$\boxed{4} \times 5 = 20$	$20 \div 5 = \boxed{4}$
$4 \times \boxed{5} = 20$	$20 \div 4 = \boxed{5}$
$\boxed{5} \times 4 = 20$	$20 \div 4 = \boxed{5}$
$5 \times \boxed{4} = 20$	$20 \div 5 = \boxed{4}$

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portunity to investigate the relation between division and multiplication. Direct them to complete each equation on the page. When this has been done, discuss how the four related equations in each row are related.

Supplemental Experiences

▶ Have the children use their plastic numerals and operation and relation symbols to make related multiplication and division equations. Direct them to lay out the numerals 2, 3, and 6 and the ×, ÷, and = signs. Ask them to tell how many true statements they can make using these symbols. Have the children record the different equations they have made, on the chalkboard:

$$2 \times 3 = 6 \qquad 6 \div 3 = 2$$
$$3 \times 2 = 6 \qquad 6 \div 2 = 3$$

Review the commutative property which relates the two multiplication equations and the inverse concept which relates the division equations to the multiplication equations. Ask the children to tell whether division is commutative.

Repeat the activity with other numerals such as 2, 4, and 8 or 5, 1, and 5.

Four Operations

Work Pages S-367 and S-368

Objectives

To evaluate the children's understanding of addition, subtraction, multiplication, and division.

To evaluate the children's ability to select the correct operational sign, +, −, ×, or ÷, in a given equation.

Fundamentals

The child has been developing an understanding of the four fundamental operations in arithmetic—addition, subtraction, multiplication, and division. He has learned that there are some interesting relations between different pairs of these operations:

$$\begin{tikzcd} \text{Addition} \arrow[r, leftrightarrow, "\text{inverse}"] \arrow[d, "\text{repeated}"'] & \text{Subtraction} \arrow[d, "\text{repeated}"] \\ \text{Multiplication} \arrow[r, leftrightarrow, "\text{inverse}"] & \text{Division} \end{tikzcd}$$

Now the child will be asked to apply his skills and knowledge to find the missing operational symbol in a variety of equations. He should learn to try different possibilities as he works problems such as $2 \triangle 2 = 4$. The plus sign will make this statement true, but there may be another symbol which also completes the equation. The child should be encouraged to examine each idea critically and to be creative in his attack on numbers.

Readiness for Understanding

Understanding of addition, subtraction, multiplication, and division.

Understanding of the inverse relation between addition and subtraction and between multiplication and division.

Developmental Experiences

▶ Write the four signs +, −, ×, and ÷ on the chalkboard. Ask the children to tell what operation is indicated by each sign. Lead them to discuss the different situations in which each operation might be used. Help the children summarize the relations which exist between different pairs of these operations. They should be able to say that subtracting a number is the inverse of adding that number; that dividing by a number is the inverse of multiplying by that number. They should recall that multiplication can be thought of as repeated addition and that division can be thought of as repeated subtraction.

Tell the children that you would like them to use some of the many ideas they have learned about addition, subtraction, multiplication, and division. Write on the chalkboard several equations in which the operational symbol is missing.

$$5 \triangle 1 = 5 \qquad 4 \triangle 2 = 2$$
$$5 \triangle 1 = 4 \qquad 2 + 2 + 2 = 3 \triangle 2$$
$$5 \triangle 1 = 6 \qquad 1 \times 4 = 4 \triangle 1$$

Direct the children's attention to the first equation. Call upon some child to read the equation in the way he thinks it should be completed. He might say, "Five divided by one equals five." Have the other children tell whether this is now a true statement. Ask whether there might be another solution to this problem. Encourage some child to say that "five times one equals five" is also a true statement. In a similar fashion lead the children to examine each of the suggested examples and others which are similar.

For children who are very adept at determining the operational symbol which completes an equation, prepare some problems which require more thought. Write examples on the chalkboard such as

$$10 \triangle 2 = 15 \triangledown 3 \qquad 5 \triangle 3 = 3 \triangledown 5$$
$$1 \triangle 3 = 12 \triangledown 4 \qquad 9 \triangle 3 = 3 \triangledown 1$$
$$2 \triangle 5 = 10 \triangledown 3 \qquad 8 \triangle 4 = 4 \triangledown 2$$

those illustrated. Direct the children's attention to the first problem. Tell them they must think very carefully which operational symbol might go in each of the triangles. Make it clear that the same symbol or different symbols can be used. Remind them that they must end up with the same number on either side of the equal sign. Some child might say that ten divided by two equals fifteen divided by three. Another child might say, "Ten plus two equals fifteen minus three." Proceed in a similar fashion to have the children give various solutions to the remaining problems.

▶ Present oral story problems without numbers. Have the children tell what operation must be used to find the solution to such problems as the following:

(1) On Rae-Marie's birthday a certain number of her friends came to her party. There were as many girls as boys. How many boys were at the party?

(2) The boys had fun searching for peanuts. Each boy found the same number of peanuts. How many peanuts did they find?

(3) The girls liked the pretty paper hats. After Rae-Marie gave a hat to each girl, she had some left over. How many did she have to start with?

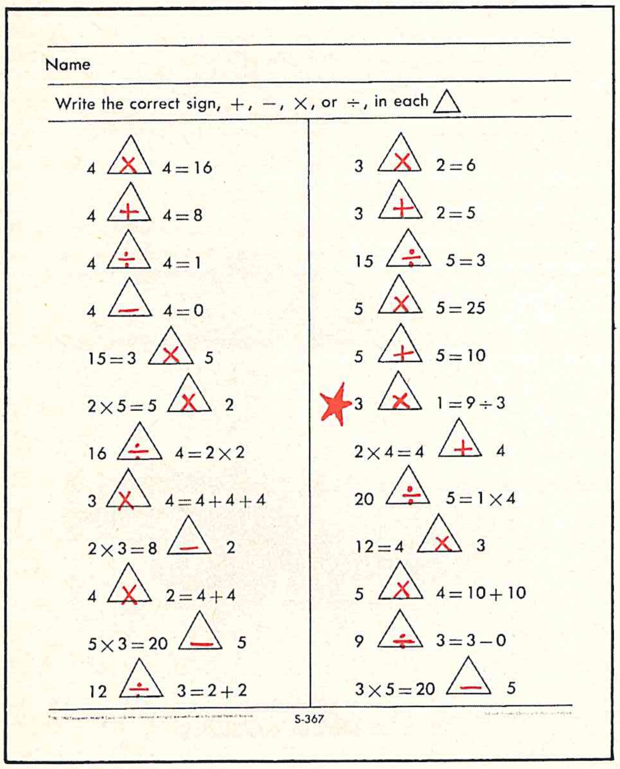
(4) Each of the guests brought a gift for Rae-Marie. Grandmother and Mother and Father gave her some gifts. How many gifts did Rae-Marie receive?

(5) When it was time for the children to go home Rae-Marie gave each of them the same number of lollipops. How many lollipops did she give to her friends?

After each story has been discussed and the correct operation indicated, have the children supply their own numbers and work out the solution to the problem.

Have the children make up some of their own stories without numbers and take turns presenting them to the class.

▶ Use work page S-367 to evaluate the children's ability to select the correct operational sign in the given equations. Direct them to write +, −, ×, or ÷ in each triangle to make the statements true. Encourage them to complete the page independently.


Name

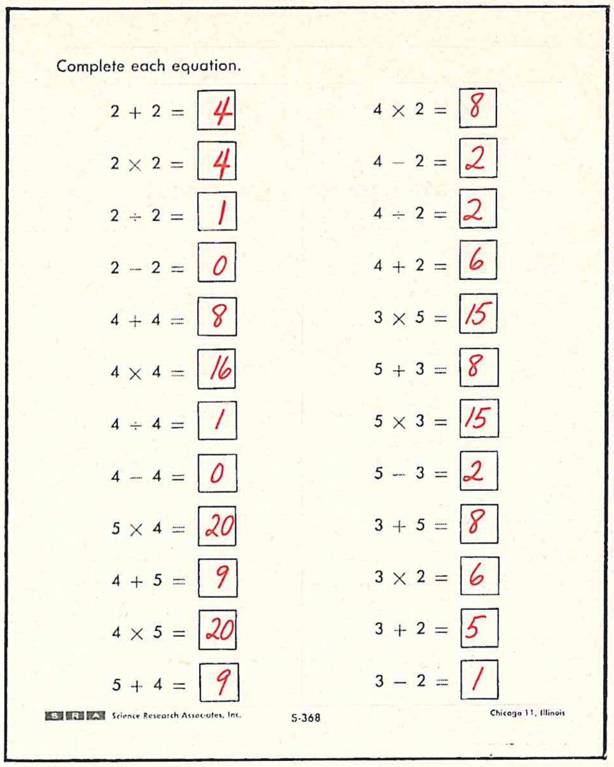
Write the correct sign, +, −, ×, or ÷, in each △

4 × 4 = 16
4 + 4 = 8
4 ÷ 4 = 1
4 − 4 = 0
15 = 3 × 5
2 × 5 = 5 × 2
16 ÷ 4 = 2 × 2
3 × 4 = 4 + 4 + 4
2 × 3 = 8 − 2
4 × 2 = 4 + 4
5 × 3 = 20 − 5
12 ÷ 3 = 2 + 2

3 × 2 = 6
3 + 2 = 5
15 ÷ 5 = 3
5 × 5 = 25
5 + 5 = 10
★3 × 1 = 9 ÷ 3
2 × 4 = 4 + 4
20 ÷ 5 = 1 × 4
12 = 4 × 3
5 × 4 = 10 + 10
9 ÷ 3 = 3 − 0
3 × 5 = 20 − 5

S-367

★ **More than one correct answer is possible for each problem.**


Complete each equation.

2 + 2 = 4
2 × 2 = 4
2 ÷ 2 = 1
2 − 2 = 0
4 + 4 = 8
4 × 4 = 16
4 ÷ 4 = 1
4 − 4 = 0
5 × 4 = 20
4 + 5 = 9
4 × 5 = 20
5 + 4 = 9

4 × 2 = 8
4 − 2 = 2
4 ÷ 2 = 2
4 + 2 = 6
3 × 5 = 15
5 + 3 = 8
5 × 3 = 15
5 − 3 = 2
3 + 5 = 8
3 × 2 = 6
3 + 2 = 5
3 − 2 = 1

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Use work page S-368 to evaluate the children's ability to recognize the different operational symbols and to perform the indicated operations. Tell them to quickly complete each equation on this page.

Supplemental Experiences

▶ Have the children create equations and inequalities in which the relation symbol is missing. Direct them to make their problems interesting by indicating different operations on each side of the circle. For example:

2 × 3 ○ 8 − 5
20 ÷ 5 ○ 7 + 3
5 ÷ 5 ○ 2 × 0
4 × 1 ○ 4 ÷ 4

Have one child write his problem on the chalkboard and call on a second child to write the correct sign, >, <, or =, in the circle. If the second child answers correctly, he may write his problem and continue the game.

▶ Pass out numeral cards bearing a variety of numerals. Place a card bearing the numeral 12 in the pocket chart. Call upon all the children who have cards which belong to the "twelve family" to read their cards and place them in the pocket chart. Repeat the activity by calling for all cards which belong to another number family.

3 × 4   10 ÷ 2   9 − 6
6 ÷ 3   8 + 4   15 − 3

The following pages are not required for this book: Pages 249-252, four additional pages without a number, or the original index.

This concludes the book's contents.